Mobility of a Simple Hinged Body in an Ideal Fluid

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1 Introduction

This work investigates the mobility of a coupled rigid body in an ideal fluid. Common belief holds that a single hinged body cannot propel itself through an ideal fluid, however there exists no well known literature that proves this point exactly.

The geometric reasoning for this is a simple argument, yet a difficult concept to understand. Based on the work from Kanso [2005], the shape space of a single hinged object is one dimensional. Specifically, the shape space is the space of relative configurations of rigid bodies, which in this case can be defined by a single variable, θ, the relative angle between the bodies. From Kanso [2005], *The configuration space has the structure of a principal bundle over the shape space X, which allows net locomotion to be cast as a geometric phase, or holonomy, over closed loops traced by the shape variables.* In this case where the shape space is one dimensional no closed loops can be traced out, hence producing no net motion. It should be noted that as $\theta \in S^1$, if θ is allowed to increase continuously, letting the rigid bodies interfere with each other, then a closed loop (S^1) can be traced out. However, we excluded this case by only allowing motions where the rigid bodies do not collide.

This work compliments the geometric phase approach by deriving the system Lagrangian for both a single hinge system in free space and for a single hinge in an ideal fluid. The effect of the fluid is analyzed and oscillatory inputs are used to demonstrate the periodic motions of the Center of Mass of the rigid bodies.

2 Motivation

Kanso [2005] demonstrates the ability for a two hinge (three rigid body) system to move in an ideal fluid. Although care is taken to analytically derive the effect of the fluid, it is unclear, at least to the author, how the fluid plays a role in geometric phase, or net locomotion. A simple question that illustrates this is "if a two hinge system can only induce a net rotation in free space, how does the addition of fluid now allow for a net translation, when the coordinates and shape space of the system remain the same?" Perhaps this is obvious, but analyzing the effect of fluid on a one hinge system, may help to clarify this, or at least explain it in more rudimentary terms.

Saffman [1966] writes conditions on mobility ('*whether a body that is initially at rest relative to the fluid can by deformation of its surface give itself a persistent velocity*'), in terms of momentum conservation, involving the added mass of the body due to its acceleration in the fluid, and the *fluid impulse* which can be interpreted as the linear momentum of the fluid. Saffman creates a general proof, showing that net motion in a homogeneous body can be produced by asymmetrical deformations, and that periodic deformations can give rise to a non-zero-mean fluid impulse. Although Saffman does not write the bodies momentum in terms of a Lagrangian or Hamiltonian, deriving momentum expressions from these will allow direct analysis of a bodies mobility.

Purcell [1977] discusses motion in low Reynolds number flow, in which viscous effects are predominate. Here motion is determined completely by the geometry of the submerged body. In this situation there exists *The Scallop Theorem* which states that if an body tries to swim by reciprocal motion, it retraces its trajectory exactly. This indicates that a scallop like object (a single hinge system), which can only make reciprocal motions, will never achieve net movement. Low Reynolds number flow and potential flow, which is at high Reynolds numbers, both are time reversible. However, potential flow is entirely dominated by inertial forces (as opposed to viscous), which creates the question: "why do flows that are dominated by completely different principles, apparently both predict the same result when applied to a single hinged system?" Again, how does the presence of the fluid (either viscous or inertial) change the dynamics of coupled bodies, and allow for net locomotion? It will be interesting to rigorously create a 'potential flow' proof of the scallop theorem.

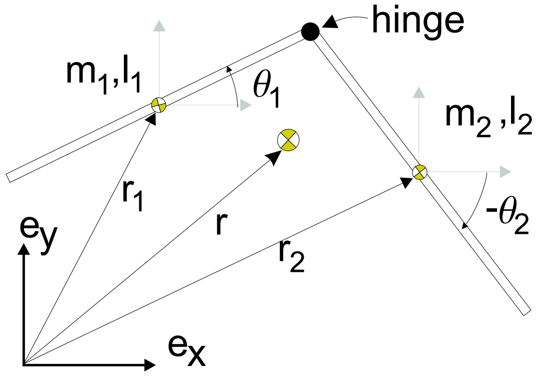

Figure 1: Simple Hinged Rods

3 Dynamics of a Free Single Hinged Rigid Body

Before analysis of a hinged rigid body in an ideal fluid is undertaken, the dynamics of a free hinged rigid body are analyzed to gain insight into the more complicated problem.

3.1 Lagrangian of Coupled Rigid Body

One of the simplest coupled rigid body problems is two rigid rods joined at one end by a simple hinge, constrained to move on a plane. This system, shown in Figure 1, is a 4-DOF system, and can be represented in generalized coordinates by θ_1, θ_2, the orientation of the rods, and r, the position of their combined center of mass (COM). The Lagrangian for this system is now derived. It should be noted that the approach, while much less general, is based on the approach in Sreenath [1988].

The center of mass (COM) of the structure r, is defined by the following:

$$(m_1 + m_2)r = m_1 r_1 + m_2 r_2 \tag{1}$$

The hinge constraint:

$$r_1 + R(\theta_1)\frac{l_1}{2}e_x = r_2 - R(\theta_2)\frac{l_2}{2}e_x \tag{2}$$

The kinetic energy of the body K_B in spatial coordinated is:

$$K_B = \frac{1}{2}m_1\dot{r_1}^T\dot{r_1} + \frac{1}{2}m_2\dot{r_2}^T\dot{r_2} + \frac{1}{2}I_1\dot{\theta_1}^2 + \frac{1}{2}I_2\dot{\theta_2}^2 \tag{3}$$

Using (1) and (2) we can write r_1 and r_2 in terms of r, the COM:

$$\begin{aligned}
r(m_1+m_2) &= m_1 r_1 + m_2\left(r_1 + R(\theta_1)\frac{l_1}{2}e_x + R(\theta_2)\frac{l_2}{2}e_x\right) && (4)\\
\Rightarrow r_1 &= r - \frac{m_2}{m_1+m_2}\left(R(\theta_1)\frac{l_1}{2}e_x + R(\theta_2)\frac{l_2}{2}e_x\right) && (5)\\
\Rightarrow \dot{r_1} &= \dot{r} - \frac{m_2}{m_1+m_2}\left(R'(\theta_1)\dot{\theta_1}\frac{l_1}{2}e_x + R'(\theta_2)\dot{\theta_2}\frac{l_2}{2}e_x\right) && (6)
\end{aligned}$$

Similarly:

$$\begin{aligned}
r_2 &= r + \frac{m_1}{m_1+m_2}\left(R(\theta_1)\frac{l_1}{2}e_x + R(\theta_2)\frac{l_2}{2}e_x\right) && (7)\\
\Rightarrow \dot{r_2} &= \dot{r} + \frac{m_1}{m_1+m_2}\left(R'(\theta_1)\dot{\theta_1}\frac{l_1}{2}e_x + R'(\theta_2)\dot{\theta_2}\frac{l_2}{2}e_x\right) && (8)
\end{aligned}$$

Note that:

$$R(\theta_i) = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \end{bmatrix} \quad (9)$$

$$\Rightarrow \frac{d}{dt}R(\theta_i) = R'(\theta_i)\dot{\theta}_i, \quad \text{where } R'(\theta_i) = \begin{bmatrix} -\sin(\theta_i) & -\cos(\theta_i) \\ \cos(\theta_i) & -\sin(\theta_i) \end{bmatrix} \quad (10)$$

Therefore,

$$\begin{aligned} e_x^T R'(\theta_i)^T R'(\theta_j) e_x &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} -\sin(\theta_i) & \cos(\theta_i) \\ -\cos(\theta_i) & -\sin(\theta_i) \end{bmatrix} \begin{bmatrix} -\sin(\theta_j) & -\cos(\theta_j) \\ \cos(\theta_j) & -\sin(\theta_j) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} && (11) \\ &= \sin(\theta_i) sin(\theta_j) + \cos(\theta_i)\cos(\theta_j) && (12) \\ &= \cos(\theta_1 - \theta_2) && (13) \\ &= 1, \text{ if } i = j && (14) \end{aligned}$$

Now,

$$\begin{aligned} \frac{1}{2}m_1 \dot{r_1}^T \dot{r_1} &= \frac{1}{2}m_1 \dot{r}^T \dot{r} - \frac{m_1 m_2}{m_1+m_2}\frac{l_1}{2}\dot{\theta}_1[\dot{r}^T R'(\theta_1)e_x] - \frac{m_1 m_2}{m_1+m_2}\frac{l_2}{2}\dot{\theta}_2[\dot{r}^T R'(\theta_2)e_x]\ldots && (15) \\ &+ \frac{1}{2}m_2\frac{m_1 m_2}{(m_1+m_2)^2}\left([R'(\theta_1)e_x]\dot{\theta}_1\frac{l_1}{2} + [R'(\theta_2)e_x]\dot{\theta}_2\frac{l_2}{2}\right)^T \left([R'(\theta_1)e_x]\dot{\theta}_1\frac{l_1}{2} + [R'(\theta_2)e_x]\dot{\theta}_2\frac{l_2}{2}\right) && (16) \end{aligned}$$

The last term is expanded, then simplified:

$$\begin{aligned} &\left([R'(\theta_1)e_x]\dot{\theta}_1\frac{l_1}{2} + [R'(\theta_2)e_x]\dot{\theta}_2\frac{l_2}{2}\right)^T \left([R'(\theta_1)e_x]\dot{\theta}_1\frac{l_1}{2} + [R'(\theta_2)e_x]\dot{\theta}_2\frac{l_2}{2}\right) && (17) \\ &= \dot{\theta_1}^2\frac{l_1^2}{4}\left(e_x^T R'(\theta_1)^T R'(\theta_1)e_x\right) + \dot{\theta_2}^2\frac{l_2^2}{4}\left(e_x^T R'(\theta_2)^T R'(\theta_2)e_x\right) + ... && (18) \\ &\quad \dot{\theta}_1\dot{\theta}_2\frac{l_1 l_2}{4}\left(e_x^T R'(\theta_2)^T R'(\theta_1)e_x\right) + \dot{\theta}_2\dot{\theta}_1\frac{l_2 l_1}{4}\left(e_x^T R'(\theta_1)^T R'(\theta_1)e_x\right) && (19) \\ &= \dot{\theta_1}^2\frac{l_1^2}{4} + \dot{\theta_2}^2\frac{l_2^2}{4} + ... && (20) \\ &\quad \dot{\theta}_1\dot{\theta}_2\frac{l_1 l_2}{4}(\cos(\theta_1-\theta_2)) + \dot{\theta}_2\dot{\theta}_1\frac{l_2 l_1}{4}(\cos(\theta_2-\theta_1)) && (21) \\ &= \dot{\theta_1}^2\frac{l_1^2}{4} + \dot{\theta_2}^2\frac{l_2^2}{4} + \dot{\theta}_2\dot{\theta}_1\frac{l_2 l_1}{4}2\cos(\theta_1-\theta_2) && (22) \end{aligned}$$

Then,

$$\begin{aligned} \frac{1}{2}m_1 \dot{r_1}^T \dot{r_1} &= \frac{1}{2}m_1 \dot{r}^T \dot{r} - \frac{m_1 m_2}{m_1+m_2}\frac{l_1}{2}\dot{\theta}_1[\dot{r}^T R'(\theta_1)e_x] - \frac{m_1 m_2}{m_1+m_2}\frac{l_2}{2}\dot{\theta}_2[\dot{r}^T R'(\theta_2)e_x]\ldots && (23) \\ &+ \frac{1}{2}m_2\frac{m_1 m_2}{(m_1+m_2)^2}\left(\dot{\theta_1}^2\frac{l_1^2}{4} + \dot{\theta_2}^2\frac{l_2^2}{4} + \dot{\theta}_2\dot{\theta}_1\frac{l_2 l_1}{4}2\cos(\theta_1-\theta_2)\right) && (24) \end{aligned}$$

Similarly:

$$\begin{aligned} \frac{1}{2}m_2 \dot{r_2}^T \dot{r_2} &= \frac{1}{2}m_2 \dot{r}^T \dot{r} + \frac{m_1 m_2}{m_1+m_2}\frac{l_1}{2}\dot{\theta}_1[\dot{r}^T R'(\theta_1)e_x] + \frac{m_1 m_2}{m_1+m_2}\frac{l_2}{2}\dot{\theta}_2[\dot{r}^T R'(\theta_2)e_x]\ldots && (25) \\ &+ \frac{1}{2}m_1\frac{m_1 m_2}{(m_1+m_2)^2}\left(\dot{\theta_1}^2\frac{l_1^2}{4} + \dot{\theta_2}^2\frac{l_2^2}{4} + \dot{\theta}_2\dot{\theta}_1\frac{l_2 l_1}{4}2\cos(\theta_1-\theta_2)\right) && (26) \end{aligned}$$

Therefore:

$$\begin{aligned} \frac{1}{2}m_1 \dot{r_1}^T \dot{r_1} + \frac{1}{2}m_2 \dot{r_2}^T \dot{r_2} &= \frac{1}{2}(m_1+m_2)\dot{r}^T \dot{r}\ldots && (27) \\ &+ \frac{1}{2}(m_1+m_2)\frac{m_1 m_2}{(m_1+m_2)^2}\left(\dot{\theta_1}^2\frac{l_1^2}{4} + \dot{\theta_2}^2\frac{l_2^2}{4} + \dot{\theta}_2\dot{\theta}_1\frac{l_2 l_1}{4}2\cos(\theta_1-\theta_2)\right) && (28) \end{aligned}$$

The kinetic energy K_B from (3) is then:

$$K_B = \frac{1}{2}(m_1+m_2)\dot{r}^T\dot{r} + \frac{1}{2}\frac{m_1m_2}{m_1+m_2}\left(\dot{\theta_1}^2\frac{l_1^2}{4} + \dot{\theta_2}^2\frac{l_2^2}{4} + \dot{\theta_2}\dot{\theta_1}\frac{l_2l_1}{4}2\cos(\theta_1-\theta_2)\right) + \frac{1}{2}I_1\dot{\theta_1}^2 + \frac{1}{2}I_2\dot{\theta_2}^2 \tag{29}$$

Note that this is a relatively simple system as the kinetic energy is is decoupled in the COM coordinate r and the orientation coordinates, θ_1 and θ_2. In fact, the kinetic energy is invariant under rotations (where θ_1 and θ_2 are shifted by the same amount), and under translations. This point is noted as later when fluid is added to the system, it appears that the system may not be invariant when rotated.

In an attempt to relate a simple hinged system to a swimming organism, a potential constraint is added to keep the rods from colliding. In (30) and (31), two potentials based on the relative orientations of the rods are presented. (30) is a easily implementable potential, where (31) may better model some systems, as it represents the body not having an internal resistive spring force for some of its range of movement. Keeping the rods from colliding can be guaranteed by arbitrarily increasing the spring constant k.

$$V_B = k(\theta_1-\theta_2)^2 \tag{30}$$

$$V_B = \begin{cases} k(\theta_1-\theta_2-\theta_{tol})^2 & \theta_1-\theta_2 \geq \theta_{tol} \\ 0 & -\theta_{tol} < \theta_1-\theta_2 < \theta_{tol} \\ k(\theta_1-\theta_2+\theta_{tol})^2 & \theta_1-\theta_2 \leq -\theta_{tol} \end{cases} \tag{31}$$

The Lagrangian of the system is then:

$$L = K_B - V_B \tag{32}$$

$$L = \frac{(m_1+m_2)}{2}\dot{r}^T\dot{r} + \frac{1}{2}\frac{m_1m_2}{m_1+m_2}\left(\dot{\theta_1}^2\frac{l_1^2}{4} + \dot{\theta_2}^2\frac{l_2^2}{4} + \dot{\theta_2}\dot{\theta_1}\frac{l_2l_1}{4}2\cos(\theta_1-\theta_2)\right)\dots \tag{33}$$

$$+ \frac{1}{2}I_1\dot{\theta_1}^2 + \frac{1}{2}I_2\dot{\theta_2}^2 - k(\theta_1-\theta_2)^2 \tag{34}$$

To simulate and analyze the motion of the system the Lagrangian is converted into a Hamiltonian via the Legendre transform:

$$p = \frac{dL}{d\dot{q}} = \begin{bmatrix} \frac{dL}{d\dot{r}} \\ \frac{dL}{d\dot{\theta}_1} \\ \frac{dL}{d\dot{\theta}_2} \end{bmatrix} = \begin{bmatrix} (m_1+m_2)\dot{r} \\ \frac{m_1m_1}{2(m_1+m_2)}\left(\dot{\theta}_1\frac{l_1^2}{2} + \dot{\theta}_2\frac{l_1l_2}{2}\cos(\theta_1-\theta_2)\right) + I_1\dot{\theta}_1 \\ \frac{m_1m_1}{2(m_1+m_2)}\left(\dot{\theta}_2\frac{l_2^2}{2} + \dot{\theta}_1\frac{l_1l_2}{2}\cos(\theta_1-\theta_2)\right) + I_2\dot{\theta}_2 \end{bmatrix} \tag{35}$$

The Hamiltonian of the System is then:

$$H(q,p) = p^T\dot{q} - L(q,\dot{q}) \tag{36}$$

Mathematica is used to solve the Legendre transform for $\dot{q}$ in terms of p, and derive the Hamiltonian. Using Hamilton's Equations, it is easy to see that the Center of Mass of the Structure does not move:

$$\dot{r}_x = \frac{dH}{dp_{r_x}} = \frac{p_{r_x}}{m_1+m_2} \tag{37}$$

$$\dot{r}_y = \frac{dH}{dp_{r_y}} = \frac{p_{r_y}}{m_1+m_2} \tag{38}$$

$$\dot{p}_{r_x} = -\frac{dH}{dr_x} = 0 \tag{39}$$

$$\dot{p}_{r_y} = -\frac{dH}{dr_y} = 0 \tag{40}$$

This is a well known result, however the mathematical process is valuable, and serves as a guide for dealing with the more complicated case when fluid is present.

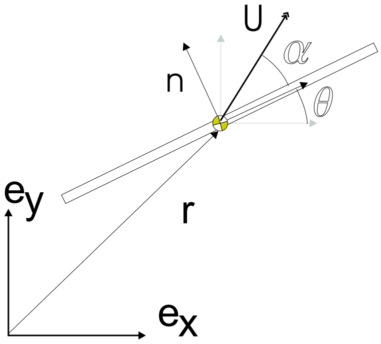

Figure 2: Kinetic Energy of a Flat Plate In Potential Flow

4 Coupled Body moving though ideal fluid

Now that the dynamics of a free simple hinged body are understood, the effect of an ideal fluid surrounding this body can be analyzed.

4.1 Potential flow around a flat plate

There are many books that discuss potential flow solutions for simple objects submerged in a flow. Lamb [1932] is a reference that has been referred to many times. In particular Kanso [2005] used Kirchhoff's form of the potential, where the velocity potential is expressed as a linear function of the submerged solid's body velocities. While many books discuss the velocity potential, Milne-Thomson [1950] is one of the few to explicitly write the Kinetic Energy of the fluid in terms of the body variables.

Kinetic Energy of a ellipsoid moving through an ideal fluid:

$$K_f = 1/2\rho\pi U^2(b^2\cos\alpha^2 + a^2\sin\alpha^2) + 1/16\rho\pi\dot{\theta}^2(a^2 - b^2)^2 \tag{41}$$

where the ellipse has axes $2a$, $2b$, moving with velocity U in the direction defined by the angle α w.r.t the body frame. This is depicted in Figure 2 in the case where the y-axis of the ellipsoid is set to zero, forming a flat plate.

It should be noted that when the ellipsoid is shrunk to a plate, the velocity of the fluid at the edges becomes infinite during rotational motion. The kinetic energy of the plate is shown in (42).

$$K_f = 1/2\rho\pi(U\sin\alpha)^2(L/2)^2 + 1/16\rho\pi\dot{\theta}^2(L/2)^2 \tag{42}$$

Note that the $U\sin\alpha$ term is just the body velocity in the vertical direction.

$$U\sin\alpha = \dot{r}\cdot n = [-\sin\theta, \cos\theta]\dot{r} \tag{43}$$

The kinetic energy written in terms of the spatial velocity coordinates is then as follows:

$$K_f = 1/2\rho\pi([-\sin\theta, \cos\theta]\dot{r})^2(L/2)^2 + 1/16\rho\pi\dot{\theta}^2(L/2)^2 \tag{44}$$

4.2 Lagrangian of single hinged system in potential flow

Combining the kinetic energy of the body from (3), with the kinetic energy of each plate individually in the flow, gives a first approximation to the true system. This is referred to as decoupled fluid interaction in Kanso [2005]. The Lagrangian for this system is:

$$L = \frac{(m_1+m_2)}{2}\dot{r}^T\dot{r} + \frac{1}{2}\frac{m_1m_2}{m_1+m_2}\left(\dot{\theta_1}^2\frac{l_1^2}{4} + \dot{\theta_2}^2\frac{l_2^2}{4} + \dot{\theta_2}\dot{\theta_1}\frac{l_2l_1}{4}2\cos(\theta_1-\theta_2)\right)\dots \tag{45}$$

$$+ \frac{1}{2}I_1\dot{\theta_1}^2 + \frac{1}{2}I_2\dot{\theta_2}^2 \tag{46}$$

$$+ 1/2\rho\pi([-\sin\theta_1, \cos\theta_1]\dot{r}_1)^2(L_1/2)^2 + 1/16\rho\pi\dot{\theta}_1^2(L_1/2)^2 \tag{47}$$

$$+ 1/2\rho\pi([-\sin\theta_2, \cos\theta_2]\dot{r}_2)^2(L_2/2)^2 + 1/16\rho\pi\dot{\theta_2}^2(L_2/2)^2 \tag{48}$$

This equation can then be written in terms of the center of mass $r = [r_x, r_y]^T$ using (4) and (7):

$$
\begin{aligned}
L &= \frac{1}{2}\left(I_1\dot\theta_1^2 + \dot\theta_2^2 I_2 + \left(\dot r_x^2 + \dot r_y^2\right)(m_1+m_2) + \frac{m_1 m_2\left(\dot\theta_1^2 L_1^2 + 2\dot\theta_1\dot\theta_2 L_2\cos(\theta_1-\theta_2)L_1 + \dot\theta_2^2 L_2^2\right)}{8(m_1+m_2)}\right) && (49)\\
&+ \frac{1}{64}\pi\rho\,(\,\dot\theta_1^2 L_1^2 + \dot\theta_2^2 L_2^2 && (50)\\
&+ \frac{2L_1^2\left(\dot\theta_1 L_1 m_2 + \dot\theta_2 L_2\cos(\theta_1-\theta_2)m_2 + 2\dot r_x(m_1+m_2)\sin(\theta_1) - 2\dot r_y(m_1+m_2)\cos(\theta_1)\right)^2}{(m_1+m_2)^2} && (51)\\
&+ \frac{2L_2^2\left(\dot\theta_2 L_2 m_1 + \dot\theta_1 L_1\cos(\theta_1-\theta_2)m_1 - 2\dot r_x(m_1+m_2)\sin(\theta_2) + 2\dot r_y(m_1+m_2)\cos(\theta_2)\right)^2}{(m_1+m_2)^2}\;) && (52)
\end{aligned}
$$

The point in showing this Lagrangian is to show the coupling between the COM velocity coordinate $\dot r$, and the orientation coordinates θ_1 and θ_2. What is interesting to point out, is that this coupling means that the Lagrangian is no longer invariant under a rotation of θ_1 and θ_2 by the same amount, if $L_1 \neq L_2$.

4.3 Zero Mobility with Symmetric Movements

This section will show how symmetrical oscillations of a single hinge system cause no net motion (over one period).

First note that as the Lagrangian does not contain any COM coordinate terms (r), that the partial derivative of the Lagrangian with respect to this coordinate is zero:

$$\frac{\partial L}{\partial r} = 0 \tag{53}$$

Using Euler-Lagrange Equations:

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial L}{\partial \dot r} = \frac{\partial L}{\partial r} = 0 && (54)\\
\Rightarrow \frac{\partial L}{\partial \dot r} = p_r = \text{constant} && (55)
\end{aligned}
$$

Computing $\frac{\partial L}{\partial \dot r}$ shows the following structure:

$$\frac{\partial L}{\partial \dot r} = A(\theta_1,\theta_2)\dot r + B(\theta_1,\theta_2,\dot\theta_1,\dot\theta_2) = \mathit{constant} \tag{56}$$

Setting this constant to zero, allows us to directly calculate $\dot r$:

$$\dot r = -A^{-1}(\theta_1,\theta_2)B(\theta_1,\theta_2,\dot\theta_1,\dot\theta_2) \tag{57}$$

Note that A will always be invertible, simply as the p_r momentum must always depend on the translational velocity. (This can also been seen directly from the $\frac{\partial L}{\partial \dot r}$ expression). Substituting in prescribed symmetric oscillations for θ_1 and θ_2 allows direct integration of $\dot r$.

Letting:

$$
\begin{aligned}
\theta_1 &= \theta_o + \alpha\cos\omega t && (58)\\
\theta_2 &= -\theta_o - \alpha\cos\omega t && (59)\\
& && (60)
\end{aligned}
$$

This kind of motion, shown in Figure 3 with θ_o set to zero, can represent either "Eel like motion" or "Clam like motion", by changing θ_0 appropriately.

Integrating $\dot r$ in with respect to time over one period shows that not net motion is achieved.

$$
\begin{aligned}
\dot r &= -A^{-1}(\theta_1,\theta_2)B(\theta_1,\theta_2,\dot\theta_1,\dot\theta_2) && (61)\\
r &= \int_0^{2\pi/\omega} -A^{-1}(\theta_o+\alpha\cos\omega t, -\theta_o-\alpha\cos\omega t)B(\theta_o+\alpha\cos\omega t, -\theta_o-\alpha\cos\omega t, -\omega\alpha\sin\omega t, \omega\alpha\sin\omega t) && (62)\\
&= 0 && (63)
\end{aligned}
$$

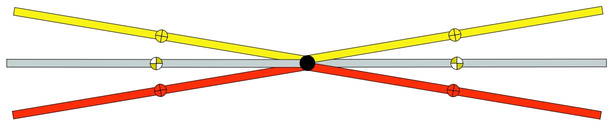

Figure 3: Symmetrical oscillatory motion of hinged body

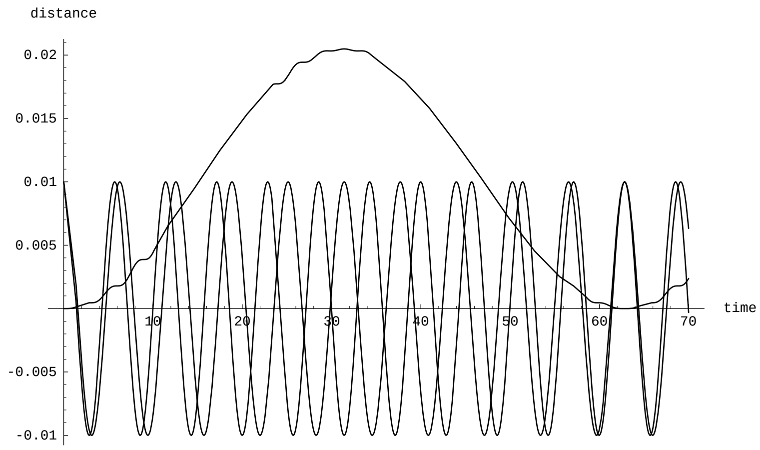

Figure 4: The Center of Mass coordinate r_x when excited by asymmetric oscillations. (θ_1 and θ_2 are the sine wave functions)

4.4 Asymmetrical Movements

As stated in Saffman [1967] asymmetrical movements are required to cause a persistent motion. A asymmetrical motion that can be made by a single hinge systems is to have θ_1 oscillate at one frequency, and then force θ_2 to oscillate at a fraction of the frequency. The author does not see how such motion could be actuated internally, but the result would indeed be asymmetrical in any inertial frame. It is interesting to note that the motion from this kind of actuation produces no net motion over the common multiple of both periods. However, if it is assumed that θ_1 and θ_2 can be independently specified, it appears as though net motion could be achieved through choosing different motion patterns. Figure 4 shows the x-translation of the center of mass over an interval of time. Indeed, if looked at on a short time scale, it appears as though net motion has been achieved.

5 Conclusions and Geometric Concepts- Justifications for Continuation

After presenting this material [05/31/2005], J. Marsden indicated that the fundamental reason why a single hinge cannot move is due to the one dimensional shape space, which cannot trace out an area, hence produces no geometric phase. Time constraints have prevented a full understanding of this, yet I believe that much is to be gained from showing the effects of geometric phase in conjunction with more rudimentary dynamical methods. In particular I there are several questions/actions that I wish to address in the near future:

- If θ_1 and θ_2 can be independently specified, are there motion patters that can create net movement? This could be argued by finding one such pattern, or rigorously showing that the shape space is still one dimensional.
- Determine an analytical method for adding coupled fluid dynamics.
- Understand the relation between geometric phase and dynamic phase, and show concretely how these change in the presence of a fluid, both for 1 hinge and for 2 hinge structures. Specifically address the question, "why does a 2 hinge structure achieve net translational motion (geometric phase) only in the presence of a fluid, while only being able to produce a net rotational motion in free space?"

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