Dirac Structures and the Legendre Transformation for Implicit Lagrangian and Hamiltonian Systems

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Abstract

We investigate mechanical systems with degenerate Lagragians in the context of Dirac structures. We first show an induced Dirac structure D_{Δ_Q} can be defined on T^*Q by a given constraint distribution $\Delta_Q \subset TQ$ and demonstrate how an implicit Lagrangian system (L, Δ_Q, X) can be constructed in the case that a given Lagrangian L is degenerate and with a vector field X on T^*Q. Then, we introduce a generalized Legendre transformation to define a Hamiltonian H_P on a constraint momentum space $P = \mathbb{F}L(\Delta_Q)$ and also define a generalized Hamiltonian H on the Pontryagin bundle $TQ \oplus T^*Q$ by incorporating primary constraints in the sense of Dirac. Thus, we develop an implicit Hamiltonian system (H, Δ_Q, X) from the degenerate Lagrangian, that is, a Hamiltonian analogue of an implicit Lagrangian system. Further, we illustrate the equivalence between implicit Lagrangian and Hamiltonian systems in the context of the generalized Legendre transformation, where we also clarify the duality relation between the Legendre map and the primary constraints. We shall illustrate an example of L-C circuits, which is a typical physical system of a degenerate Lagrangian with constraints in both contexts of implicit Lagrangian and Hamiltonian systems.