Set Oriented Numerics in Dynamics and Optimization

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Abstract

Over the last years so-called *set oriented* numerical methods have been developed in the context of the numerical treatment of dynamical systems. The basic idea is to cover the objects of interest—for instance *invariant manifolds* or *invariant measures*—by outer approximations which are created via adaptive multilevel subdivision techniques. These schemes allow for an extremely memory and time efficient discretization of the phase space and have the flexibility to be applied to several problem types.

In this talk we will show how to make use of these set oriented numerical techniques for the solution of *multiobjective optimization* problems. In these problems several objective functions have to be optimized at the same time. For instance, for a perfect economical production plan one wants to simultaneously *minimize cost* and *maximize quality*. As indicated by this example the different objectives typically contradict each other and therefore certainly do not have identical optima. Thus, the question arises how to approximate the "optimal compromises" which, in mathematical terms, define the so-called *Pareto* set. In order to make our set oriented numerical methods applicable we construct a dynamical system which possesses the Pareto set as an attractor.

The corresponding numerical techniques will be applied to a couple of real world applications such as the optimization of an active suspension system for cars or the design of a spacecraft trajectory from Earth to the Lagrange point $L2$. The latter example is mainly motivated by the ESA mission Darwin, and here the conflicting objectives are *flight time* and *energy consumption*.