Titan Mission Trajectory Design

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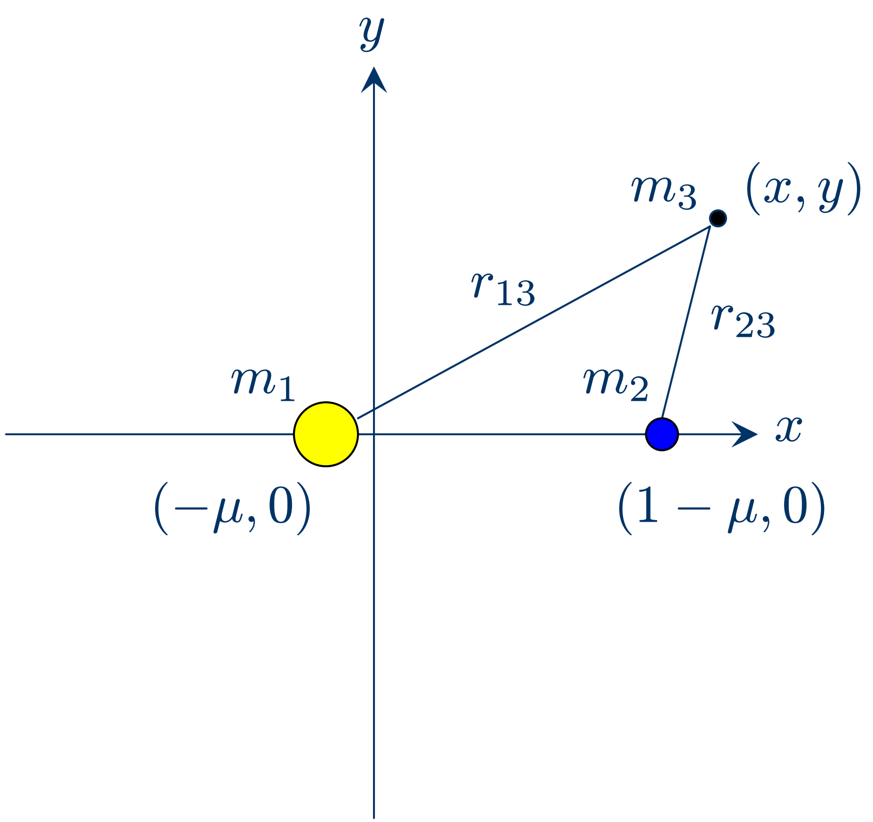
Outline

- The circular restricted three-body problem (CR3BP)
- Invariant manifolds in the CR3BP
- Discrete Mechanics and Optimal Control (DMOC)
- Application:
  - Shoot the Moon
  - Saturnian moon tour

The Circular Restricted Three-Body Problem (CR3BP)



$$
\begin{aligned}
m_1 &= 1-\mu \\
m_2 &= \mu \\
G &= 1 \\
\omega &= 1
\end{aligned}
$$

The Circular Restricted Three-Body Problem (CR3BP)

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}$$

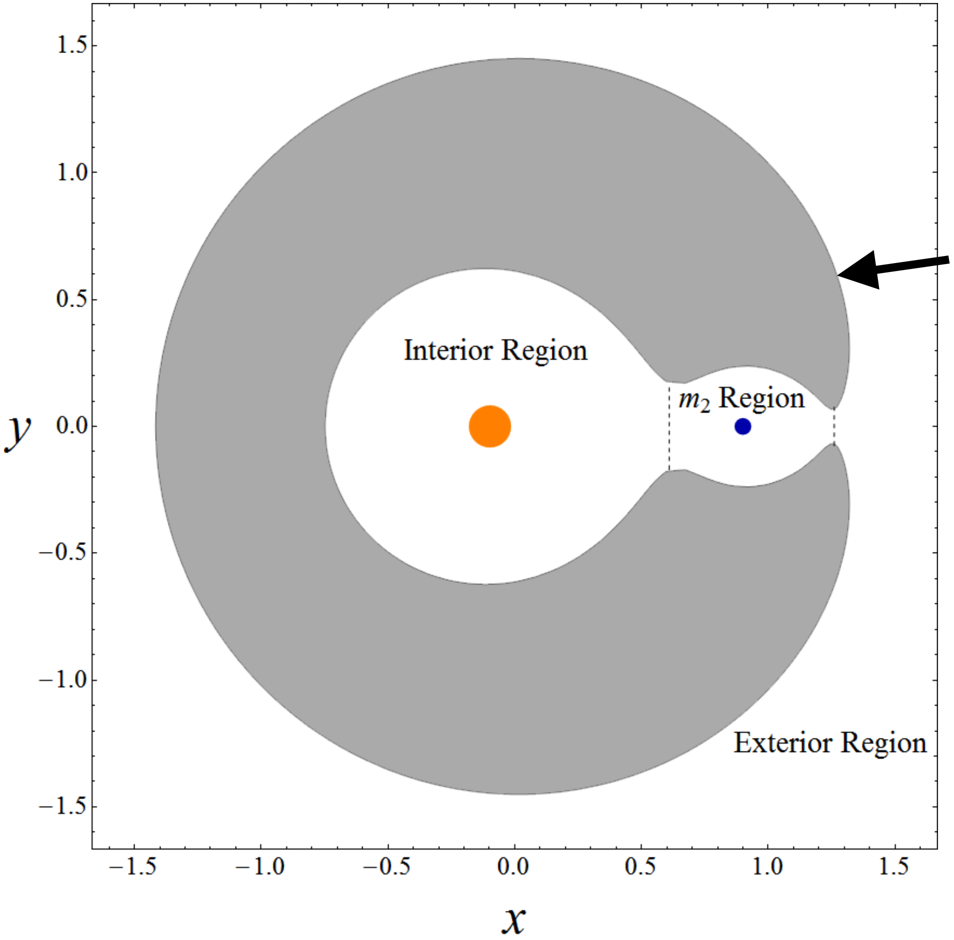
$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y}$$

$$\Omega(x, y) = \frac{x^2 + y^2}{2} + \frac{1 - \mu}{\sqrt{(x + \mu)^2 + y^2}} + \frac{\mu}{\sqrt{(x - 1 + \mu)^2 + y^2}}$$

Constant of motion:

$$E = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \Omega(x, y)$$

The Circular Restricted Three-Body Problem (CR3BP)

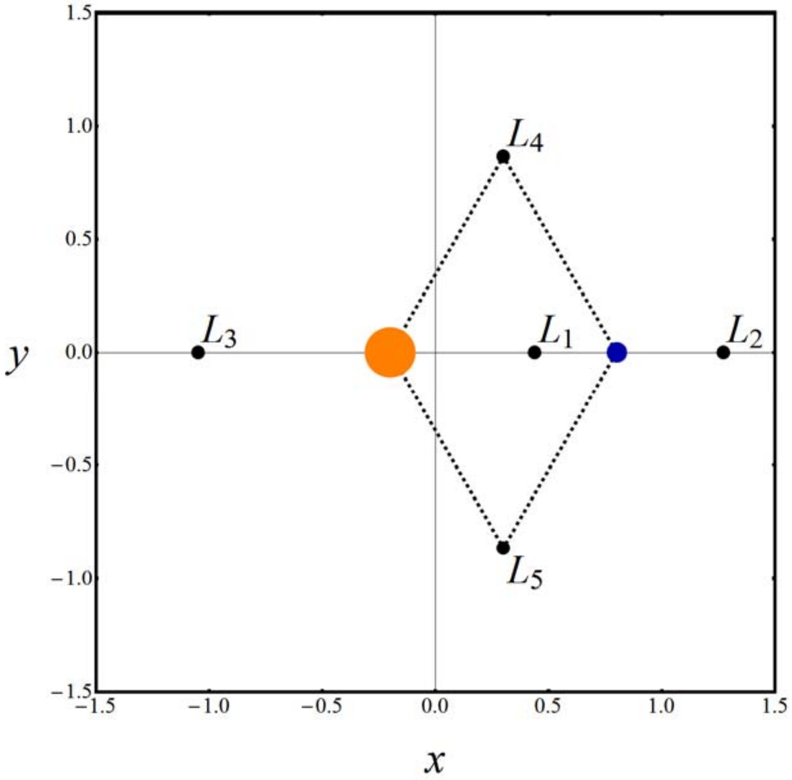


Energetically "forbidden" regions

The Circular Restricted Three-Body Problem (CR3BP)

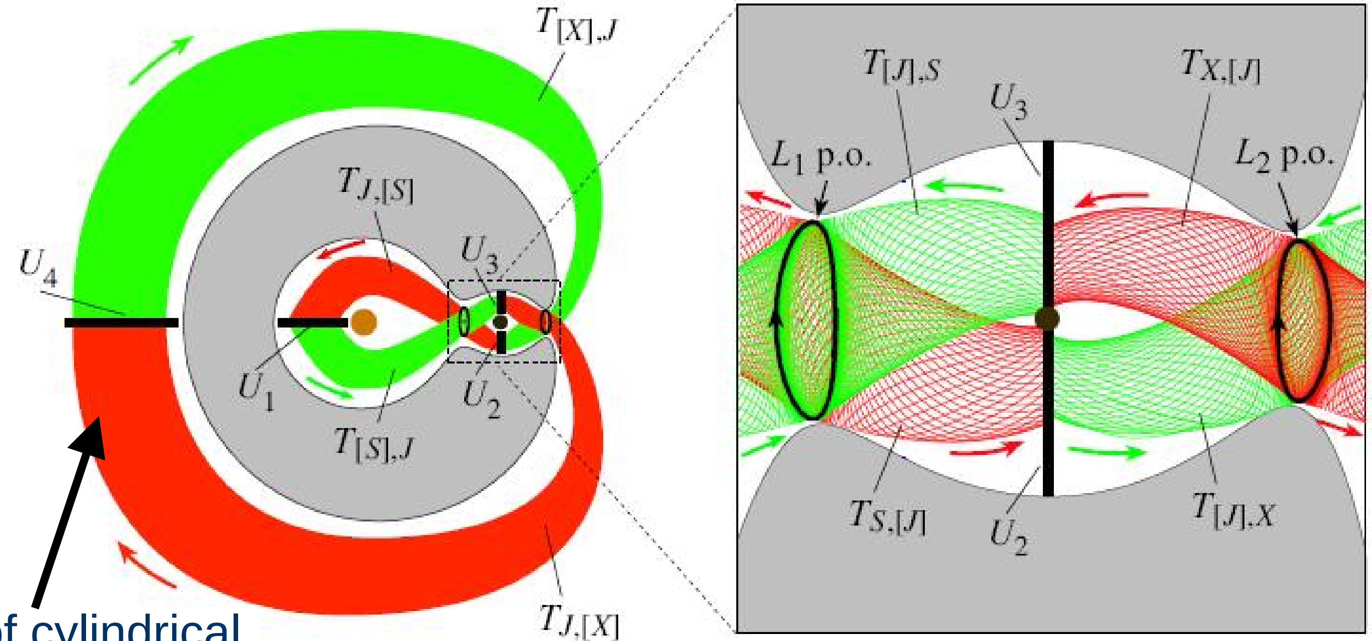
Lagrange points L_i, $i = 1, 2, 3, 4, 5$

Note the positions of
L_1 and L_2



Invariant Manifolds

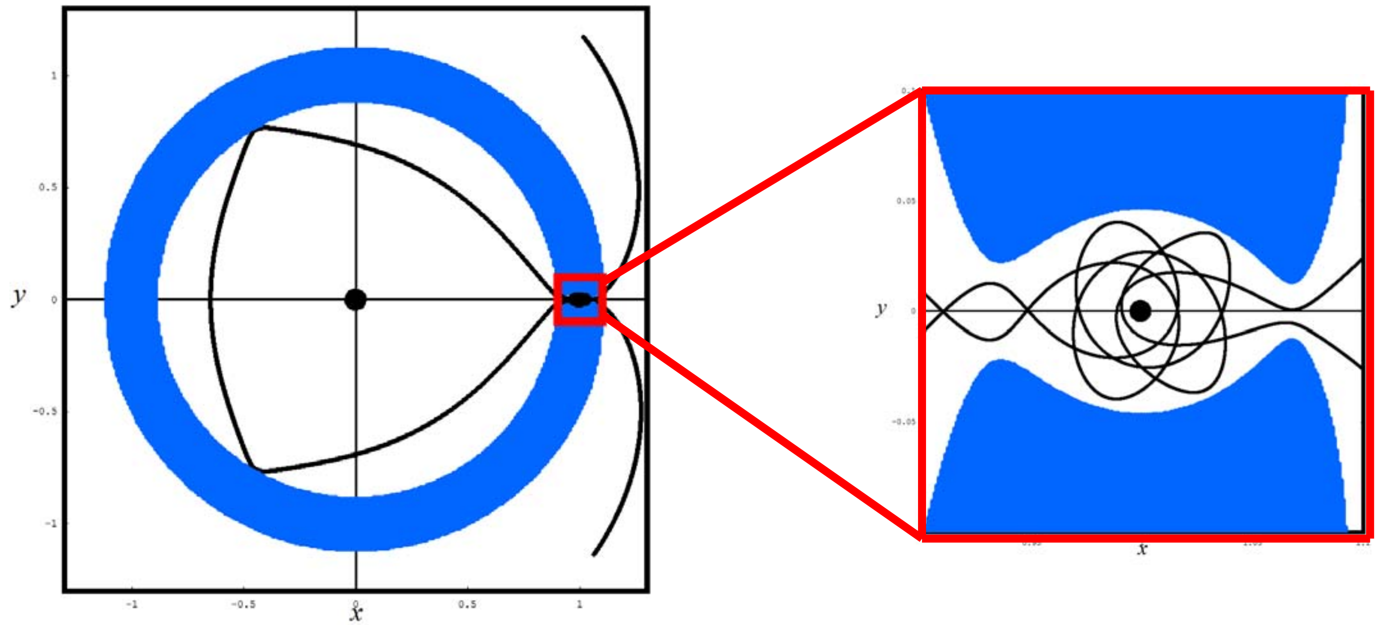
Stable and unstable manifolds of the L_1 and L_2 Lyapunov orbits *belonging to a particular energy surface*



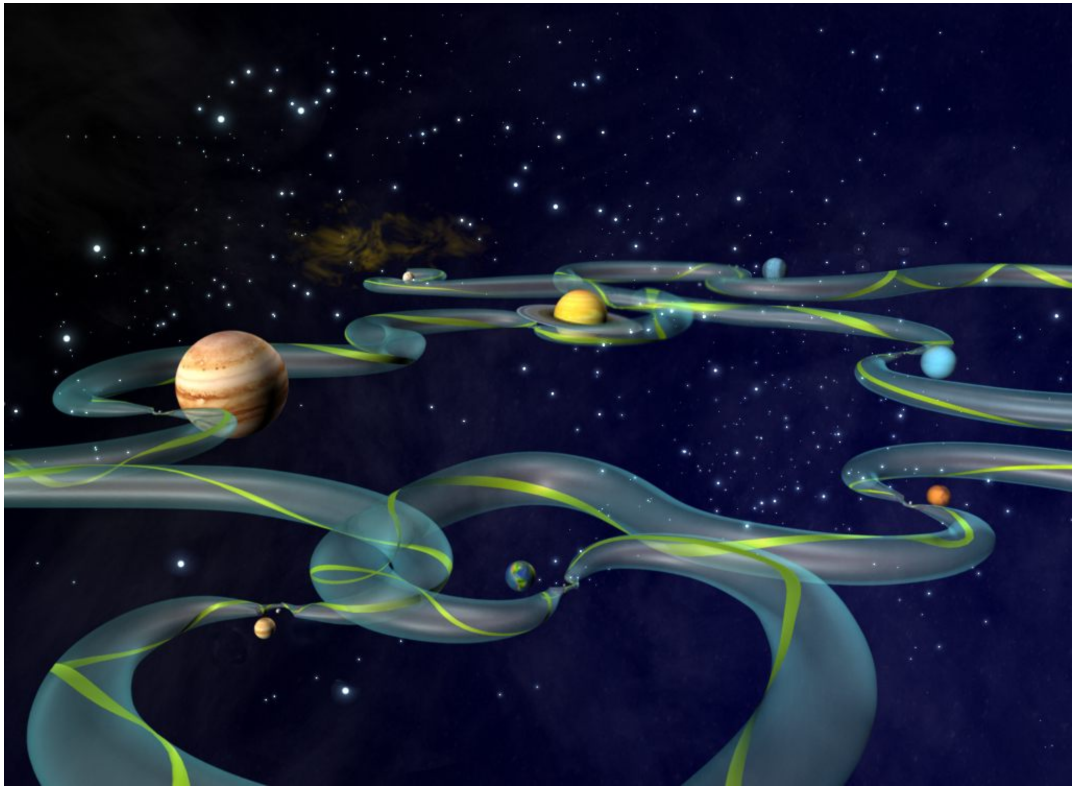
Projection of cylindrical tubes onto position space

Ross, S.D., “Cylindrical Manifolds and Tube Dynamics in the Restricted Three-Body Problem" (PhD thesis, California Institute of Technology, 2004), 109.

Orbits with Prescribed Itineraries

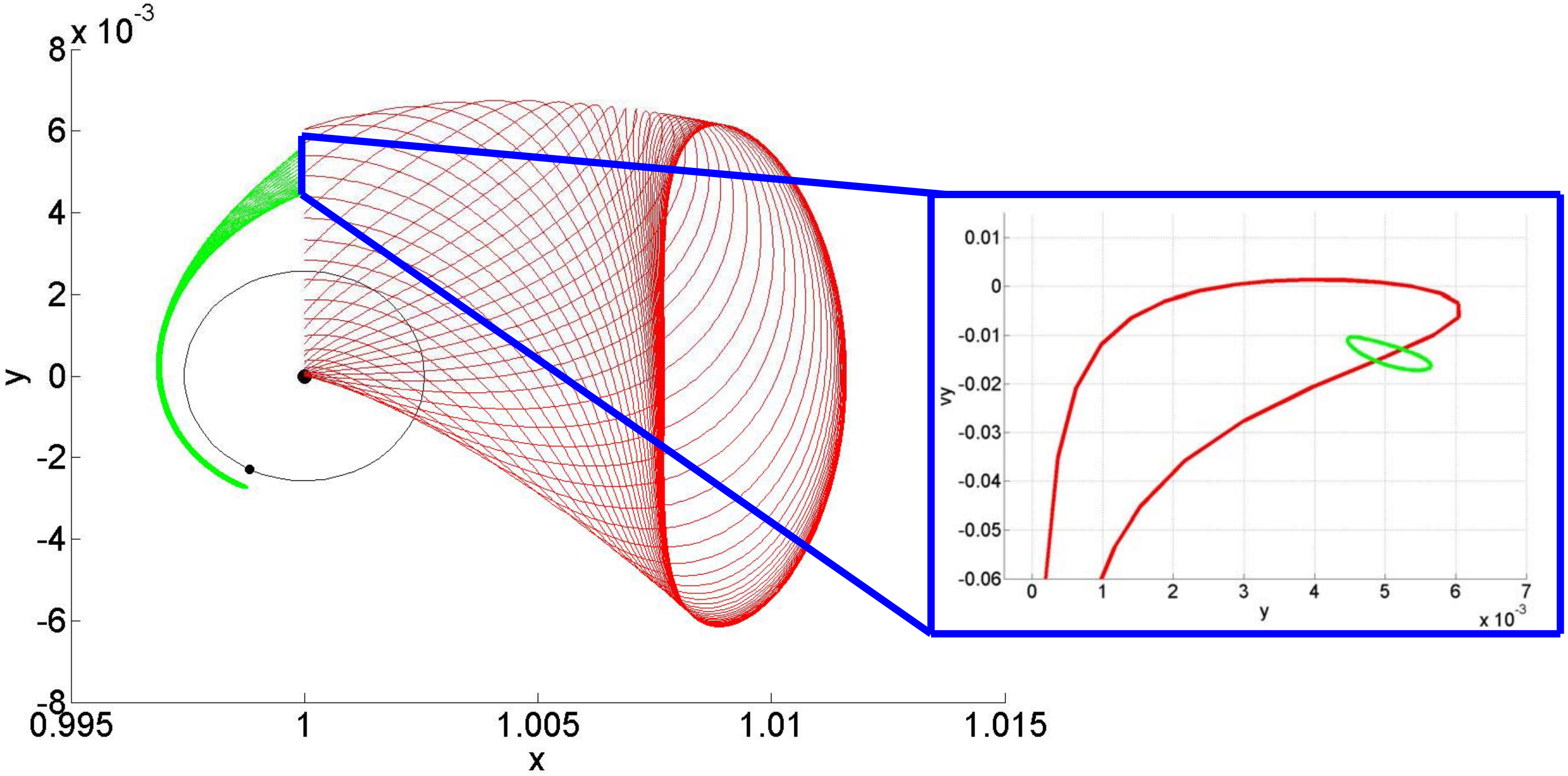


Interplanetary Transport Network

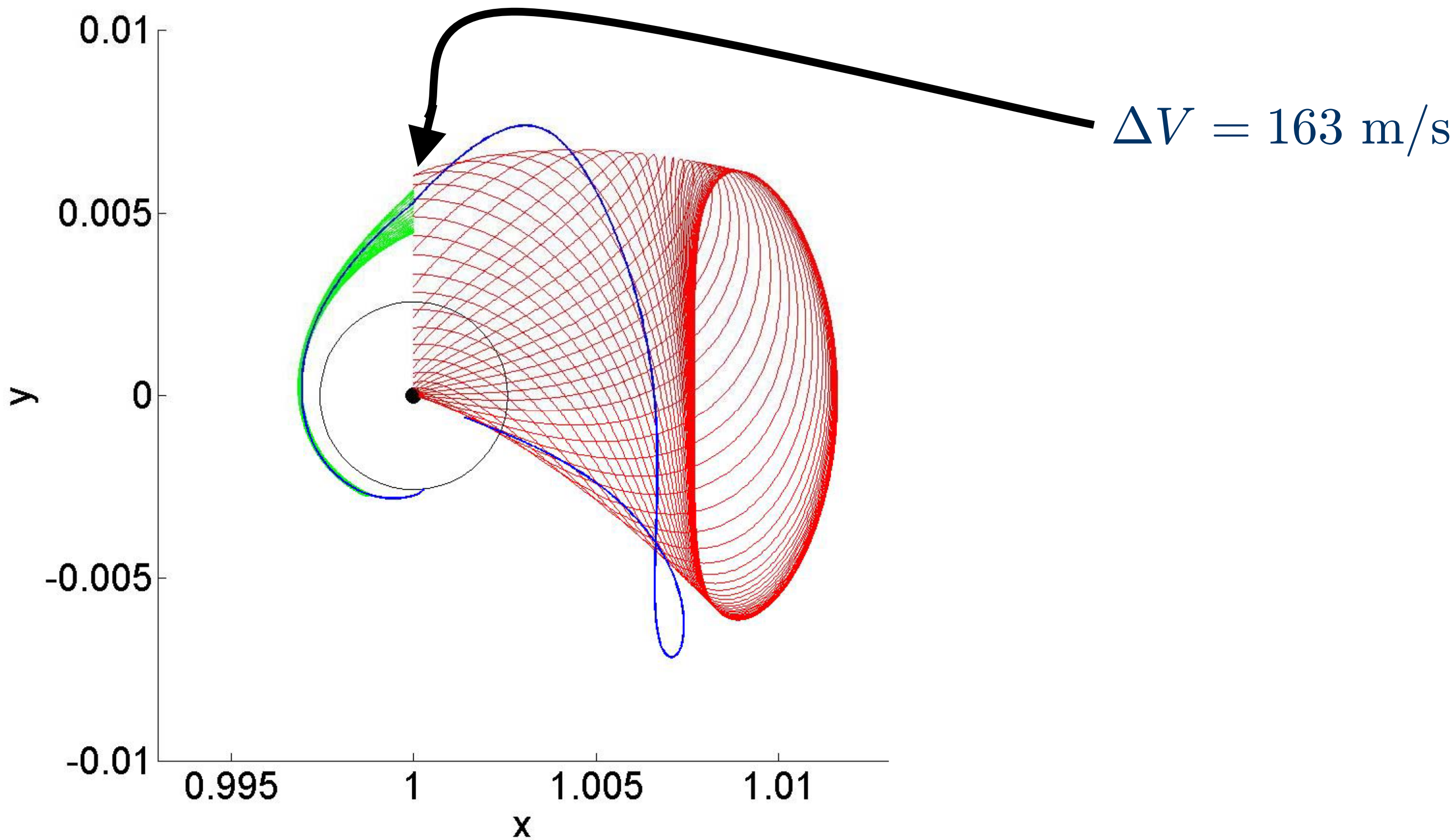


http://www.jpl.nasa.gov/releases/2002/release_2002_147.html

Shoot the Moon



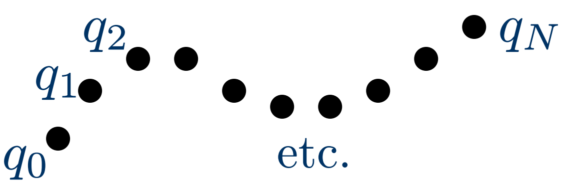
Shoot the Moon



DMOC

Minimize:

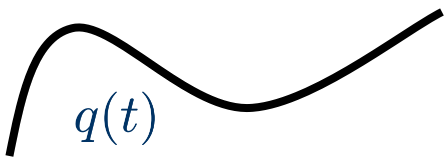
$$\Delta V = \sum_{k=0}^{N-1} ||f_k|| \, \Delta t$$

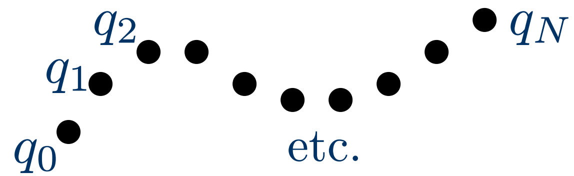


Subject to:

$$D_2 L_d(q_{k-1}, q_k, \Delta t) + D_1 L_d(q_k, q_{k+1}, \Delta t) + \underbrace{f_{k-1}^{+} + f_k^{-}}_{\text{Thrust}} = 0$$

Variational Integrators





Continuous:
Extremize the integral

$$\int_0^T L(q, \dot{q})\,\mathrm{d}t$$

Arrive at the
Euler-Lagrange equations

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}} = 0$$

Discrete:
Extremize the sum

$$\sum_{k=0}^{N-1} L_d(q_k, q_{k+1}, \Delta t)$$

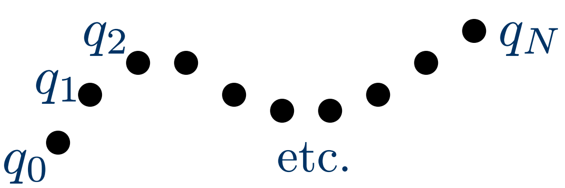
Arrive at the discrete
Euler-Lagrange equations

$$D_2 L_d(q_{k-1}, q_k, \Delta t) + D_1 L_d(q_k, q_{k+1}, \Delta t) = 0$$

DMOC

Minimize:

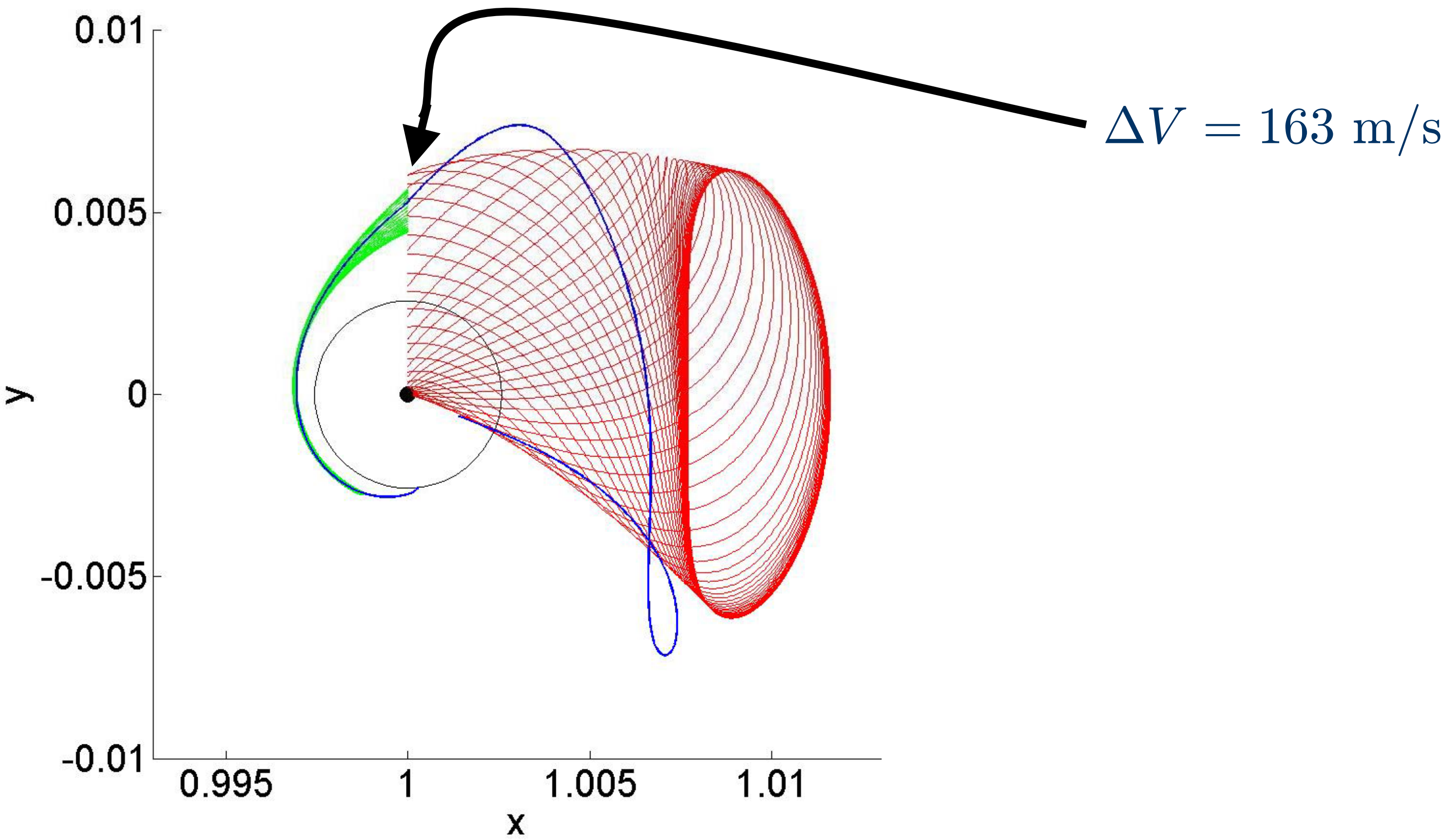
$$\Delta V = \sum_{k=0}^{N-1} ||f_k|| \, \Delta t$$



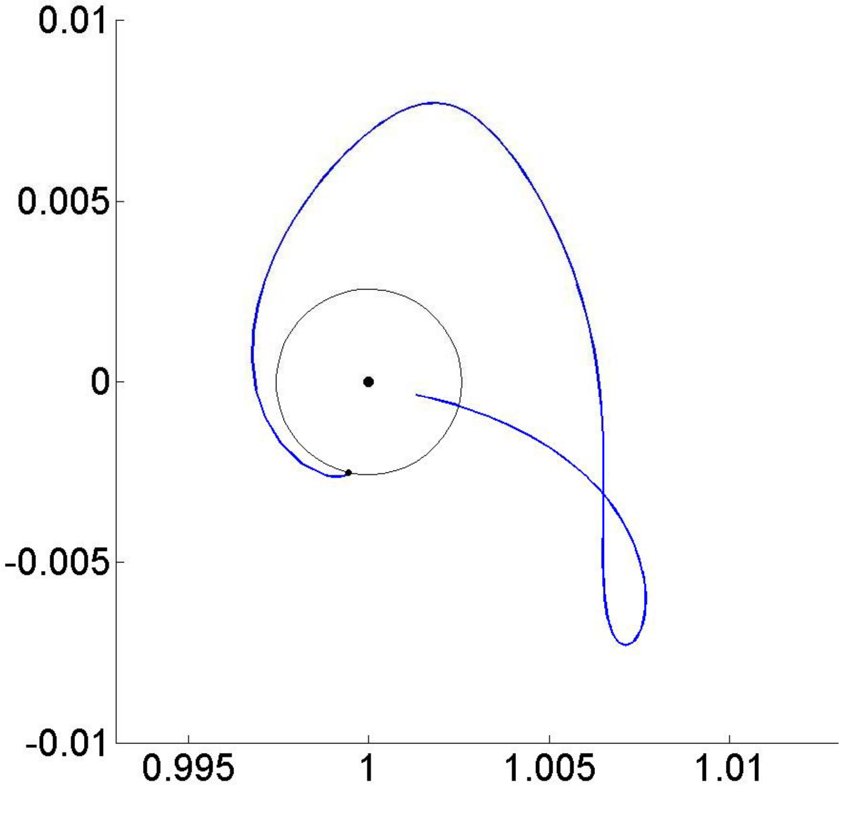
Subject to:

$$D_2 L_d(q_{k-1}, q_k, \Delta t) + D_1 L_d(q_k, q_{k+1}, \Delta t) + \underbrace{f_{k-1}^{+} + f_k^{-}}_{\text{Thrust}} = 0$$

Shoot the Moon



Shoot the Moon

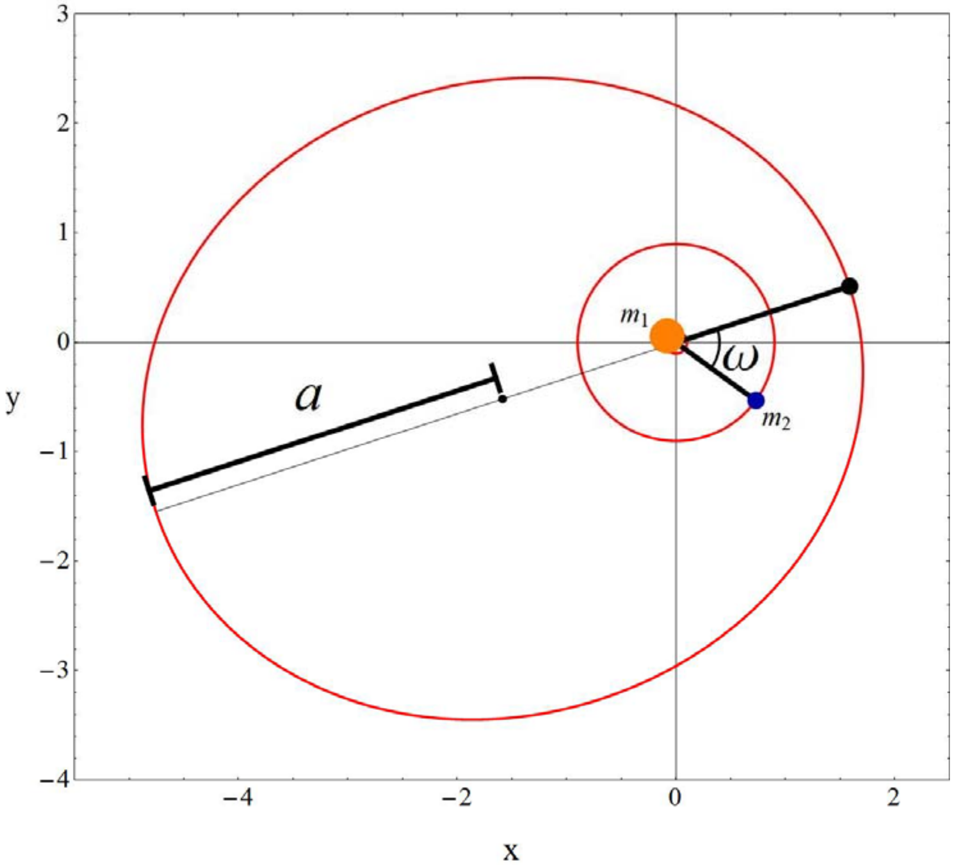


$$\Delta V = 17 \text{ m/s}$$

Saturnian Moon Tour

- The invariant manifolds of the various Saturn-Moon-spacecraft three-body systems do not intersect.

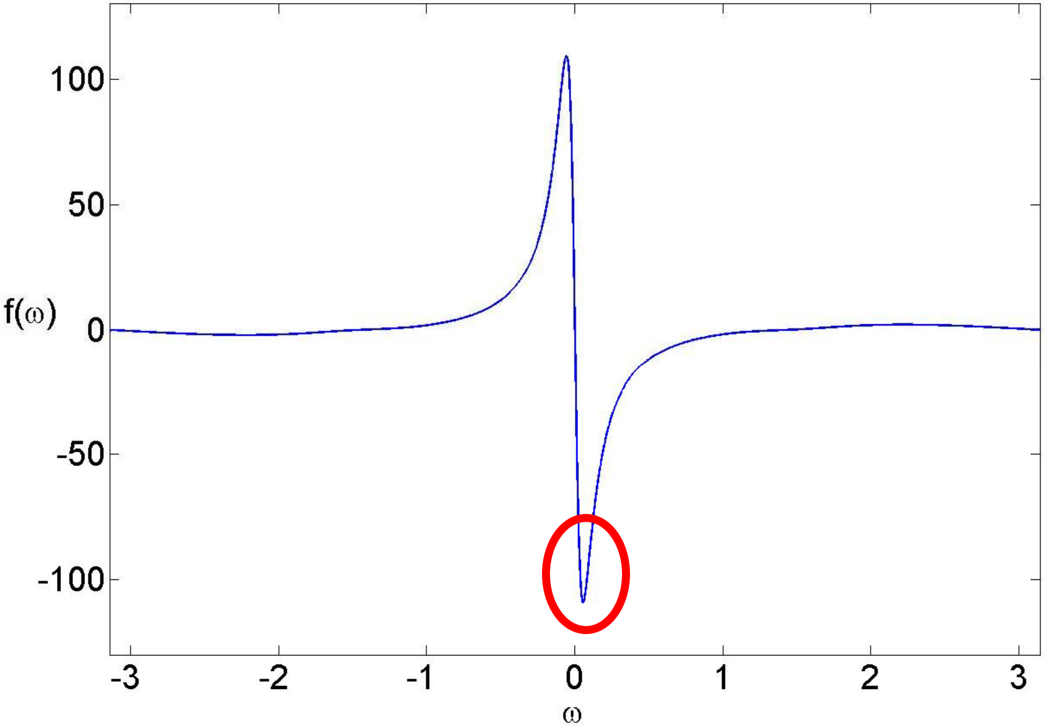
Saturnian Moon Tour



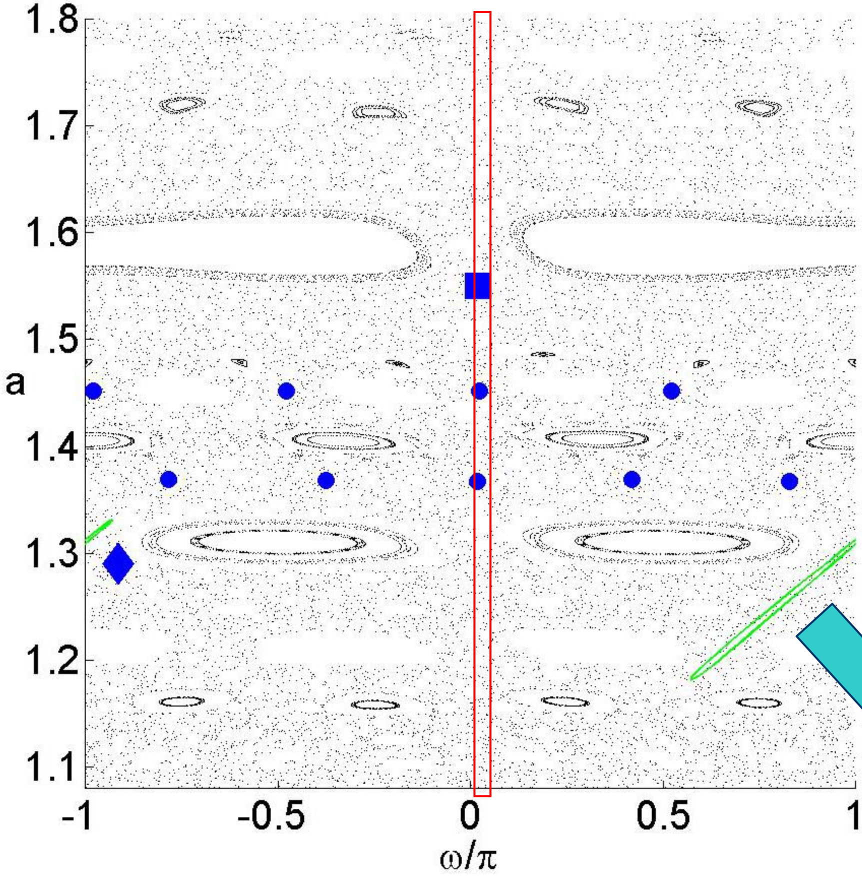
$$K = -\frac{1}{2a}$$

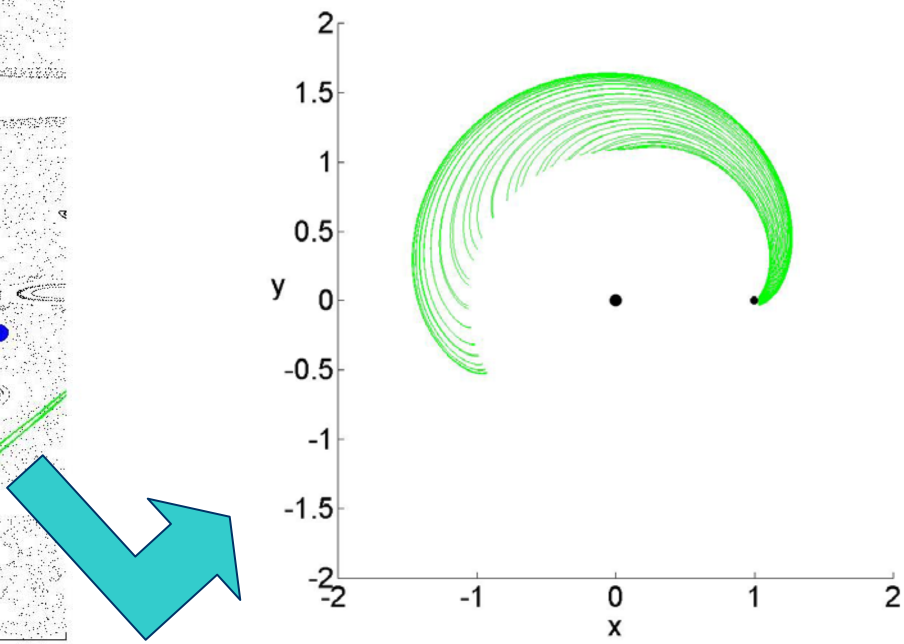
Saturnian Moon Tour

$$\begin{pmatrix} \omega_{n+1} \\ K_{n+1} \end{pmatrix} = \begin{pmatrix} \omega_n - 2\pi(-2K_{n+1})^{-3/2} \mod 2\pi \\ K_n + \mu f(\omega_n) \end{pmatrix}$$

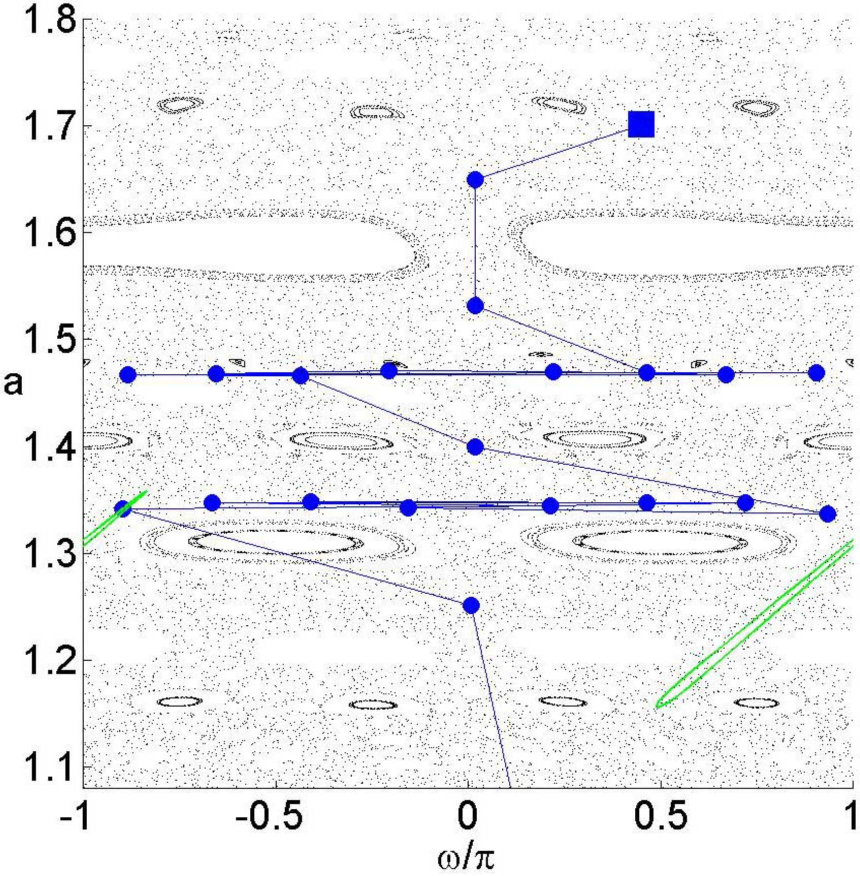


Saturnian Moon Tour



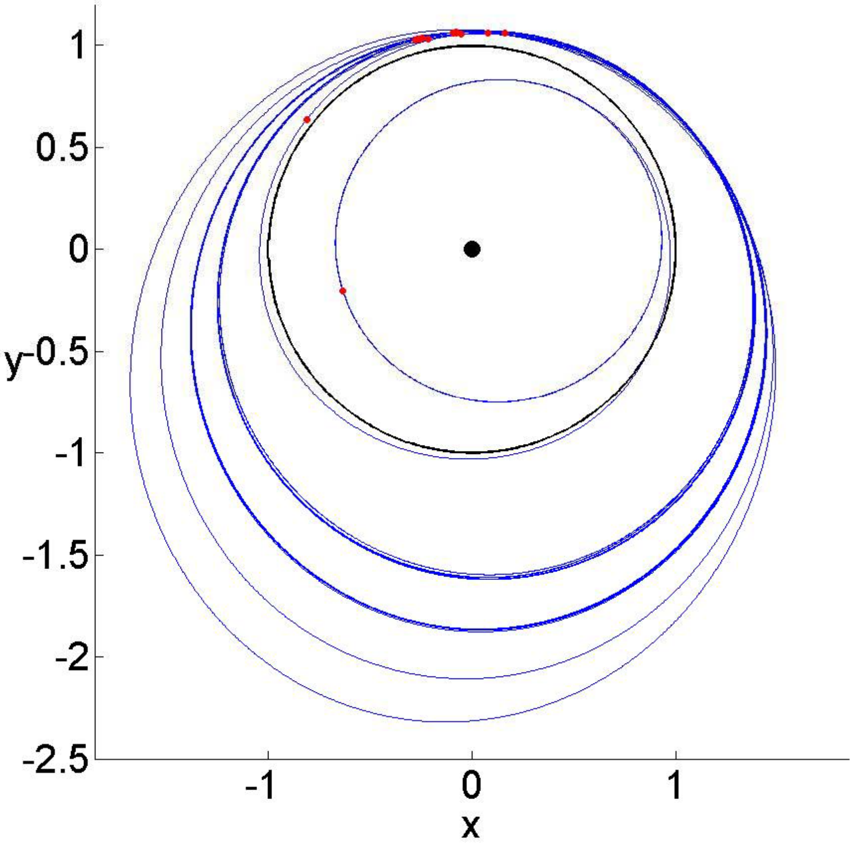


Saturnian Moon Tour



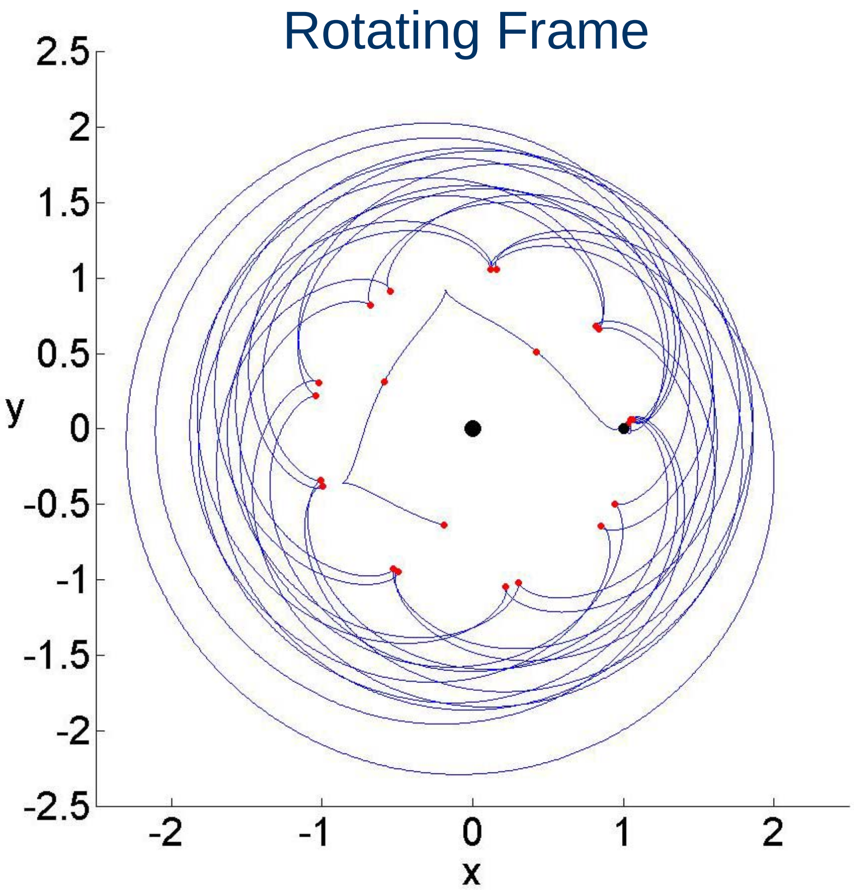
$$\Delta V = 13 \text{ m/s}$$

Saturnian Moon Tour



$$\Delta V = 13 \text{ m/s}$$

Saturnian Moon Tour



$$\Delta V = 13 \text{ m/s}$$

Further Study

- Continuation to inner moons of Saturn
- Comparison with standard trajectory design techniques
- Analysis of trade-off between Delta-V vs. time-of-flight

Acknowledgments

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