CDS 212 - Introduction to Modern Control
Homework # 4

Date Given: October 23rd, 2003
Date Due: October 30th, 2003, in class

P1. DFT: Chapter 6, Exercise 4.

P2. DFT: Chapter 6, Exercise 5.

P3. DFT: Chapter 6, Exercise 6.

P4. DFT: Chapter 6, Exercise 7.

P5. [***Zhou 6.5 and 6.7***] Show that a non-minimum phase system cannot be stabilized by a very high-gain controller. Show also that an unstable system cannot be stabilized by a very low gain controller. (Hint: use the root locus method).

P6. [***Zhou 6.9***] Let a plant P have a zero on the right half-plane at $z = a$ (where a is real). Let $Y = (I + PK)^{-1}D$ be the output due to a unit step disturbance D starting at $t = 0$. Suppose we want the output $y(t)$ to stay in the following time-template

$$|y(t)| \leq \begin{cases} M & 0 \leq t < T \\ m & t \geq T \end{cases}$$

where $M > 1 > m > 0$, and $T > 0$. Show that for a stabilizing controller to meet these specifications we must have

$$aT \geq \ln\left(\frac{M-m}{M-1}\right)$$

Explain the tradeoffs in this problem.

P7. [***Zhou 6.9***] Let K be a stabilizing controller for the nominal plant

$$P = \frac{s-\alpha}{(s-\beta)(s+\gamma)}$$

where $\alpha > 0$, $\beta > 0$, and $\gamma \geq 0$. Suppose $|S(jw)| \leq \delta < 1$ for all $w \in [-w_0, w_0]$. Find a lower bound for $\|S\|_\infty$ and calculate the lower bound for $\alpha = 1$, $\beta = 2$, $\gamma = 10$, $\delta = 0.2$, and $w_0 = 1$.