CDS 212 - Introduction to Modern Control
Homework # 1

Date Given: October 2, 2003
Date Due: October 9, 2003

P1. Show that if $x \in \mathbb{R}^n$, the quantity defined by:

$$\|x\|_p := \left(\sum_{k=1}^{n} |x_k|^p \right)^{\frac{1}{p}}$$

is a norm, for $1 \leq p < \infty$ (Hint: try using Hölder's inequality). What happens when $p < 1$?

P2. A set $S \subset \mathbb{R}^n$ is *convex* if

$$x, y \in S \Rightarrow \lambda x + (1-\lambda) y \in S, \qquad \text{for all } \lambda \in [0,1].$$

Show that if $\|\cdot\|$ is a norm then the unit ball $\{x \in \mathbb{R}^n : \|x\| \leq 1\}$ is convex.

P3. Let $A \in \mathbb{C}^{n \times n}$. The matrix norm *induced* by a vector p-norm is defined as:

$$\|A\|_p := \sup_{x \in \mathbb{C}^n, x \neq 0} \frac{\|Ax\|_p}{\|x\|_p}.$$

(a) Show that the expression above is actually a norm on $\mathbb{C}^{n \times n}$, by verifying the defining properties.

(b) Let $\lambda_{\max}(M)$ denote the largest eigenvalue of the hermitian matrix M. Show that for the case $p = 2$, $\|A\|_2 = \sqrt{\lambda_{\max}(A^* A)}$.

P4. DFT: Chapter 2, Exercise 1.

P5. Prove that the relation

$$\langle f, g \rangle_2 = \int_{-\infty}^{+\infty} g(t)^* f(t) dt$$

satisfies all axioms of the inner product.

P6. DFT: Chapter 2, Exercise 2 (u_2 and u_9 only).

P7. DFT: Chapter 2, Exercise 9.

P8. DFT: Chapter 2, Exercise 13.

P9. Prove entries (1,2), (2,1) and (2,2) in Table 2.2 of DFT.