Titan Wind Analysis using Lagrangian Coherent Structures

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Titan has long been an interesting proposition for a mission—it is the only moon in the solar system to have a dense atmosphere, it has a hydrocarbon precipitation system, and it is hypothesised to have liquid water beneath its surface. One way of exploring Titan would be by use of an autonomous unmanned Montgolfière. With the limitations in control of this in mind, we use Lagrangian Coherent Structures (LCS) to analyse the Titan Weather Research and Forecasting (TitanWRF) wind model looking for atmospheric regions to avoid in deployment of such a balloon. The LCS are calculated using the NEWMAN code on the CITerra supercomputer. It is found that at the likely mission arrival date (~2030) a Montgolfière could be trapped if deployed above 60° latitude. A preliminary robustness analysis is then done to determine the usefulness of this prediction in the circumstance of the winds encountered differing from the model. This shows that even a 2% random noise level on the wind field data will destroy the LCS. However, the noise profile used is not proven to be physically likely, and so further research with different noise profiles is necessary for a full picture of how useful the LCS are in autonomous or pre-programmed balloon control.

Titan, the largest moon of Saturn, is unique, in that it is the only place in the solar system other than Earth to have stable surface bodies of liquid (1), hydrocarbon seas including methane and ethane. Beneath its surface a liquid water-ammonia mix is hypothesised to exist. It is also the only moon in the solar system known to have a dense atmosphere (2). Titan is also a 'super-rotator', i.e. the bulk atmosphere rotates much faster than its surface, as on Venus. Usually this results in strong Westerlies (winds travelling from West to East) at the poles and weak Easterlies (winds travelling from East to

West) at the equator, the weak Easterlies being due to the solid-body rotation of Titan, and hence the increased speed of the surface near the equator relative to the winds (3).

Due to being almost ten AU from the Sun, Titan only receives around 1% of the solar flux of Earth, so its tropospheric temperature is only around 70 – 94K (4), with a typical surface temperature being in the low nineties. Notable features of Titan include hydrocarbon rain and possible cryovolcanoes (3) (5).

Aside from the geological reasons of interest, there is also an exobiological interest in the moon; the conditions under the surface, where liquid water is thought to exist mixed with ammonia (6), are thought to be very similar to those on Earth in the early stages of life (7).

These coupled opportunities for multi-disciplinary science make a mission to Titan very attractive. The recent discovery of water vapour and complex hydrocarbons venting from the southern regions of Enceladus (another of Saturn's moons) (8), the possibility that this may be the cause of Saturn's E-ring, and the fact that both Enceladus and Titan could be visited in a single mission, then make such an event a matter of 'when' as opposed to 'if'.

There are a number of ways to explore Titan; the Titan Saturn System Mission (TSSM) recently proposed jointly by ESA and NASA consisted of an orbiter and two Titan exploration probes—a lander that would splashdown in one of the methane seas, and a Montgolfière balloon to explore the atmosphere (9) (10) (11) (12). The possibility to explore the moon of another world by balloon is unique to Saturn, and offers an unprecedented amount of mobility, however an intelligent control system will be necessary to ensure maximum use is gained from it.

We decided to build on the work done by Han Bin Man (3) and Sarah Sherman (9), and use Lagrangian Coherent Structures (13) (14) to look for regions that must be avoided by such a control system to ensure that the Montgolfière does not become stuck in any region of the atmosphere. Due to the operational range of the balloon in the frigid temperatures of the atmosphere of Titan, and the likely arrival date of the mission due to priority being given to the Europa Jupiter System Mission (EJSM), we have restricted our analysis to the lower 20km of the atmosphere, and the wind fields of Titan around 2030. This corresponds roughly to the first half of Titan's Southern-summer (Northern-winter).

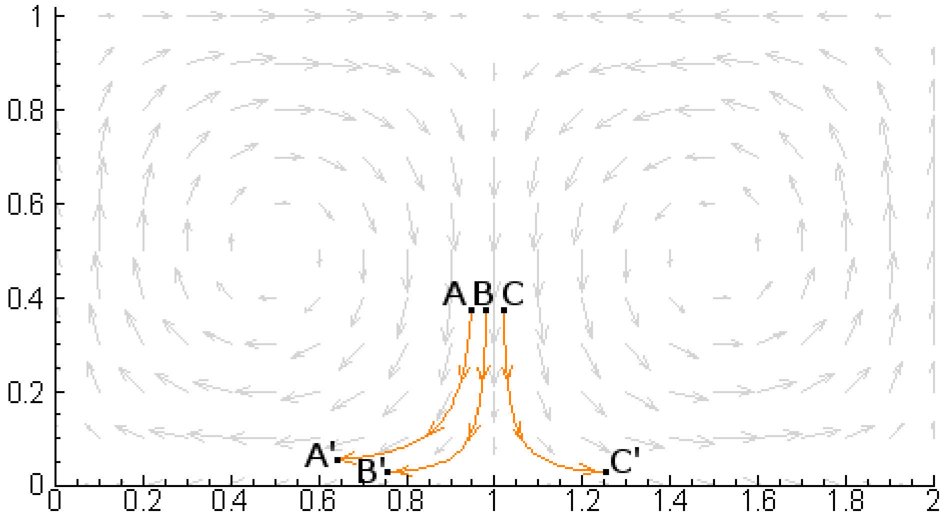
In short, Lagrangian Coherent Structures are ridges in the scalar Finite-Time Lyapunov Exponent (FTLE) field (13). The FTLEs are generated by integrating the divergence between particles in a flow-field, as they are transported by the flow over some suitable period of time. What this in effect means is that they are the boundaries between distinct, in-this-case atmospheric, regions; boundaries across which there is minimal particle flux. The difference between these and the separatrices found in classical chaos theory is that in this case the field is time-dependent; the winds in the atmosphere change over time. Due to the length of the mission (six Earth-months), and the existence of planetary waves in the atmosphere with periods of Earth-weeks to Earth-months (15), a time-independent approximation of the atmosphere is not useful.

Titan rotates on its axis once every 15 Earth-days and 22 Earth-hours; due to being tidally locked this is the same period as its rotation around Saturn. To avoid confusion and the unwieldy use of 'Titan-' or 'Earth-' throughout the report, the author will use the notation 'Tear' for a Titan-year, 'Tay' for a Titan-day, and 'Tour' for a Titan-hour (defined as 1/24 of a Tay). A Tear is of course the same length of time as a Saturn-year (approximately 30 Earth-years). 'Year', 'day', and 'hour' are reserved for their Earth measurements.

A simple example of LCS using a time-independent data-set (13) —both to aid understanding and to help elucidate its usefulness—follows (diagrams originally produced by Dr Shawn Shadden[i]):

Example:

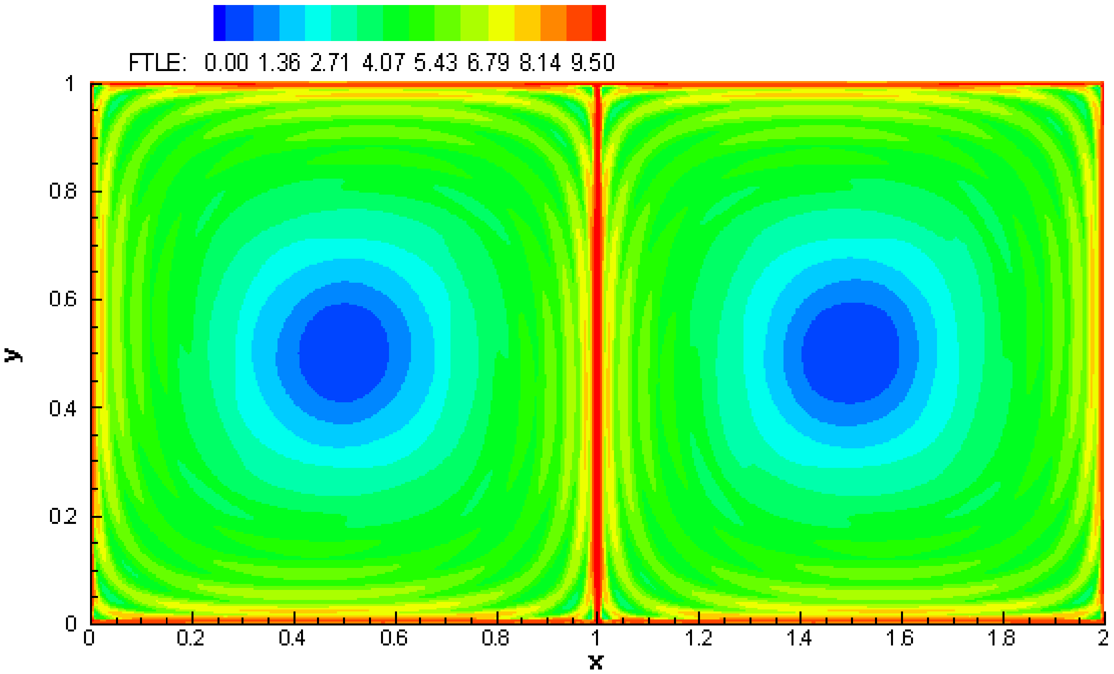
Figure 1a) Double-Gyre velocity field



Take any two particles in a double-gyre wind-field (Figure 1a). For most pairs, if the particles started close together, they will stay close together for a reasonable period of time (e.g. particles A and B). However, in certain places a pair of particles just as far apart will rapidly diverge, either immediately or after a short period of time (e.g. particles B and C). If we look at all possible pairs of particles (or some discrete approximation thereof), then we can find the boundaries from which particles to either side of will diverge, and hence the different regions of dynamical activity.

The red lines in Figure 1b are then the separatrices; the ridges in the FTLE field around which particles separate. These ridges are referred to as Lagrangian Coherent Structures, or LCS. When dealing with time-dependent fields, these structures still exist, but they move depending on the starting integration time, and may get very complicated with very long integration periods.

Figure 1b) Double-Gyre LCS plot



It should be noted, that whilst these LCS separate different dynamical regions, there are many places along an LCS where even a slight nudge would move the traveller from one dynamical region to another; the presence of an LCS simply means that it is very unlikely for the traveller to pass between them without some action external to the flow field.

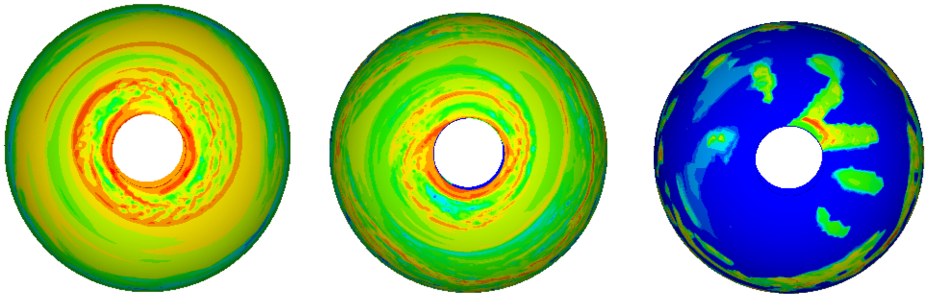
This is where LCS become useful to the autonomous control of a balloon. Since a balloon is likely to have only vertical control, it could very easily become trapped in one dynamical region of the atmosphere. However, if the on-board computer knew where the LCS of the region it was in were, and what regions the LCS were bordering, it would be able to descend or ascend at the correct time to move between regions, thus giving it a lot more control over its movement. An aerobot would have even more choice, as horizontal actuation could enable it to move between regions in all directions. Recent research has shown that a small amount of actuation is enough to access most of the surface of Titan (16), hence in the case of using a Radioisotope Thermoelectric Generator (RTG), the need to use LCS would be minimal; however, if fuel needed to be conserved, then efficient use of LCS could be crucial.

Locations of LCS

To calculate the LCS of Titan's atmosphere in early Southern-summer we used the NEWMAN code (17) [ii], provided by Dr Philip du Toit. The code was parallelised on the CITerra supercomputer and integrated for a period of just over six months. We used a variety of starting times, all around or just after the expected mission dates; however the dominant features were the same for each of them. We limited our analysis to the lower-half of Titan's troposphere (0 – 20km altitude), as, due to the temperature gradient, above some level (possibly as low as 10 or 12km) methane or some other hydrocarbon ice may form on the balloon.

Our results showed that for the period of interest, there appear to be two main LCS in Titan's atmosphere, both in the Northern hemisphere. The more prominent of the two encircles the Northern pole at around 60 degrees latitude, and descends from at least 20km in height to between 10 and 5km from the surface. It can be seen in Figure 2a, but better in Figures 2c and 2d.

Figure 2a) Northern hemisphere FTLE plots at various altitudes



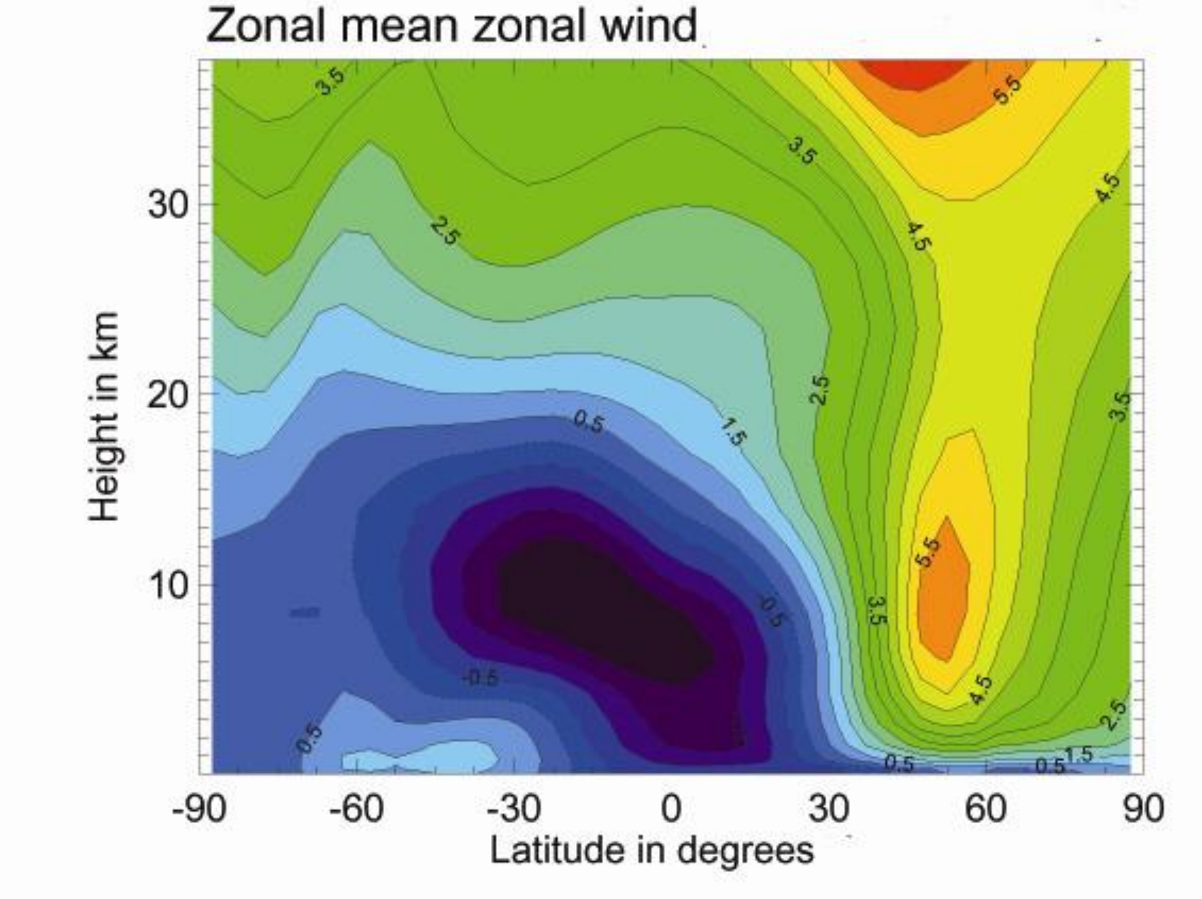
North Pole: FTLE at 10km FTLE at 5km FTLE at 1km

Red is used for high values of the FTLE-field (in these cases the LCS of the region), whereas blue is for low values. A quite obvious LCS can be seen around the Northern pole at 10km, and is beginning to disappear by 5km altitude. By 1km there is little discernable structure around the pole.

The less prominent of the two LCS is a spiral, and can be seen best in Figure 2a at 10km. It starts just inside of the Northern-circle LCS at around 60 degrees latitude and extends to around 45 degrees latitude. These spiral LCS around the North pole have been noted before on Titan at lower altitudes (around 2.5km) in previous work done by Sarah Sherman (9).

Titan's tropospheric circulation is dominated by a single Hadley cell during Southern-summer, whereby 'air' (a mixture of 98.4% nitrogen and 1.6% methane and other more complicated hydrocarbons) rises in the Southern hemisphere and descends at mid to high latitudes in the Northern hemisphere (18). The Coriolis force then deflects the winds heading towards the equator West, and winds heading towards the poles East. The combination of the Hadley cell, atmospheric super-rotation, and Coriolis force results in the pattern of zonal (West to East) winds shown in Figure 2b, whereby there are two counter-rotating winds; within the lower 10km those above 30 degrees of latitude are Easterlies, and those below 30 degrees of latitude are Westerlies. The Easterlies are in general stronger than the Westerlies; in the lowest 20km of the atmosphere the fastest Easterly is over 5.5 metres per second, whereas the fastest Westerly is only just over 2 metres per second.

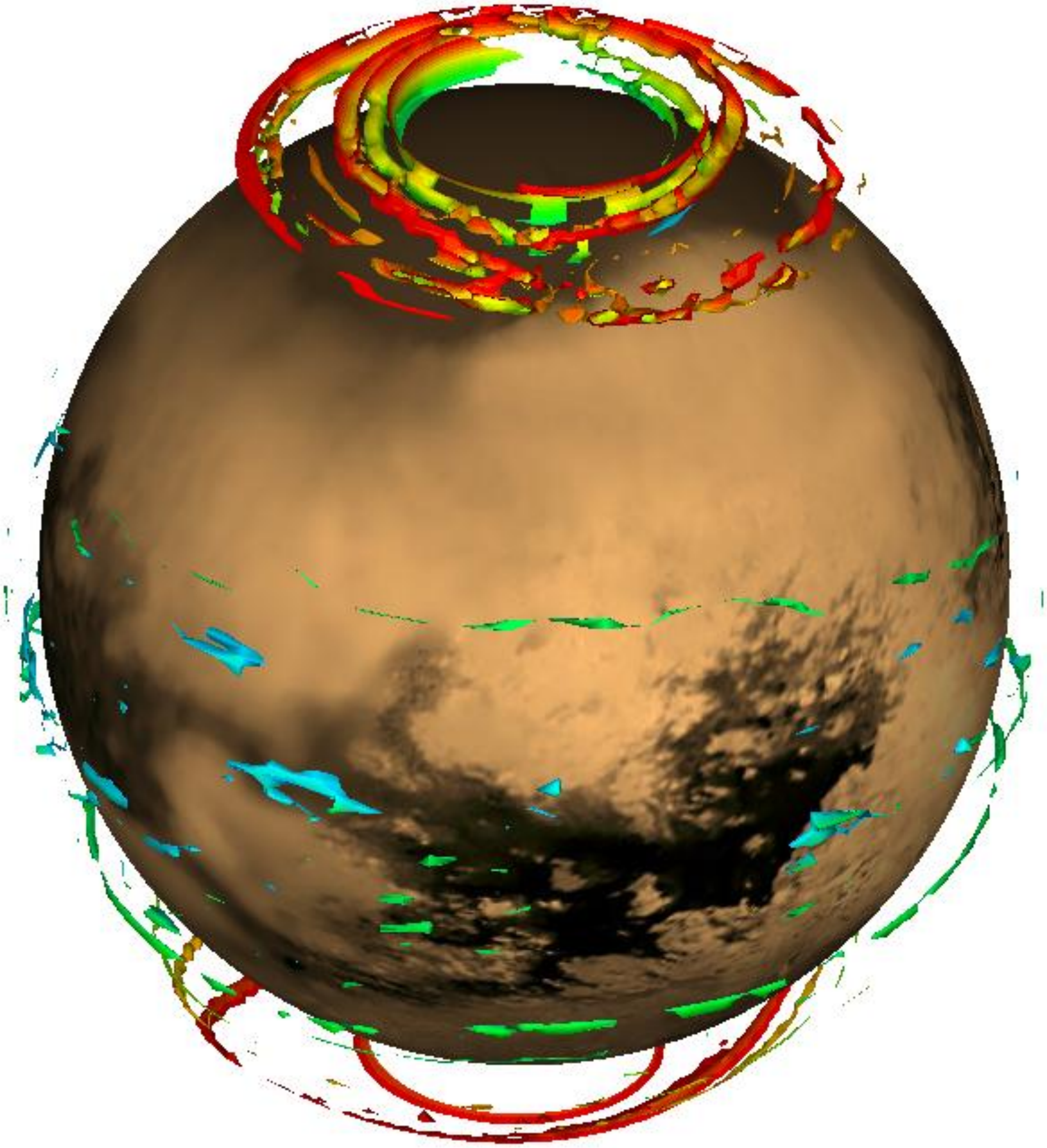
Figure 2b) Wind speed in the longitudinal direction averaged over all longitudes for each latitude for Titan Southern-summer



All speeds are in metres per second. Positive values are for Easterlies, negative values for Westerlies. (Zonal being Westerly.)

The Northern-circle LCS exists just North of the strongest winds, separating the pole with its relatively weaker winds from the fastest blowing Easterlies. The tail of the Northern-spiral LCS however extends to the other side of the fastest winds, effectively separating the two counter-rotating winds. This would suggest that although a balloon could be trapped at the North pole by the circular structure, if it were to be deployed in the opening of the spiral, it should be able to escape. Simulations in which highlighted particles in the atmosphere (drifters) were followed as they move with the winds provide further evidence for this, showing that near this region many drifters initially rotating in an Easterly direction undergo a change in direction and end up travelling in a Westerly direction in the equatorial winds. Drifters within the Northern-circle LCS however do not leave the pole, and simply circle round for the duration of the simulation.

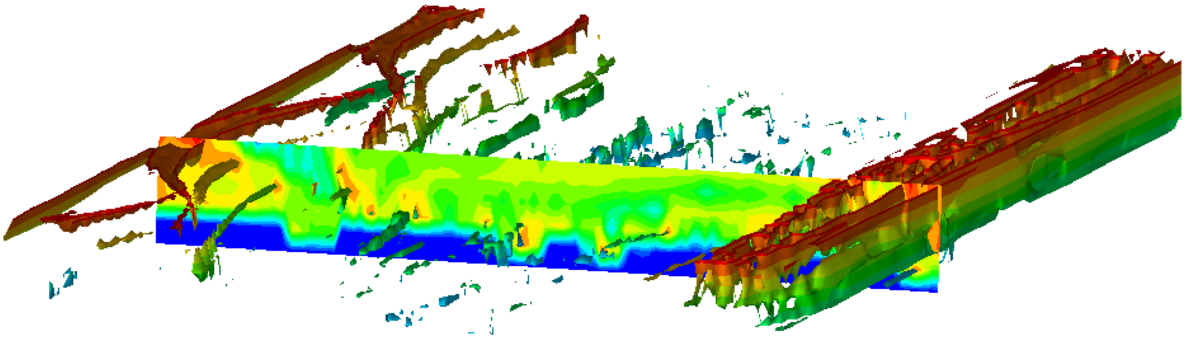
Figure 2c) a 3D FTLE iso-surface displayed on a model of Titan showing the LCS around the North pole



Red is used for 20km, dark blue for 0km. Altitudes above Titan are not in scale with model of Titan (else they would all appear flat). LCS at the North pole is quite prominently displayed.

However, using the opening in the spiral, either by deployment or navigation, is made almost impossible by dint of the fact that this LCS rotates, and so the exact location of the opening would not be foreknown. Even if the TitanWRF (19) wind model were perfect, due to the existence of planetary-scale waves with a variety of periods, the exact state of the moon at a certain time would need to be known in order to calibrate the model to correctly set the phase of the waves.

Figure 2d) a 3D FTLE isosurface; the prominent LCS of Titan's Southern-summer, from -80 degrees to +80 degrees



On the FTLE iso-surface, the colour is used to indicate the altitude for ease of gauging relative positions of structure, with red indicating 20km and deep blue indicating 0km. On the slice-through, red is used for high values of FTLE and blue for low values; the iso-surface passes through some of the reddest regions on the slice-through. The bands immediately next to the North and South pole must be discounted; they are an artefact of the integration process without wind data for the poles, however the thicker structure at the North pole is real.

A more promising route may be by escaping beneath the LCS at the pole. If a balloon were to descend below 5km (ideally to around 1km) at as low a latitude (i.e. as far South) as it could manage, then it would now be in a different dynamical region of the atmosphere—the same region as the central band of air, and should no longer be trapped at the pole. However, some surface features on Titan do extend to 1 or 2km in altitude, so it may not be possible to do this without running the risk of the Montgolfière crashing.

With an aerobot with limited fuel, but a wish to speed things up somewhat, the results suggest a different approach, simply that when close to either of the LCS at any height, use a small period of actuation to pass through it. When in either region, there will be interesting science targets, and given enough time the aerobot should reach them, but for it to pass between the polar and equatorial regions without actuation is very unlikely, and hence that should be its priority for fuel-usage, in a limited-fuel scenario.

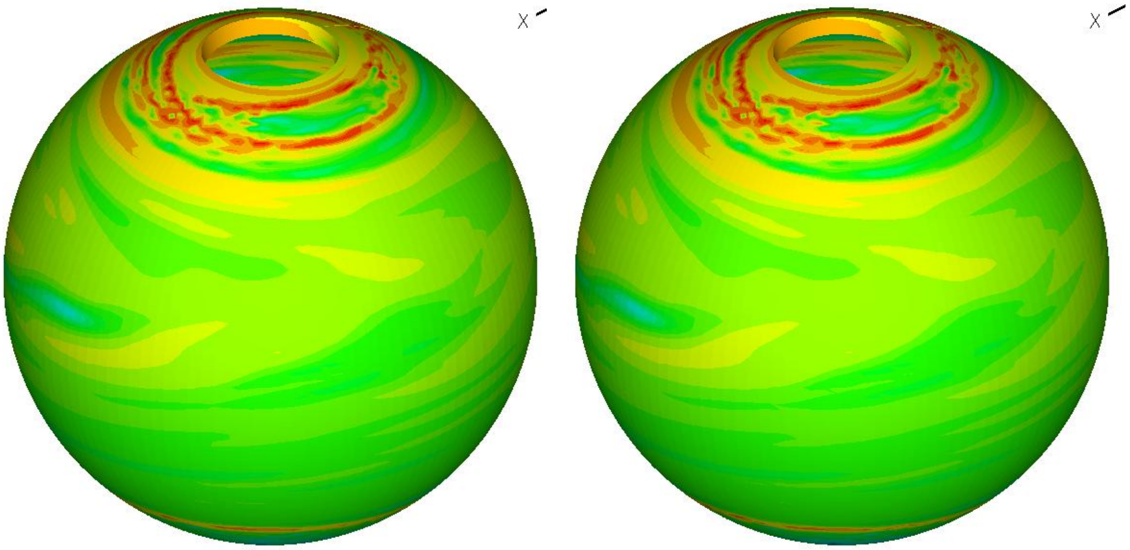
The results also show a very low FTLE value near the surface over the whole planet. This suggests that an aerobot with horizontal actuation would find that it would have more control over its motions if it were to descend to this level, since a low FTLE value means a low local divergence of nearby particles, and hence a reduced dependency of path upon position. I.e. a small variation in start location is unlikely to result in a vastly different destination after actuation.

Robustness of LCS

To test the robustness of the LCS, i.e. how resistant they were to small changes in the wind-field data, we added a different small random value to each component of each wind vector weighted by the relevant component of the mean wind velocity at that latitude. To do this we used a Gaussian profile for the random data of mean 0 and standard deviation 1, multiplied the value for each data point by the relevant component of the average wind velocity at that latitude, and then by a variable between 0.005 and 2 (this was the same for each point; the variation was between runs and so as to see the effect of increasing randomness). This was added to the initial data set (thus causing both a change in magnitude and direction of the wind vectors), and the NEWMAN code run as before.

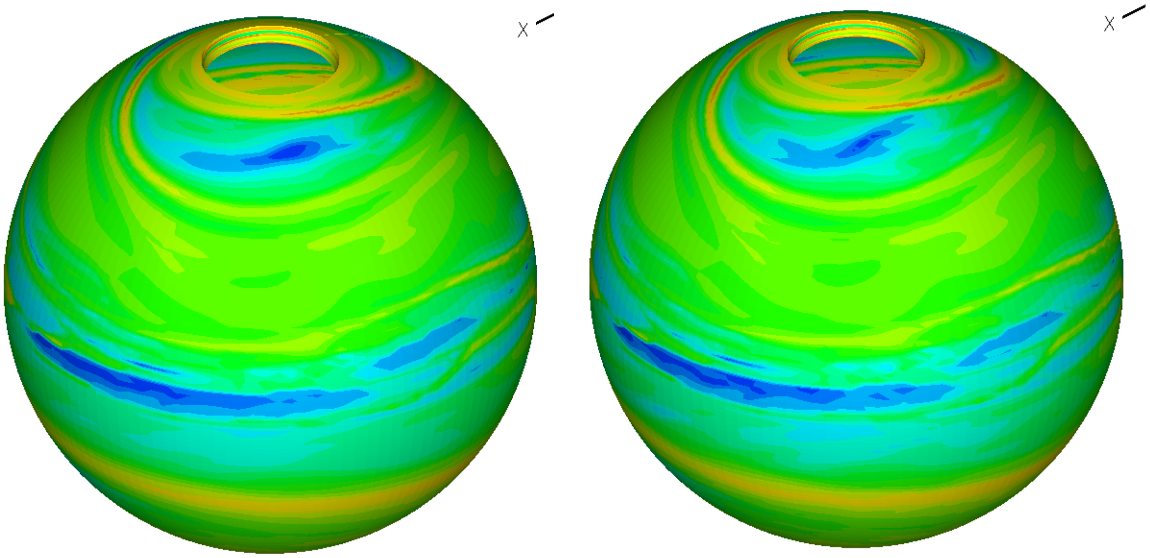
The reason for the weighting by mean wind velocity at each latitude was two-fold. Firstly, to avoid data points that just happened to be lower in magnitude than those around them having a lower deviation, and secondly because the latitudinal variation in velocities is far greater than any longitudinal variation, and we did not have the computing power to centre the probability distribution for each point on itself within a reasonable time-frame.

Figure 3a) FTLE plots at 10km with varying amounts of noise



LCS at 10km with 0% noise

LCS at 10km with 1% noise



LCS at 10km with 2% noise

LCS at 10km with 5% noise

The results were conclusive; even with a variation in each data point of only 2%, the LCS disappear almost completely.

It was expected that random noise would destroy the LCS, although perhaps not so easily. The LCS are defined by their property of having minimal flux across them; if random noise is added, then there is a fairly high chance that any particle near the boundary could have the little shove it needs to cross over to the different dynamical region. At higher altitudes this not necessarily a physical noise profile—there is no cause and effect (the noise at each time step is independent of those before and after it), and there is little to cause the smaller scale variations added by the noise—however at lower altitudes, and especially near the surface, it may have some merit. Although the cause-and-effect problem still exists, there are surface features which could have an effect similar to random noise at a smaller scale than in this model, which could in turn have the observed effect on the LCS. This however requires work to be done looking at a higher resolution wind model, and perhaps using 'typical' Titan surface features embedded in them, to see how this effects the LCS near the surface. It is also possible however that these surface features would just set up their own LCS, adding to the complexity rather than reducing it.

The Next Step

It should be noted that in the course of this experiment, we had access to only one Tear's worth of data; although enough for a preliminary analysis, having multiple Tears of data would also help in testing the robustness of LCS to the phase of planetary scale waves, as well as being able to build a statistical profile of each season (or other arbitrary time period). This would enable us to look at not only a typical Tear, but also exceptional ones—Tears in which the Southern-summer winds were far stronger than usual, or where the zero-zonal wind line only hovered around 10 degrees of latitude, for example. Indeed, the LCS observed may be vastly different, or there may even be none at all. This Titannual variation will need to be understood if the Montgolfière is to be programmed to be able to deal with whatever Titan's atmosphere may blow at it.

What also needs to be taken into account when interpreting these results is that the wind models currently do not agree one hundred percent with the observations made of the winds of Titan, in particular with the direction of surface winds—sand dunes have been imaged in the equatorial regions which suggest the direction of winds near the surface are actually the opposite of what the model predicts (i.e. Westerly as opposed to Easterly) (20). What may be more important for engineers is to develop on-board instruments for measuring wind speeds in the near vicinity of the balloon, rather than attempting to use global wind models for navigation. However, as Cassini completes more flybys, and the models are improved upon, the parameters fine-tuned, they may get to the stage of accurately predicting all the observed wind patterns.

Another avenue for further research would be to test whether LCS might still be useful for autonomous or pre-programmed control, by using different noise profiles (and especially attempting to make a more physically relevant one). The winds on Titan will almost certainly be different from any model; even if the model can perfectly describe a Tay, it will not be *the* Tay on which the Montgolfière arrives. The robustness of LCS to different-but-similar wind patterns will be of great importance if they are to be used in control applications.

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[i] A more full tutorial on LCS can be found at Dr Shadden's Caltech website: http://www.cds.caltech.edu/~shawn/LCS-tutorial/

[ii] The NEWMAN code is the available for download at: http://www.cds.caltech.edu/~pdutoit/Philip_du_Toit/Software.html