

Global perspective on balloon entrapment on Titan using Lagrangian Coherent Structures

Sarah D. Sherman¹, Jerrold E. Marsden², Philip DuToit²

1. Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544, USA

2. Control and Dynamical Systems, California Institute of Technology, Pasadena, CA 91125, USA

In order to determine the viability of a drifting balloon vehicle versus a powered balloon vehicle in Titan's atmosphere, it is critical to analyze the ambient winds and their effects on the balloon's movement. It has been proposed that extracting Lagrangian Coherent Structures (areas with a high fluid particle separation rate) from a theoretical wind model will produce time-dependent 3D surfaces depicting natural boundaries in Titan's atmosphere. These surfaces, when compiled, create a time-dependent map of accessible areas on Titan given a drifting balloon. Having this map would allow mission planners to determine if a drifting balloon could reach locations with rich scientific data. Our results indicate that the geologically-interesting North and South poles of Titan are unreachable in the season of Huygens, suggesting that using a drifting balloon would significantly limit the scientific return of the mission if it took place at that time. Therefore, it is suggested that the balloon should use powered directional control such as a propeller or a series of horizontal vents in order to reach the otherwise unreachable sites.

In the past twelve months, Saturn's Cassini satellite has made a number of highly publicized fly-bys of Saturn's largest moon, Titan. During that time, it captured images of previously undiscovered methane lakes, a monumental mountain range and evidence of methane drizzle. As a result of these new opportunities for exploration, the design of a return mission to Titan has earned the attention of NASA and the Jet Propulsion Laboratory. Because Titan is too distant for earth-based real-time control, there is a need to design a vehicle that can travel autonomously through the windy atmosphere. It is critical that we determine the feasibility of a drifting balloon versus a powered balloon with respect to navigation of interesting sites on Titan.

Using Lagrangian Coherent Structures (LCS), it will be possible to create a map of where the balloon will likely go, given initial location and time. By comparing the map of reachable sites to interesting sites, we can determine if the balloon's drifting path will be scientifically rich enough to warrant an unpowered balloon. Otherwise, a propeller or another form of thrust will be necessary to guide the balloon to an interesting site. Other questions of interest are whether the balloon will be entrapped in a particular latitudinal belt or a pole. This may put a disastrous end to the mission if the sites of entrapment are not scientifically interesting, but if they turn out to have a wealth of diverse scientific phenomena, such entrapment could be useful. It was found that weakly defined LCS surround the North and South poles, meaning that longitudinal freedom is significantly limited. It is ultimately detrimental to the mission to know that these structures exist since the balloon will never be able to travel from equatorial latitudes to either of the poles without some form of propulsion.

It was determined that LCS is the most appropriate analysis tool for estimating the effects of Titan's winds on a drifting balloon for both its robustness and its inherent precision. A sizable local error in a given input data set (in this case, a theoretical model of Titan wind velocities) would have little effect on the location of LCS. This is important given that the models we used are purely theoretical and it is assumed that there are some inherent errors. Additionally, by definition, particle flux across an LCS is exactly zero, and experimentally, it is less than 0.5% of the surrounding vector magnitude.¹

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To complete this study, it is necessary to evaluate the LCS for all locations and time for a reasonable flight envelope of about one year. Three wind models were initially used: the Newman model developed in the Geological and Planetary Sciences department at Caltech, the Laboratoire Météorologie Dynamique model from Paris, France, and the Tokano model from the Institut fuer Geophysik und Meteorologie of the University of Koln in Germany. We chose to focus on the Newman model developed locally at Caltech because the output of this model was tailored for our use, and the model had reasonable agreement with the only wind profile measured on Titan during the Huygens probe descent of 2005. The Newman model was created using heat convection, circulation, radiation, haze properties, Saturn's tidal forcing and other planetary phenomena. It incorporates only the predominant winds, analogous to trade winds on Earth, so it does not reflect any meteorological activity. The omission of meteorological wind estimates was acceptable because those small-scale effects are nearly impossible to predict, and even so, they are most likely negligible. The model is based on NCAR's Weather Research and Forecasting (WRF) model, with constants and parameterizations added to customize it for Titan.² The resultant data set consists of a three-dimensional velocity vector for 100 time intervals at 64 longitudinal points, 36 latitudinal points, and 50 altitudes.

The velocity field data is read into an LCS software package that outputs dynamically evolving 3D surfaces representing the local maximum value of the finite time Lyapunov exponent (FTLE), also known as LCS. The FTLE is a function of the eigenvalues of the velocity field represented by the equation:

$$\sigma = \frac{1}{|T|} \log \sqrt{\lambda_{max}(C)}$$

where σ is the FTLE value, T is the integration time, λ is the eigenvalue of some vector, C . Physically, FTLE measures the rate of separation between two nearby particles, so a high FTLE value represents a divergent flow and a low value represents a convergent flow. When viewed globally, an FTLE plot shows the lines (in 2D) or surfaces (in 3D) that act as boundaries between dynamically different regions of the flow. In the case of Titan's atmosphere, these surfaces represent physical boundaries that air particles will never cross because the flow is divergent at that location.

Integration time, T , may be varied to achieve better resolution of LCS, which should appear as sharp maxima in the FTLE field rather than gradual increases. The physical meaning of T is the time it takes for the tracers to diverge from an LCS. The LCS results are largely dependent on the integration time, so an experiment was conducted where FTLE was computed for the same time sample at T values ranging from 100 to 800 hrs. This is a logical range of time for Titan winds. The best LCS definition was seen at $T=500$ hrs, as shown below in Fig. 1. This value of $T = 500$ hrs was used for the remainder of the project.

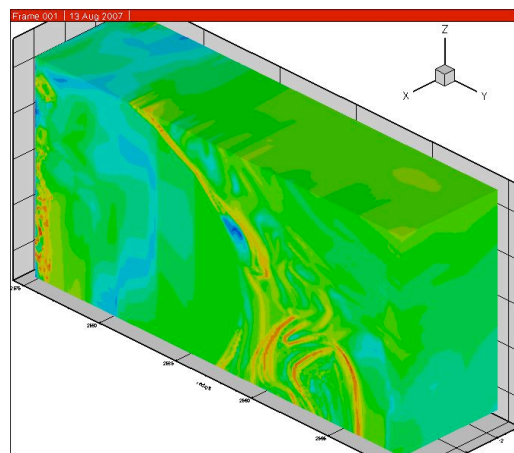


Figure 1: FTLE field optimized for LCS definition at T=500 hrs.

² Newman, Claire. "About TitanWRF" <<<http://www.planetWRF.com>>>.2006.

Because FTLE values must be computed over a mesh of points in 3D space over a length of time, a computer must perform the series of calculations. We used a computer program developed by Philip DuToit that consists of several data-formatting files in addition to the LCS calculation program itself. The final version of the program is designed to run on parallel computers so that an average computation over 500 hours takes less than 5 seconds on 24 computers. The user-accessible input file, titan.in, specifies the resolution and other parameters needed to calculate the finite time Lyapunov exponent (FTLE), as well as the option to integrate the paths of drifting particles in the wind field or plot the velocity field itself. The parameters in the titan.in file are listed in the appendix.

For an FTLE calculation, the LCS file outputs a data file containing an array of FTLE vectors for each point in time and space. These results were plotted in Tecplot, the commercial data visualization software. Tecplot gives the option of plotting FTLE on either a sphere or a Cartesian coordinate system, as shown below in Fig. 2.

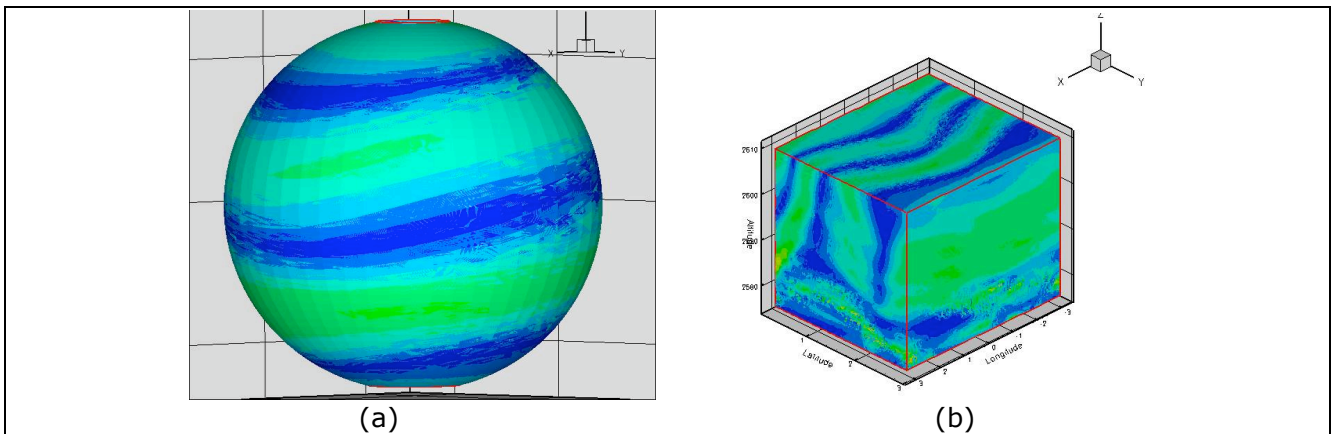


Figure 2: Tecplot graphical representation of FTLE plot in (a) spherical and (b) Cartesian coordinates.

Implementing the LCS code with the Newman model produced a plot of tracers through Titan’s atmosphere superimposed on a 3D sphere. The results produce a roughly horizontal rotational movement. This effect can be seen below in Fig. 3.

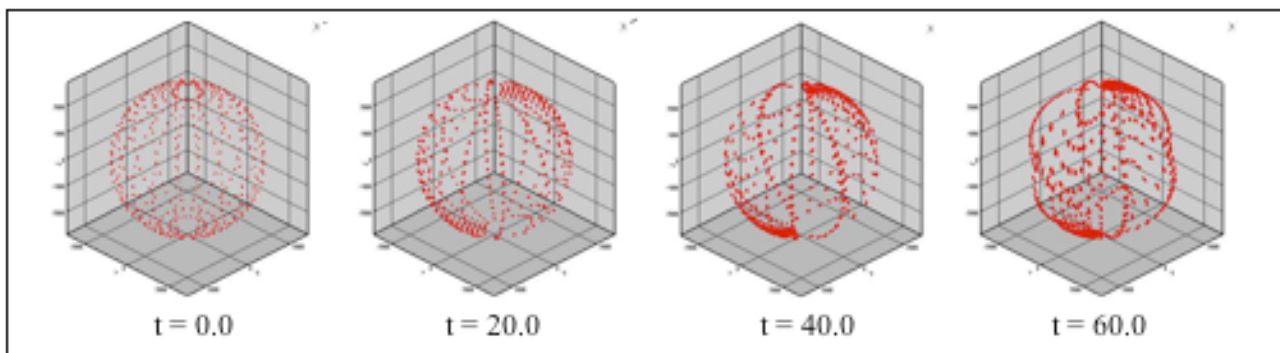


Figure 3: Tracer sequence over 60 earth hours using Newman model.

The next step in the project was to superimpose these tracers over an FTLE plot to see if their movement corresponds to the LCS. A series of three time steps in the LMD model is shown below in Fig. 4.

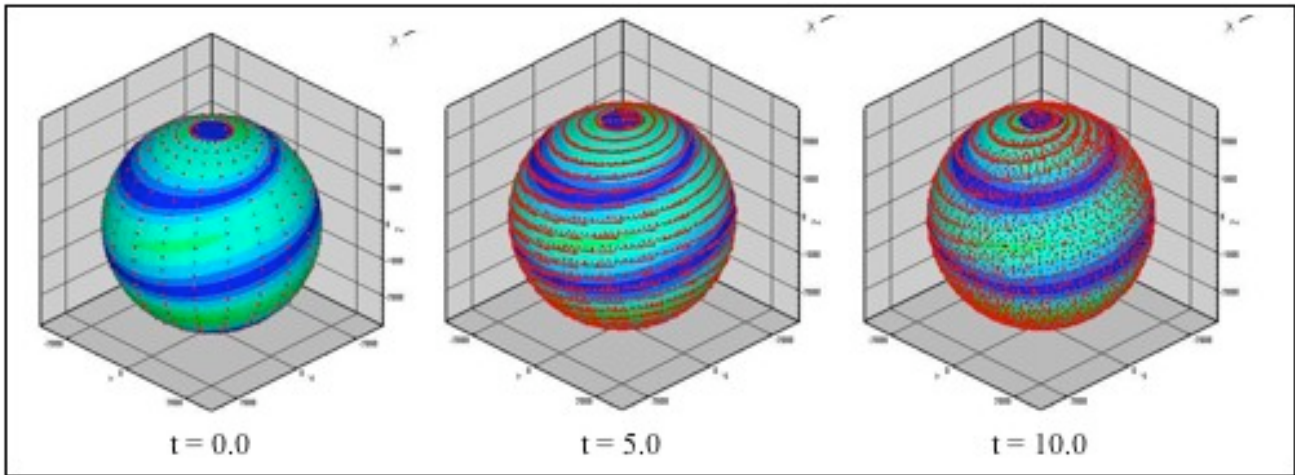


Figure 4: Tracer sequence superimposed on FTLE map over 10 earth hours using LMD model

It must be noted that there appear to be more tracers in the $t = 5.0$ and $t = 10.0$ frames. This is because in the first frame, multiple tracers exist at the same latitude and longitude but at different altitudes and over time, they separate giving a more populated appearance. From an initial glance, it also appears that the tracers follow the general curves of the LCS, which bend up to the right. Upon closer examination, however, it does not appear that the tracers stay confined in any particular region. This may be because the time scale only goes to 10 earth hours. To examine the effects of a longer time step, we can see the same visualization at 50 earth hours as shown in Fig. 5.

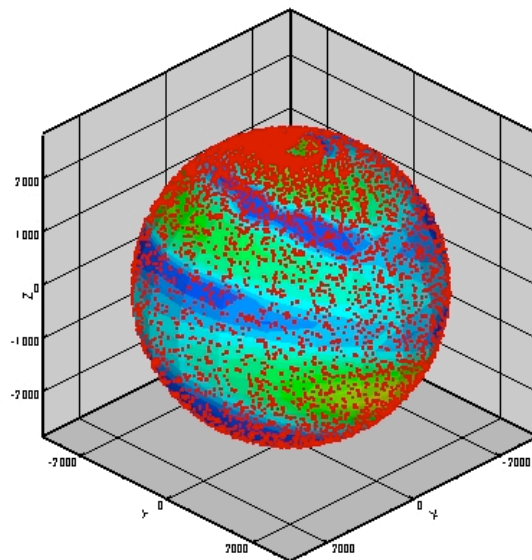


Figure 5: Tracer sequence superimposed on FTLE map after 50 earth hours using LMD model.

Since it is hard to visualize FTLE in three dimensions as shown above in Fig. 2, slices were taken out of the previous result to show the underlying structure. Because of topographical obstructions at altitudes lower than 0.5 km and the danger of methane icing above 15 km, the range of interest for the balloon is from 0.5 km to 15 km. Slices were made at 0.5 km, 1 km, 5 km and 10 km. These can be seen below in Fig. 6.

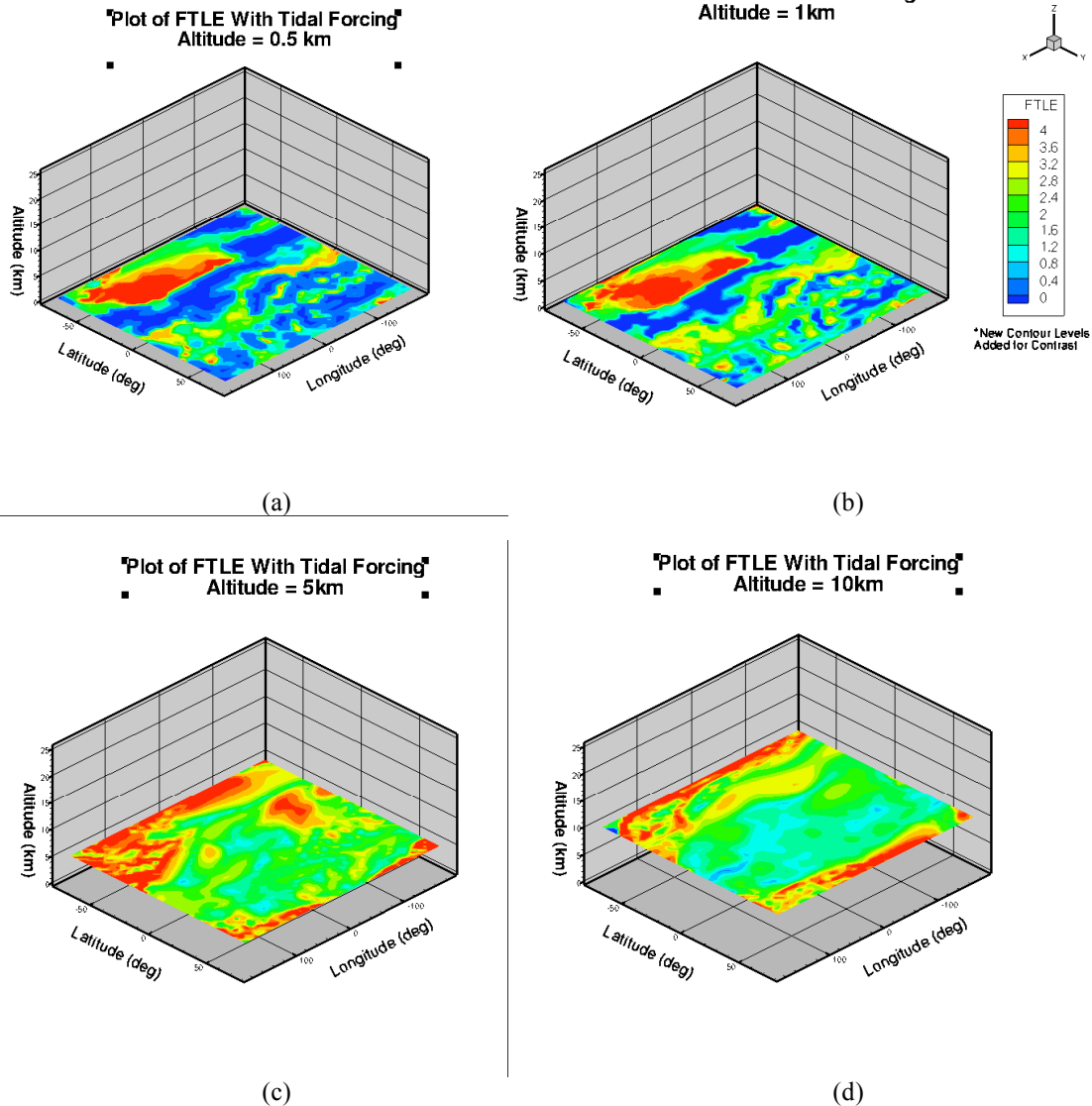


Figure 6: FTLE field at $T = 500$ hrs shows different LCS definition at varying altitudes (a) 0.5 km (b) 1 km (c) 5 km (d) 10 km.

It can be seen that at lower altitudes, there appears to be little definition of LCS, which appears as the redder area. This can be explained by the fact that winds at such low altitudes are weak and do not display characteristics similar to trade winds. For this reason, it is unlikely that the lowest altitude sample can provide much insight as to how to operate a vehicle near the ground. As altitude increases, more LCS structure emerges, most notably at 5 km and 10 km. The highest FTLE values are closest to the poles, showing that a balloon beginning its journey anywhere from -75 to 75 degrees latitude will remain in that region. It is encouraging that it is unlikely a balloon will get “trapped” at one of the poles, but it is also discouraging that this implies that the North and South pole are unattainable by a purely drifting balloon.

It will be useful to analyze and optimize the control effort to cross an LCS to enter a new region, either for the sake of finding new scientifically interesting sites or for avoiding undesirable entrapment.

References

1. Haller, G. “Lagrangian coherent structures from approximate velocity data.” *Physics of Fluids*. 2 May 2002: Vol. 14 , Issue 6.
2. Newman, Claire. “About TitanWRF” <<<http://www.planetWRF.com>>>. 2006.

Appendix: Parameters listed in titan.in input file for LCS code

Parameters for Titan 3D File ...		FTLE_XMax	=	179.0
Parameters_Print	= 1	FTLE_XRes	= 50	
Path_Input	=	FTLE_YMin	=	-75.0
	/gpfs/mezic/Titan/Input	FTLE_YMax	=	75.0
Path_Output	= /gpfs/mezic/Titan/Output	FTLE_YRes	= 50	
Path_Work	= /gpfs/mezic/Titan/Work	FTLE_ZMin	=	0.55
FTLE_Compute	= 0	FTLE_ZMax	=	24.5
Trace_Compute	= 1	FTLE_ZRes	= 50	
Plot_Velocity	= 0	FTLE_IntTimeLength	=	500.0
Velcoity_Format	= 0	FTLE_OutFile	=	FTLE_T500.dat
V_X = y		FTLE_CombineSlides	=	0
V_Y = -sin[x]		Trace_GenerateMesh	=	1
V_Z = 0.0		Trace_InFile	=	lg3am.dat
Data_InFile	= titanvel	Trace_OutFile	=	TRACE_T500.dat
Data_NumInputFiles	= 100	Trace_MeshXMin	=	-179
Data_Format	= 0	Trace_MeshXMax	=	179.0
Data_XMin	= -177.1875	Trace_MeshXRes	=	10
Data_XMax	= 177.1875	Trace_MeshYMin	=	-75.0
Data_XRes	= 64	Trace_MeshYMax	=	75.0
Data_XPeriodic = 1		Trace_MeshYRes	=	32
Data_YMin	= -87.5	Trace_MeshZMin	=	10.0
Data_YMax	= 87.5	Trace_MeshZMax	=	10.1
Data_YRes	= 36	Trace_MeshZRes	=	1
Data_YPeriodic = 0		Trace_MeshReleaseTime	=	0.0
Data_ZMin	= 0.5	Trace_MeshColor	=	1
Data_ZMax	= 25.0	Plot_OutFile	=	titanvelplot.dat
Data_ZRes	= 50	Plot_XMin	=	-180.0
Data_ZPeriodic = 0		Plot_XMax	=	180.0
Data_TRes	= 100	Plot_XRes	=	15
Data_TDelta	= 50.0	Plot_YMin	=	-75.0
Data_TMin = 0		Plot_YMax	=	75.0
Data_TPeriodic = 0		Plot_YRes	=	15
Atmos_Set	= 1	Plot_ZMin	=	0.55
Atmos_Radius	= 2575.0	Plot_ZMax	=	24.5
Atmos_Spherical	= 0	Plot_ZRes	=	15
Atmos_AltScale = 1.0		Parallel_Direction	=	1
Output_T1	= 0.0	Boundary_UsePolygon	=	0
Output_TRes	= 100	Boundary_InFile	=	
Output_TDelta	= 10.0			mbybdylatlontighttec.dat
Int_Method	= 0	Vent_U	=	10
Int_Accuracy	= 0.1	Vent_V	=	0
Int_MinTimeStep	= 0.1	Vent_W	=	0
Int_MaxTimeStep	= 1.0	Vent_T	=	200
Time_Direction	= 1	Vent_D	=	500
FTLE_XMin	= -179.0			