Fast Lie Algebra Variational Integrators

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Abstract

In this talk I will present a fast, Lie algebra variational integrator. Due to its variational construction, the method is discrete spatial angular momentum and symplectic two-form preserving. As a consequence the discrete energy remains bounded for long-time integrations. Moreover, the method is explicit, and hence computationally efficient, and easy to implement.

This method is an instance of a family of Newmark algorithms on Lie Algebras which generalize certain methods for $\mathfrak{so}(3)$ proposed and investigated by Simo & Vu-Quoc [1988]; Simo & Wong [1991] to any Lie algebra. This family comes from discretizations of variational principles on trivialized Lie groups specifically the Euler-Poincaré Variational Principle [Marsden & Scheurle, 1993]. This view affords a simple and unified procedure to analyze the geometric structure of these Lie-Newmark methods and improve them.

By discrete Noether's theorem, it is shown that all of these algorithms preserve a discrete spatial angular momentum. An extension of the variational proof of symplecticity to trivialized Lie groups provides a natural way to assess the symplectic nature of the algorithms. This discrete proof of symplecticity leads to some negative results on symplecticity of existing Lie-Newmark methods. The paper then proposes a new Lie-Newmark method which leads to a positive result on symplecticity.

Numerical simulations on the free rigid body confirm the positive and negative results on discrete energy, spatial angular momentum, and symplectic-form preservation. Moreover quantiative comparisons to the current state-of-the-art through work-precision diagrams reveal that this new geometric Lie algebra integrator is fast and efficient.

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