

## Index

- A**
- ABC flow, 39
  - Abelian, 271
    - Lie group, 278
  - Abelian Lie group
    - structure theorem, 279
  - abstract
    - Legendre transform, 185
  - abstract wave equation, 113
  - acceleration, 62
  - action, 185, 189, 190, 202, 229
    - canonical, 365
    - coadjoint, 311, 443
    - functional, 226
    - Hamiltonian, 379
    - integral, 62, 214, 229
    - left, 173, 309
    - Lie algebra, 317, 367
      - lifted, 406
      - proper, 311
      - right, 173, 309
      - transitive, 310
  - transitive Hamiltonian, 463
  - $\text{Ad}^*$  invariance, 462
  - Ad operator, 275
- 
- ad operator, 314
  - adjoint, 172
    - operator, 275
    - representation, 310, 443
  - adjoint action
    - infinitesimal generator, 313
  - affine group, 444, 452, 458
  - algebra, 143
    - Galilean, 411
    - isotropy, 313
    - Lie, 9, 85, 143, 161, 270
    - Poisson, 335
    - stabilizer, 313
    - symmetry, 313
    - symplectic, 294
    - Virasoro, 437
  - algebraic approach to the Lie derivative, 138
    - algebraic definition
      - momentum map, 370
    - algebraic geometry, 59
    - algebraic sectors, 102
  - almost
    - complex manifold, 153
    - complex structure, 153

- alternating, 129
- alternation operator, 130
- amended potential, 252, 261, 263, 516
- annoeba propulsion, 51
- analytical solution
  - free rigid-body problem, 498
- angle
  - Euler, 7, 493
- angular
  - momenta, 118
  - momentum, 7, 115, 119, 195, 265, 368, 373, 381, 390, 411
  - velocity, 250, 484
- angular momentum
  - in body frame, 486
  - in space, 487
  - spatial, 488
- angular momentum for
  - linear elastodynamics, 119
  - Schrödinger equation, 118
- wave equation, 117
- angular momentum sphere, 55
- angular velocity
  - body, 6, 432
  - convective, 485
  - precession, 499
  - spatial, 485
- anholonomy, 51
- annihilator, 451
- anomaly, 380
- anticommutative property of the wedge product, 131
- antihomomorphism, 333
- antisymmetry, 11
  - area, 145
  - argument
    - convexity, 37
- Arnold
  - diffusion, 99
  - method, 39
- associated linear map, 63
- associative property of the wedge product, 131

- astatic load, 394
  - atlas, 122
    - maximal, 122
  - attitude, 55
    - control, 58
    - matrix, 503
  - augmented Hamiltonian, 40
  - automatic smoothness, 278
  - automorphism
    - inner, 275, 310
    - Poisson, 366
  - autoparallel, 199
  - averaging, 59, 255, 316
    - method, 234
    - principle, 259
    - theorem, 255
  - averaging connection, 51
  - axis
    - intermediate, 506
- B**
- ball
    - in rotating hoop, 210
    - in slowly rotating planar hoop, 256
  - Banach–Lie
    - group, 267, 268
  - base integral curve, 186, 189
  - basis
    - canonical, 68
  - beam, 98
  - Benjamin–Ono equation, 98
  - Berry–Hannay phase, 256
  - beyond all orders, 100
  - Bianchi IX models, 99
  - bifurcation, 354
    - bifurcation, 43, 90, 511
    - Hamiltonian, 41, 59
    - Hamiltonian Krein–Hopf, 45
    - Hamiltonian pitchfork, 43, 90
    - Hamiltonian saddle node, 101
    - Poincaré–Andronov–Hopf, 30
    - resonance, 45
    - subharmonic, 100
  - bifurcation problems, 101

- bilinear form, 63
- pull-back, 71
- push-forward, 72
- bilinear property of the wedge product, 131
  - uct, 131
- block diagonal, 40
- body
  - angular velocity, 6, 432
  - coordinate system, 483
  - coordinates, 482
  - ellipsoid of inertia, 491
  - pseudo-rigid, 40
  - representation, 489
  - rigid, 6, 265
  - velocity, 483
- body angular momentum, 13, 17
- body frame
  - angular momentum, 486
  - principal axis, 486
- Boltzmann, 9
- bordered Hessian, 235
- bracket
  - canonical, 83
  - charged fluid, 23
  - commutator, 271, 272
  - compressible fluid, 27
  - computing, 275
  - cubic, 332
  - frozen Lie–Poisson, 330
  - heavy top, 17
  - ideal fluid, 20, 329
  - Jacobi–Lie, 20, 84, 138
  - KdV, 117, 120, 330
  - Lie, 270
  - Lie–Poisson, 11, 327, 328, 416
  - Maxwell–Poisson, 26
  - Maxwell–Vlasov, 22, 28
  - MHD, 23
  - Pauli–Born–Infeld, 26
  - Poisson, 65, 82, 115, 160, 168, 234
  - Poisson–Vlasov, 26, 329
  - quadratic, 331
  - rigid-body, 329, 352, 487
  - Schouten, 353

- Schrödinger, 116
  - symplectic, 328
  - Toda lattice, 330
  - bracket of generators, 316
  - breaking symmetry, 43
  - broken symmetry, 18, 91
  - buckled forced beam, 98
  - bundle
    - colangent, 5, 128, 165
    - pseudo-sphere, 197
    - tangent, 3, 123
    - unit circle, 9
- C**
- C, Neumann problem, 242
  - calculus
    - fundamental theorem, 141
    - integral theorems, 141
  - canonical, 150, 299, 339
    - action, 365
    - basis, 68
    - bracket, 83
    - coordinates, 149, 274
    - diffeomorphism, 151
    - form, 66, 67
    - Hamilton equation, 106
    - map, 13, 71
    - momentum, 375
    - momentum map, 403
    - one-form, 166, 227, 386
    - rearrangement, 448
    - symplectic form, 66, 165
    - transformation, 69, 70, 210
    - two-form, 165
  - canonical coordinate
    - Hamilton’s equation, 76
  - canonical equation
    - Hamilton’s, 63
  - canonical transformation
    - group, 26, 266
  - cart
    - planar pendulum, 263
  - Cartan
    - connection, 258
    - magic formula, 139, 144, 356

Cartan magic formula, 139  
 Cartan-Hammy-Berry  
 connection, 259  
 Casimir, 8  
 function, 8, 21, 35, 335, 396,  
 414, 462, 464, 488, 507,  
 514  
 Poisson-Vlasov, 336  
 rigid-body, 35, 335  
 two-dimensional fluid, 35  
 Casimir function  
 momentum map, 413  
 cat  
 falling, 53  
 catastrophe  
 cusp, 92  
 caustic point, 218  
 Cayley transform, 79, 80  
 Cayley-Klein parameter, 302, 490  
 celestial mechanics, 59  
 center, 335, 462  
 center of mass, 482  
 central extension, 283, 400, 407,  
 437  
 centralizer, 462  
 centrifugal  
 force, 248, 251  
 potential, 252  
 chain rule, 124  
 change of variable, 142  
 chaos, 47, 498  
 Hamiltonian, 59  
 chaotic  
 dynamics, 30, 100  
 solution, 506  
 characteristic  
 distribution, 344  
 field, 344  
 charge, 23, 201, 266  
 density, 24  
 fluid bracket, 23  
 particle, 200  
 charge-driven Maxwell equations,  
 24  
 chart, 121

compatible, 122  
 coordinates, 121  
 Lie group, 268  
 projective, 157  
 Cherry's example, 34  
 Christoffel  
 map, 196  
 symbol, 196  
 circle  
 unit, 278  
 circulation theorem  
 Kelvin's, 266  
 Chirant's theorem, 407  
 class  
 cohomology, 399  
 conjugacy, 306, 310  
 similarity, 310  
 classical  
 commutation relation, 14  
 field theory, 105  
 Hamilton's equation, 76  
 Lie group, 283  
 Routhian, 261  
 Stokes' theorem, 142  
 classical group  
 commutativity, 319  
 classical mechanics, 117  
 invariance group, 299  
 Clebsch, 6  
 potential, 6, 21, 23  
 Clebsch-Monge representation, 22  
 closed  
 $k$ -form, 132  
 locally, 125  
 subgroup, 280  
 coadjoint  
 action, 311, 443  
 isotropy, 323, 451  
 orbit, 14, 443, 488  
 representation, 311  
 coadjoint orbit, 488  
 example, 444  
 symplectic structure, 453  
 tangent vector, 451  
 theorem, 453

coadjoint orbit compatibility  
 Lie-Poisson, 459  
 coadjoint orbit covering theorem,  
 369, 463  
 coadjoint orbits that are not sub-  
 manifolds, 449  
 cocycle, 397  
 cocycle, 397  
 Gelfand-Fuchs, 437  
 coefficient  
 connection, 262  
 curvature, 262  
 cohomology, 397  
 class, 399  
 de Rham, 142  
 second, 397  
 coisotropic, 74  
 immersion, 160  
 collective  
 function, 404  
 collective Hamiltonian theorem, 404  
 collisionless Boltzmann equation,  
 24  
 collisionless plasma, 23  
 commutation relation, 378, 388  
 classical, 14  
 commutator, 117  
 bracket, 271, 272  
 operator, 86  
 commutator lemma  
 momentum, 384  
 commute  
 Poisson, 8, 35, 83  
 compact group  
 momentum map, 380  
 compatible, 121  
 charts, 122  
 complex  
 Hilbert space, 78, 82, 154  
 manifold, 153  
 $n$ -space, 68  
 polar decomposition, 300  
 projective space, 154  
 structure, 152, 246  
 variable, 64

complex form  
 Hamilton's equation, 64  
 complex general linear group, 299  
 complex Hilbert space  
 projectivized, 52  
 complex manifold  
 almost, 153  
 complex nonlinear Schrödinger equa-  
 tion, 115  
 complex polar decomposition  
 theorem, 326  
 complex special linear group, 300  
 complex structure  
 almost, 153  
 complex variable, 49  
 components, 63, 123  
 compressible  
 flow, 26  
 fluid, 266  
 compressible fluid  
 bracket, 27  
 Hamiltonian, 27  
 computing brackets, 275  
 condition  
 Lagrangian, 186  
 resonant, 98  
 configuration, 2, 432, 481  
 fluid, 19  
 manifold, 181  
 reference, 432, 481  
 space, 148, 265, 482  
 conjugacy class, 306, 310  
 conjugate  
 momentum, 4, 252  
 point, 198  
 variable, 68  
 conjugation, 275  
 connection, 201, 203  
 Cartan, 258  
 Cartan-Hammy-Berry, 259  
 coefficient, 262  
 Hammy-Berry, 53  
 one-form, 201  
 connectivity  
 classical group, 319

- conservation
  - energy, 5, 76, 158
  - momentum, 233
  - momentum map, 372
  - total angular momentum, 8
- conservation laws, 3
- conserved quantity, 115
- constant
  - structure, 10
- constant of the motion, 14
- constrained motion
  - potential field, 238
- constraint
  - Dirac, 242
  - Dirac theory, 186
  - force, 88, 241
  - holonomic, 219, 236, 246
  - Ln, 21, 60, 431
  - rolling, 208
- construction
  - Poincaré, 57
  - Routhian, 237
- contact form, 225
- continuum mechanics, 107
- contraction, 131
- contragredient representation, 311
- contravariant antisymmetric
  - $g$ -tensor, 353
- control, 49, 58, 60
- attitude, 58
- convective
  - angular velocity, 485
  - velocity, 483
- convex neighborhood, 198
- convexity argument, 37
- coordinate representation
  - Poisson map, 342
- coordinate system
  - body, 483
  - spatial, 483
- coordinates
  - body, 482
  - canonical, 149, 274
  - chart, 121
  - guiding center, 407

- Jacobi–Haretu, 167
  - material, 481
  - representation, 341
  - space, 13
  - spatial, 481
  - spherical, 122, 128
- Corolis**
- force, 248, 251
  - term, 262, 263
- cosymplectic structure, 341**
- cotangent**
- bundle, 5, 128, 165
  - Hf, 170, 386, 391
- cotangent bundle**
- momentum map, 383
  - reduction theorem, 177
- coupled rigid bodies, 46**
- planar, 54
- covariant derivative, 198, 203**
- Levi-Civita, 199
- covering map, 463**
- crank, 54**
- criterion**
- Lagrange–Dirichlet, 36
  - Routh–Hurwitz, 43
- critical**
- point, 40, 125
  - rotation rate, 90
  - value, 125
- critical action**
- Hamilton’s principle, 181
  - principle, 181, 222, 223
- cross product, 135, 290**
- cubic bracket, 332**
- curl, 135**
- current, 23**
- curvature, 201**
- coefficient, 262
- curve**
- base integral, 186, 189
  - integral, 127
- cusp catastrophe, 92**
- cyclic variable, 215, 260**
- cylinder, 148
  - elliptic, 514
- cylindrical coordinates**
- parabolic, 516

**D**

- damped, forced sine–Gordon equation, 98
  - Darboux theorem, 148
    - relative, 150  - de Rham cohomology, 142
  - decks, 463
  - decomposition
    - orthogonal, 21
  - definite
    - positive, 284, 300
  - definition
    - integral, 141
  - deformation
    - Lie algebra, 471
  - degree, 354
  - density, 67, 106
  - charge, 24
  - derivation, 15
  - identity, 160
  - property, 355
- derivative, 124**
- covariant, 198, 203
  - directional, 83, 128, 137
  - exterior, 131
  - fiber, 183
  - functional, 5, 11
  - intrinsic, 359
  - Lie, 83, 137, 138
  - variational, 5
- description**
- Eulerian, 19
  - Lagrangian, 19
- determinant, 285**
- map, 277
- diagonal**
- block, 40
- diffeomorphism, 151**
- canonical, 151
  - equivariant, 315
  - group, 266, 309, 323
  - local, 124

differentiable, 124

map, 124

 $n$ -manifold, 122

structure, 122

differential, 128

differential equation

partial, 140

differential form of degree  $k$ , 129

differentiation

fiber, 227

diffusion

Arnold, 99

dimension, 125

Dirac

bracket formula, 243

constraint, 242

integral, 141

rule, 385

Dirac theory

constraint, 186

direction of gravity, 17

directional derivative, 83, 128, 137

dissipation

double bracket, 506

dissipation function

Rayleigh, 209

dissipation induced instability, 39, 41

dissipative

Lagrangian system, 209

weakly, 209

distance, 30

distribution

characteristic, 344

disturbance

infinitesimal, 32

divergence, 135, 140

electric field, 394

theorem, 142

dot product, 135

double bracket dissipation, 506

double pendulum

planar, 47

drift

spherical, 29, 46

gyroscope, 57

- dual
  - basis, 63
  - Lie algebra homomorphism, 375, 391
  - pair, 391
- dually
  - vector space, 67
- Duffing equation, 93, 516
  - forced damped, 96
- Duflo–Vergne theorem, 323
  - dynamic
    - phase, 53
  - dynamic definition of the Lie derivative, 137
  - dynamic phase, 53
  - dynamical system, 12, 30
    - chaotic, 30, 100
    - fluid, 20
    - left trivialization, 423
    - Lie–Poisson reconstruction, 425
    - Lie–Poisson reduction, 421
    - reconstruction, 421, 485
    - reduction, 351, 421
    - right-body, 432, 515
    - rational, 6
    - underwater vehicle, 60
    - vortex, 59, 99
- E**
  - effective action, 310
  - Ehresfest, 8
  - eigenvalue, 80
  - infinitesimally symplectic, 297
    - movement, 46
    - splitting, 46
  - elastic constant, 107
    - elasticity, 395
    - tensor, 113
  - elastodynamics, 114
    - linear, 113
  - electric field, 23, 24, 200
    - divergence, 394
  - electrodynamics, 24
  - electromagnetic field, 201
- electrostatics, 24
  - elementary elliptic function
    - Jacobi, 498
  - ellipsoid
    - energy, 55, 492, 512
    - revolution, 411
  - ellipsoid of inertia
    - body, 491
    - space, 491
  - elliptic
    - cylinder, 514
    - point, 100
  - elliptic function
    - Jacobi, 502
  - elliptic function of time, 501
  - embedding, 126
    - quasi-regular, 127
  - energetics, 39
    - energy, 185, 202, 227, 233
    - conservation, 5, 76, 158
    - ellipsoid, 55, 492, 512
    - function, 74, 183
    - kinetic, 62, 107, 496
    - potential, 62, 107
    - total, 63, 107
  - energy of the reduced trajectory, 56
    - energy–Casimir method, 37, 503
  - energy–momentum method, 38, 39, 263
    - equation
      - abstract wave, 113
      - collisionless Boltzmann, 24
      - Duffing, 93, 516
      - Euler, 18, 20, 26, 488
      - Euler–Lagrange, 2, 62, 89, 182, 187, 202, 207
      - Euler–Poincaré, 7, 10, 361, 430
      - first variation, 81
      - Hamel, 441
      - Hamilton’s, 4, 65, 157
      - Hamilton–Jacobi, 210, 212, 229
      - Jacobi, 194
      - KdV, 22, 29, 41, 109, 247, 437
- equation of motion, 17, 88, 231
  - Poisson bracket form, 162, 334
  - weak form, 106
- equilibrium
  - relative, 40, 263
  - solution, 90
  - state, 30
- equivariant curves, 123
  - equivariance, 379, 396
  - diffomorphism, 315
  - global, 401
  - infinitesimal, 14, 378, 396
  - map, 315
  - momentum map, 378, 379
- equivariance implies infinitesimal equivariance, 379
  - equivariant Darboux theorem, 149, 325
- equivariant momentum maps are Poisson, 403
  - essential  $G$ -smooth part, 322, 323
- Eutclidean group, 17, 325, 472
  - plane, 377, 467
  - special, 377
- Euler
  - angle, 7, 493
  - equation, 18, 20, 26, 488
- force, 251
  - rigid-body equation, 6
  - theorem, 290
- Euler–Lagrange equation, 2, 62, 89, 182, 187, 202, 207
  - history of, 231
  - reduced, 261
- Euler–Poincaré
  - equation, 7, 10, 361, 430
  - KdV equation, 438
  - reconstruction, 436
  - reduction, 435
  - reduction of dynamics, 485
  - Euler–Poincaré equation, 430, 435
    - history of, 430
  - Euler–Rodrigues
    - parameterization, 306
- Eulerian
  - description, 19
  - quantity, 481
  - velocity, 483
- exact  $k$ -form, 182
  - exact sequence, 370
  - example
    - Cherry’s, 34
    - coadjoint orbit, 444
    - momentum map, 389
    - resonance, 33
    - tangent vector, 452
  - existence and uniqueness theorem, 127
  - existence of solution, 120
  - expectation value, 109
  - exponential map, 273, 274, 278
  - exponentially small, 100, 101
  - extended
    - phase space, 170, 210
    - point transformation, 383, 388
  - extension
    - central, 283, 400, 407, 437
    - left-invariant, 417
    - method, 460
    - right, 271
    - right-invariant, 417
    - exterior

- derivative, 131
  - force, 205
  - exterior force one-form, 206
  - external
    - force, 482
    - variable, 53
- F**
- f*-related, 277
  - fathful action, 310
  - falling
    - cat, 53
  - feedback law, 58
  - fiber, 124
    - derivative, 183
    - differentiation, 227
    - translation, 176
  - fiber optics, 51
  - fiber translation
    - momentum, 387
  - fibration
    - Hopf 50, 179, 306, 307
  - fictionous force, 251
  - field, 3, 105
    - characteristic, 344
    - electric, 23, 24, 200
    - force, 206
    - magnetic, 23, 24, 200
    - multivector, 353
    - q*-vector, 353
    - vector, 127
    - Yang–Mills, 179, 201
  - field theory
    - classical, 105
    - Hamiltonian, 105
    - relativistic, 15
  - fifth problem
    - Hilbert's, 278
  - first integral, 60
  - first mode, 98
  - first Poisson structure, 331
  - first variation, 37, 503, 508
  - equation, 81
  - first Whitehead lemma, 371
  - flag manifold, 156

- fla, 134
- flexible attachment, 58
- flow, 81, 127
  - compressible, 26
  - Hamiltonian, 80
  - linear, 81
  - nonlinear, 81
  - property, 127
- flow property
  - time-dependent, 127
- fluid, 18, 266
  - compressible, 266
  - configuration, 19
  - dynamics, 20
  - incompressible, 18, 266
  - kinematics, 18
  - rigid body, 58
  - variational principle, 60
- fluid on a rotating sphere, 257
- foliation, 140
  - force, 62
  - centrifugal, 248, 251
  - constraint, 88, 241
  - Cortolfs, 248, 251
  - Euler, 251
  - exterior, 205
  - external, 482
  - fictionous, 251
  - field, 206
  - Lagrangian, 206
- force law
  - Lorentz, 178
- forced beam
  - buckled, 98
- forced damped Duffing equation, 96
- forced pendulum, 47
- form
  - bilinear, 63
  - canonical, 66, 67
  - contact, 225
  - differential, 129
  - Hamiltonian, 233
  - normal, 337
  - Poisson, 65

- second fundamental, 241
  - symplectic, 66
  - volume, 135, 139
- formally
- stable, 37
  - unstable, 39
- formula
- Cartan magic, 139, 144
  - Dirac's bracket, 243
  - phase, 256
  - Rodrigues', 291
- Fourcault pendulum, 51, 256, 259
- frame
- inertial, 62, 88, 254
  - rotating, 249
- free action, 310
- free boundary problem, 41, 58
- free rigid body, 31, 265
- Lagrangian, 483
- symmetric, 57
- free rigid-body problem
- analytical solution, 498
- friction, 39, 87
- Probenius' theorem, 140
- frozen Lie–Poisson bracket, 330
- Rubini–Sturdy metric, 154, 157
- function
- Casimir, 8, 21, 35, 335, 396, 414, 462, 464, 488, 507, 514
  - collective, 404
  - energy, 74, 183
  - generating, 174, 175, 211, 231
  - group, 336, 337
  - Hamiltonian, 74
  - left-invariant, 416
  - momentum, 383
  - momentum translation, 392
  - Poincaré–Melnikov, 48, 94
  - pull-back, 71
  - push-forward, 71
  - space of, 83
  - wave, 68
- functional
- action, 226

- derivative, 5, 11
  - functional derivative
    - partial, 76, 106
  - fundamental Poisson bracket, 83
  - fundamental theorem
    - calculus, 141
- G**
- G*-smooth part
    - essential, 322, 323
  - Gallilean
    - algebra, 411
    - group, 321, 326, 409
    - gauge group, 25, 266
  - Gauss–Bonnet theorem, 57
  - Gauss–Hertz principle
    - least curvature, 9
  - Gelfand–Fuchs cocycle, 437
  - general
    - Hamilton's equation, 74
    - linear group, 268, 283
  - generalized kernel, 80
  - generating function, 174, 175, 211, 231
  - generator
    - bracket of, 316
    - infinitesimal, 313, 365
    - geodesic, 195, 197
    - motion, 3
    - sprey, 196, 239
    - totally, 242
  - geometric phase, 49, 51, 53, 253, 256
  - locomotion, 52
  - geometry
    - Hamilton–Jacobi theory, 216
    - Poisson, 5
    - symplectic, 5
  - $GL(n, \mathbb{R})$ 
    - Lie algebra, 271
  - global
    - equivariance, 401
    - Poincaré lemma, 136
    - globally Hamiltonian, 379
    - graded Jacobi identity, 354

gradient, 135, 202  
 Grassmannian manifold, 156  
 gravity, 88  
 Green's theorem, 141, 145  
 group  
   affine, 444, 452, 458  
   Barnack–Lie, 267, 268  
   canonical transformation, 26, 266  
   complex general linear, 209  
   complex special linear, 300  
   diffeomorphism, 266, 309, 323  
   Euclidean, 17, 325, 472  
   function, 336, 337  
   Galilean, 321, 326, 409  
   gauge, 25, 266  
   general linear, 268  
   Heisenberg, 283, 401, 479, 516  
   homomorphism, 276  
   infinite-dimensional, 267, 323  
   isometry, 320  
   Lie, 267  
   Lorentz, 172, 321, 326  
   orthogonal, 287  
   projective unitary, 408  
   real symplectic, 293  
   rotation, 366, 444, 452, 457  
   semidirect product, 388  
   special linear, 285  
   special orthogonal, 288  
   special unitary, 301  
   symmetry, 265, 310  
   symplectic, 72, 294  
   tangent, 282  
   transformation, 336  
   unitary, 301  
   variable, 53  
   vector, 267, 274  
   volume-preserving diffeomor-  
   phism, 266  
   volume-preserving transforma-  
   tion, 18  
 guiding center coordinates, 407  
 gyroscope dirfl, 57  
 gyrosopic

instability, 45  
 system, 39  
**H**  
 Haar measure, 282, 380  
 half twist, 506  
 Hamel equation, 441  
 Hamilton equation  
   canonical, 106  
 Hamilton's equation, 4, 65, 75, 77, 157  
   canonical, 63  
   canonical coordinate, 76  
   classical, 76  
   complex form, 64  
   general, 74  
 Hamilton's principle, 2, 220, 226, 232  
   critical action, 181  
   phase space, 224  
   variational, 2, 221, 434  
 Hamilton–Jacobi  
   equation, 210, 212, 229  
   equation in Lie–Poisson form, 429  
   theorem, 213  
 Hamilton–Jacobi theory, 176  
   geometry, 216  
 Hamiltonian, 4, 24, 74, 157, 182, 389  
   Krein–Hopf bifurcation, 45  
   action, 379  
   augmented, 40  
   bifurcation, 41, 59  
   chaos, 59  
   compressible fluid, 27  
   field theory, 105  
   flow, 80  
   form, 233  
   function, 74  
   globally, 379  
   heavy top, 17, 507  
   hyperregular, 190  
   KdV, 120  
   linear vector field, 77

locally, 158, 366  
 Maxwell–Vlasov, 28  
 mechanics, 1  
 pitchfork bifurcation, 43, 90  
 rigid-body, 8  
 saddle node bifurcation, 101  
 vector field, 12, 64, 74, 85, 162, 168, 333, 361, 412  
 Hamiltonian flow  
   momentum function, 385  
   properties, 338  
 Hamiltonian flows are Poisson, 338  
 Hamiltonian system transforma-  
   tion, 158  
 Hannay's angles, 53  
 Hannay–Berry  
   connection, 53  
   phase, 259  
 harmonic oscillator, 65  
 hat map, 289  
 Hausdorff, 122  
 heavy top, 16, 41, 45, 99, 266  
   bracket, 17  
   Hamiltonian, 17, 507  
   stability, 507  
   stability theorem, 510  
 Heisenberg group, 283, 401, 479, 516  
   helicity, 335  
   Hermitean, 300  
   inner product, 68, 78  
   metric, 153  
 Hessian  
   bordered, 235  
   Hilbert space, 152  
   complex, 78, 82, 154  
   unitary group, 322  
 Hilbert's fifth problem, 278  
 Hille–Yosida theorem, 120  
 history of  
   Euler–Lagrange equation, 231  
   Euler–Poincaré equation, 430  
   momentum map, 369  
   Poisson structure, 336  
 Hodge decomposition theorem, 446  
 Hodge star operator, 133, 134  
 holonomic constraint, 219, 236, 246  
 homology, 51, 256  
 homoclinic orbit, 93, 94, 100, 112  
 homogeneous space, 319  
 homomorphism, 362  
   group, 276  
   Lie algebra, 276, 277  
   Lie group, 276  
 homotopy operator, 136  
 hook notation, 131  
 hoop, 87  
   particle, 92, 253  
   rotating, 43  
 Hopf fibration, 50, 179, 306, 307  
 Hopf–Rinow theorem, 198  
 horizontal  
   one-form, 205  
   part, 203  
   horseshoe, 94  
   hyperbolic saddle, 94  
   hyperregular, 188  
   Hamiltonian, 190  
   Lagrangian, 186

**I**

ideal flow, 39  
 ideal fluid bracket, 20, 329  
 identities for vector fields and forms, 143  
 identity  
   derivation, 160  
   graded Jacobi, 354  
   Jacobi, 6, 9–11, 85, 143, 161, 163  
   Jacobi–Schouten, 357  
   Leibniz, 11  
   Lie–Schouten, 358  
   Poisson bracket–Lie derivative, 162  
 immersion, 125  
   cosmropic, 160  
   injective, 126  
   Poisson, 344  
   theorem, 126

- incompressibility, 236
- incompressible fluid, 18, 266
- induced by subgroup
  - momentum map, 377, 391
- inertia
  - moment, 6, 486
  - tensor, 486
- inertia tensor
  - moment, 433
- inertial frame, 62, 88, 254
- infinite-dimensional
  - group, 267, 323
- infinitesimal
  - disturbance, 32
  - equivariance, 14, 378, 306
  - generator, 313, 365
  - Poisson automorphism, 366, 412
  - rotation, 368
  - symplectic transformation, 78
  - variation, 220
- infinitesimal generator
  - adjoint action, 313
- infinitesimally equivariant
  - momentum map, 379
- infinitesimally symplectic
  - eigenvalue, 297
- initial condition, 30
- injective immersion, 126
- injectively immersed
  - submanifold, 126
- inner automorphism, 275, 310
- inner product
  - Hermitian, 68, 78
  - space, 67
- instability
  - dissipation induced, 39, 41
  - gyroscopic, 45
- instantaneous rotation vector, 249
- integrability, 22
- integrable, 140
  - system, 15, 59
- integral
  - action, 62, 214, 229
  - curve, 127

- definition, 141
  - invariant, 151
  - Lagrange-d'Alembert principle, 207
  - integral theorems of calculus, 141
  - integrals in involution, 60
  - integrator
    - symplectic, 176
  - interior product, 75, 131, 353
  - intermediate axis, 506
  - internal energy function, 27
  - internal variable, 53
  - intrinsic derivative, 359
  - invariable plane, 491, 493
  - invariance
    - $Ad^*$ , 462
    - Poisson form, 232, 234
  - invariance group
    - classical mechanics, 299
  - invariant
    - integral, 151
    - left, 269
    - right, 270
    - vector field, 269
  - inverse function theorem, 124
  - inversion map, 267
  - involution, 83, 140
  - isometry, 308
    - group, 320
  - isotropic, 74, 119
  - isotropically maximal, 74
  - isotropy, 310
  - isotropy algebra, 313
  - coadjoint, 323, 451
- J**
- Jacobi
    - elementary elliptic function, 498
    - elliptic function, 502
    - equation, 194
    - identity, 6, 9–11, 85, 143, 161, 163
    - metric, 204

- principle of least action, 224
- Jacobi–Hartn coordinates, 167
- Jacobi–Lie bracket, 20, 84, 138
- Jacobi–Schouten identity, 357
- Jacobian, 140
- matrix, 71

**K**

- $k$ -form, 129
- closed, 132
- Kaluza–Klein
  - Lagrangian, 201
  - system, 200
- KAM
  - theorem, 36
  - theory, 511
  - torus, 99
- KdV
  - bracket, 117, 120, 330
  - equation, 22, 29, 41, 109, 247, 437

- Euler–Poincaré form, 438
- Hamiltonian, 120
- Lie–Poisson form, 439
- symplectic structure, 109
- Kelvin’s circulation theorem, 21, 35, 266

- kernel
  - generalized, 80
- kinematics
  - fluid, 18
- Lie groups, 489
- kinetic energy, 62, 107, 496
- rigid body, 7

- Klein–Gordon equation, 108
- Korteweg–de Vries equation, 109
- Kostant’s coadjoint orbit covering theorem, 463
- Kronecker symbol, 356
- Kähler manifold, 153
- strong, 154

- L**
  - $L^2$  pairing, 329
  - Lagrange multiplier theorem, 234

- Lagrange top, 507
- Lagrange-d'Alembert principle, 205, 208
- integral, 207
- local, 206
- Lagrange–Dirichlet criterion, 36
- theorem, 36
- Lagrange–Poincaré equation, 440, 441
- Lagrangian, 2, 25, 62, 74, 181
- condition, 186
- description, 19
- force, 206
- free rigid body, 483
- hyperregular, 186
- Kaluza–Klein, 201
- Lie–Poisson reconstruction, 428
- manifold, 175
- mechanics, 1, 181
- nongenerate, 184
- one-form, 184, 227, 228
- quantity, 482
- regular, 184
- rigid-body, 7, 433, 485
- submanifold, 216
- subspace, 218
- system, 186
- two-form, 184, 227
- vector field, 186, 227
- velocity, 483

- Lagrangian system
  - dissipative, 209
- lattice
  - Toda, 453, 459
- law
  - Lorentz force, 24
  - Newton’s second, 61
- leaf
  - symplectic, 345, 347
- least curvature
  - Gauss–Hertz principle, 9
- left
  - action, 173, 309
  - invariant, 269



- lift, 173
  - reduction, 422
  - translation map, 267
- left and right translations
  - momentum map, 392
- left extension, 270
- left trivialization
  - dynamics, 423
- left-invariant
  - extension, 417
  - function, 416
  - vector field, 269
- Legendre transform, 183, 227, 437
  - abstract, 185
- Legend equation, 97
- Leibniz
  - identity, 11
  - rule, 328
- lemma
  - momentum shifting, 176
  - Poincaré, 133, 136, 144
- Levi-Civita covariant derivative, 199
- Liapunov theorem, 32
- Lie
  - algebra, 9, 85, 143, 161, 270
  - bracket, 270
  - derivative, 83, 137, 138
  - group, 267
  - subgroup, 279
- Lie algebra
  - action, 317, 367
  - deformation, 471
  - $GL(n, \mathbb{R})$ , 271
  - homomorphism, 276, 277
  - Lie group, 270
  - Lie algebra homomorphism
    - dual, 375, 391
  - Lie derivative, 137
    - notation, 83
    - Poisson tensor, 343
    - theorem, 137
  - Lie derivative of the Poisson tensor along a Hamiltonian vector field, 358
- Lie group, 265
  - Abelian, 278
  - chart, 268
  - classical, 283
  - homomorphism, 276
  - kinematics, 489
  - Lie algebra, 270
  - Lie subalgebra, 85
- Lie subgroup
  - regular, 279
- Lie's third fundamental theorem, 280, 337
- Lie–Poisson
  - bracket, 11, 327, 328, 416
  - bracket restriction, 459
  - coadjoint orbit compatibility, 439
  - equation, 11, 12
  - Hamilton–Jacobi equation, 429
  - KdV equation, 439
  - reconstruction, 427
  - reconstruction in Lagrangian form, 428
  - reconstruction of dynamics, 425
  - reduction, 415
  - reduction of dynamics, 421, 485
  - reduction theorem, 415
  - reduction using momentum function, 419
  - ton, 419
- Lie–Schubert identity, 358
- Lie–Weinstein theorem, 348
- lift
  - coaugent, 170, 386, 391
  - left, 173
  - right, 173
  - vertical, 185, 203, 240
- lifted action, 406
  - momentum map, 386
- Lin constraint, 21, 60, 431
- line integral, 145
- line of nodes, 494
- linear
  - elastodynamics, 113
- flow, 81
- Hamiltonian vector field, 77
- momentum, 115, 118, 265, 372, 389
  - Poisson bracket, 364
  - transformation, 63
- linear group
  - general, 283
- linear map
  - associated, 63
  - linear momentum, 118
  - total, 373
- linear momentum for linear elastodynamics, 119
- linearized equation, 32
- linearly stable, 32
- linkages, 54
- Lionville
  - measure, 149, 363
  - volume, 149, 163
- Lionville–Arnold theorem, 214
- liquid drop, 41
  - rotating, 44
- load, 394
  - astatic, 394
- local
  - diffeomorphism, 124
  - Lagrange–d'Alembert principle, 206
  - operator, 132
  - local 1-to-1 theorem, 125
  - local onto theorem, 125
  - locally
    - closed, 125
    - Hamiltonian, 158, 366
    - locally Hamiltonian vector field, 158, 412
    - locomotion
      - geometric phase, 52
      - in microorganisms, 33
    - long axis, 505
  - Lorentz
    - force law, 24, 178
    - group, 172, 321, 326

## M

- magie formula of Cartan, 139
  - magnetic
    - field, 23, 24, 200
    - monopole, 146, 179
    - term, 176, 177, 262, 406, 478
  - magnetic field
    - particle, 178, 226
  - magnetohydrodynamics (MHD), 266
    - bracket, 23
  - Magnet–Weinstein, 360
  - manifold, 121
    - complex, 153
    - configuration, 181
    - Kähler, 153
    - Lagrangian, 175
    - Poisson, 11, 327, 328
    - stable, 94
    - symplectic, 147
    - unstable, 94
    - volume, 149
- map
  - canonical, 13, 71
  - Christoffel, 196
  - covering, 463
  - determinant, 277
  - differentiable, 124
  - equivariant, 315
  - exponential, 273
  - hat, 289
  - inversion, 267
  - momentum, 13, 14, 25, 367, 394
  - plasma to fluid, 27
  - Poincaré, 94
  - Poisson, 13, 339, 362
  - reduction, 51
  - rigid-body, 13
  - space, 220
  - symplectic, 69, 71, 150
  - Martin, 14
  - mass, 23, 62, 408
  - matrix, 3, 262
  - mass density
    - spatial, 487

- material
  - coordinates, 481
  - picture, 489
  - point, 481
  - velocity field, 19
- matrix, 63
  - altitude, 503
  - Jacobian, 71
  - mass, 3, 262
  - skew-Hermitian, 301
  - symplectic, 64, 72
  - upper triangular, 448
- matrix group
  - momentum, 374, 390
- Maurer–Cartan
  - equation, 280, 456
  - structure equation, 280
- maximal
  - atlas, 122
  - isotropic, 74
- maximally isotropic, 175
- Maxwell
  - equation, 24, 266, 395
- Maxwell–Bloch system, 516
- Maxwell–Poisson bracket, 26
- Maxwell–Vlasov
  - bracket, 22, 28
  - equation, 266
- Hamiltonian, 28
- system, 22
- measure
  - Haar, 282, 380
  - Liouville, 149, 363
- Mécanique Analytique, 231, 232, 431
- mechanical system reduction, 51
- mechanics
  - Hamiltonian, 1
  - Lagrangian, 1, 181
  - nonholonomic, 208
- Melnikov
  - method, 92
  - vector, 99
- meson, 108
- method
  - Arnold, 39
  - averaging, 234
  - energy–Casimir, 37, 503
  - energy–momentum, 38, 39, 263
  - extension, 460
  - Melnikov, 92
  - Peinart–Melnikov, 47, 48, 94, 498, 506
  - restriction, 460
  - separation of variables, 215
- metric, 30
  - Rubini–Sturdy, 154, 157
  - Hermitian, 153
  - Jacobi, 204
- micromotor, 51
- middle axis, 505
- molecular dynamics, 51
- moment
  - inertia, 6, 486
  - inertia tensor, 433
- moment map, 15
- momenta
  - angular, 118
  - moments of inertia
  - principal, 486
- momentum
  - angular, 7, 115, 119, 195, 265, 368, 373, 381, 390, 411
  - canonical, 375
  - commutator lemma, 384
  - conjugate, 4, 252
  - conservation, 233
  - Fiber translation, 387
  - function, 383
  - linear, 115, 118, 265, 372, 389
  - map, 13, 14, 25, 367, 394
  - matrix group, 374, 390
  - phase space, 148, 173, 482
  - shift, 176, 292
  - translation, 393
  - momentum function
  - Hamiltonian flow, 385
  - momentum map, 15, 28, 375
  - algebraic definition, 370

- canonical, 403
  - Casimir function, 413
  - compact group, 380
  - conservation, 372
  - cotangent bundle, 383
  - diagram, 368
  - equivariance, 378, 379
  - example, 389
  - history of, 369
  - induced by subgroup, 377, 391
  - infinitesimally equivariant, 379
  - left and right translations, 392
  - lifted action, 386
  - projective representation, 376
  - subalgebra, 376
  - momentum shifting lemma, 176
  - momentum translation
    - function, 392
  - monopole
    - magnetic, 146, 179
  - Morse theory, 49
  - motion, 19, 482
    - geodesic, 3
    - pendulum, 515
    - periodic, 506
    - rotational, 361
  - movement of eigenvalues, 45
  - moving system, 257
  - multilinear, 129
  - multiple jump, 39
  - multisymplectic geometry, 59
  - multivector, 353
  - field, 353
- N**
- $n$ -body problem, 370
  - $n$ -dimensional torus, 278
  - $N$ -particle system, 372
  - natural projection, 123
  - neighborhood, 122
  - convex, 198
  - neighborhood theorem
  - regular Lagrangian, 193
  - Newton's
    - equation, 232
    - second law, 3, 61, 62
    - Newton's law, 2
    - rotating frame, 249
    - nilpotent, 283
    - node
      - line of, 494
    - Noether
      - quantity, 13
      - theorem, 372
    - non-integrability, 99
    - nonconjugate solution, 193
    - nongenerate, 66
    - Lagrangian, 184
    - strongly, 66
    - weakly, 66
  - nonholonomic
    - mechanics, 208
    - system, 60, 338
  - velocity constraint, 442
  - nonlinear
    - flow, 81
  - oscillator, 65
  - Schrödinger equation, 114, 115
  - stability, 29
  - wave, 59
  - nonperiodic Toda lattice, 331
  - normal
    - form, 337
    - subgroup, 319
  - notation
    - hook, 131
    - Lie derivative, 83
    - pull-back, 71
    - numerical
      - algorithm, 80
      - analysis, 59
      - integration, 59
      - integrator, 214
- O**
- off-center rotating hoop, 91
  - one-dimensional transport equation, 111
  - one-form, 75, 129
  - canonical, 166, 227, 386

- connection, 201
  - horizontal force, 206
  - horizontal, 205
  - Lagrangian, 184, 227, 228
  - one-parameter subgroup, 273
  - one-to-one resonance, 50
  - open mapping theorem, 66
  - operator
    - adj, 314
    - adjoint, 275
    - alternation, 130
    - commutator, 86
    - Hodge star, 133, 134
    - local, 132
    - orbit, 309
    - coadjoint, 14, 443, 488
    - homoclinic, 93, 100
    - space, 311
    - symplectic form, 476
    - tangent space of, 313
    - Toda, 448
  - ordinary differential equation, 127
  - orientable, 139
  - orientation, 139
  - oriented positively, 139
  - orthogonal, 284
    - decomposition, 21
  - orthogonal complement
  - symplectic, 72, 204
  - orthogonal group, 287
    - proper, 482
  - oscillator
    - harmonic, 65
    - nonlinear, 65
- P**
- pair
    - dual, 391
  - parabolic cylindrical coordinates, 516
  - parallel transported, 199
  - parameter
    - Cayley–Klein, 302, 490
  - parameterization
    - Euler–Rodrigues, 306

- part
  - horizontal, 203
  - scalar, 304
  - vertical, 304
- partial
  - differential equation, 140
  - functional derivative, 76, 106
  - particle, 173
    - hoop, 44, 92, 253
    - magnetic field, 178, 226
    - quantum, 408
    - rotating hoop, 87
    - Yang–Mills, 53
  - particle relabeling symmetry, 19
  - pass, 46
  - passively rotating system, 219
  - path space, 220
  - Pauli, 14
    - spin matrix, 302
  - Pauli–Born–Infeld bracket, 26
  - Pauli–Jost theorem, 344
  - PDE, 59
  - pendulum, 100
    - equation, 89
    - forced, 47
    - Foucault, 51, 256, 259
    - motion, 515
    - rigid body, 512
    - spherical, 100, 195, 202, 242, 260, 263
    - whirling, 89
  - pendulum on a merry-go-round, 257
  - perfect fluid, 440
  - periodic motion, 506
  - permutation
    - sign, 130
  - phase
    - Berry–Hannay, 256
    - dynamic, 53
    - formula, 256
    - geometric, 51, 53, 253, 256
    - Hannay–Berry, 259
    - portrait, 91
    - reconstruction, 54

- rigid-body dynamics, 54
  - shift, 51
  - space, 63, 148, 165
  - symmetry, 117
- phase formula
  - rigid-body, 56
- phase space, 30
  - extended, 170, 210
  - Hamilton’s principle, 224
  - momentum, 148, 173, 482
  - rotation, 405
  - translation, 406
  - velocity, 3, 482
- $\varphi$ -related, 137
- Phillips lectures, 369
- picture
  - material, 489
- planar pendulum
  - cart, 263
- Planck’s constant, 68, 109
- plane
  - Euclidean group, 377, 467
  - invariant, 491, 493
  - special linear group, 465
- plasma particle number density, 23
- plasma physics, 22
- plasma to fluid
  - map, 27
- Poisson map, 362
  - Poincaré, 7
  - lemma, 133, 136, 144
  - map, 94
- Poincaré lemma
  - global, 136
  - relative, 136
- Poincaré sphere, 9
- Poincaré–Andronov–Hopf bifurcation, 30
- Poincaré–Meinkov
  - function, 48, 94
  - method, 47, 48, 94, 498, 506
  - theorem, 95
- Poinsoot
  - construction, 57

- theorem, 490
- point
  - caustic, 218
  - conjugate, 198
  - critical, 40, 125
  - elliptic, 100
  - material, 481
  - regular, 125
  - spatial, 481
- transformation, 170
- point transformation
  - extended, 383, 388
- Poisson
  - algebra, 335
  - automorphism, 366
  - bracket, 65, 82, 115, 160, 168, 234
  - commute, 8, 35, 83
  - form, 65
  - geometry, 5
  - immersion, 344
  - manifold, 11, 327, 328
  - map, 13, 339, 362
  - reduction, 246
  - reduction theorem, 349
  - structure, 64, 328, 341
  - submanifold, 345
  - tensor, 340
  - transformation, 70
- Poisson automorphism, 412
- infinitesimal, 366, 412
- Poisson bracket
  - fundamental, 83
  - linear, 364
  - rigid-body, 8
- Poisson bracket form, 5
  - equation of motion, 162, 334
- Poisson bracket–Lie derivative identity, 162
- Poisson map, 14, 339
  - coordinate representation, 342
  - plasma to fluid, 362
  - properties, 339
  - Poisson structure
    - history of, 336

- rank, 344
- trivial, 328
- Poisson tensor
  - Lie derivative, 343
- Poisson–Darboux theorem, 348
- Poisson–Vlasov
  - bracket, 26, 329
  - Casimir, 336
  - equation, 24
- polar decomposition
  - complex, 300
  - theorem, 284
- polar wander, 57
- position vector, 173
- positive definite, 284, 300
- positively oriented, 139
- potential
  - amended, 252, 261, 516
  - centrifugal, 252
  - Clebsch, 6, 21, 23
  - energy, 62, 107
  - rotated, 250
- potential field
  - constrained motion, 238
- power, 62, 210
- power system, 100
- precession
  - angular velocity, 499
- preserving
  - volume, 140, 151
- pressure, 20, 27
- presymplectic form, 360
- primary constraint set, 237
- principal axis body Frame, 486
- principal moments of inertia, 486
- principle
  - averaging, 259
  - critical action, 181, 222, 223
  - Hamilton's, 220, 232
  - Lagrange–d'Alembert, 205, 208
  - Masupertus, 232
  - variational, 62, 105
- principle of least action
  - Jacobi's form, 224
- problem
  - free boundary, 41, 58
  - resonance, 57
  - Routh, 42
  - three-body, 47, 100
  - two-body, 263
- product, 391
  - cross, 135, 290
  - dot, 135
  - interior, 75, 131, 353
  - rule, 132
  - semidirect, 18, 325, 444, 473
  - tensor, 130
  - wedge, 130
- projection
  - natural, 123
- projective
  - chart, 157
  - representation, 323
  - space, 154, 304
  - unitary group, 323, 408
  - unitary representation, 409
- projective representation
  - momentum map, 376
- projective space
  - complex, 154
- projectivization, 154
- projectivized complex Hilbert space, 52
- propagation velocity, 108
- proper
  - action, 311
  - orthogonal group, 482
  - rotation group, 265
- property
  - deformation, 355
  - flow, 127
  - Hamiltonian flow, 338
  - Poisson map, 339
  - submanifold, 125
  - pseudo-rigid body, 40
  - pseudo-sphere bundle, 197
  - pseudo-vector, 390
  - pull-back, 131, 137
  - bilinear form, 71
  - function, 71

- notation, 71
  - vector field, 71
  - pure quaternion, 304
  - push-forward, 71, 131, 137
  - bilinear form, 72
  - function, 71
  - vector field, 71
- Q**
- $q$ -tensor
    - contravariant antisymmetric, 353
  - $q$ -vector, 353
    - field, 353
  - quadratic bracket, 331
  - quantity
    - conserved, 115
    - Eulerian, 481
    - Lagrangian, 482
    - Noether, 13
  - quantum mechanics, 52, 117, 385
  - quantum symplectic form, 68
  - quantum particle, 408
  - quasi-regular embedding, 127
  - quaternion, 303
  - pure, 304
  - quotient, 280
    - space, 311
  - quotients of Poisson manifolds, 349
- R**
- rank
    - Poisson structure, 344
    - rate of work equation, 62, 210
    - rattleback, 42
    - Rayleigh dissipation function, 209
    - real projective space, 292
    - real symplectic group, 293
    - rearrangement
      - canonical, 448
    - reconstruction, 256
    - dynamics, 421, 485
    - equation, 422, 436
    - Euler–Poincaré, 436
    - Lie–Poisson, 427
  - phase, 54
  - theorem, 425
  - reduced
    - Euler–Lagrange equation, 261
    - variational principle, 434
  - reduction, 12, 13, 349
    - dynamics, 351, 421
    - Euler–Poincaré, 435, 485
    - left, 422
    - Lie–Poisson, 415, 485
    - map, 51
    - mechanical system, 51
    - Poisson, 246
    - right, 422
    - Routh, 260
  - reference configuration, 432, 481
  - regular, 127, 227
    - Lagrangian, 184
    - Lie subgroup, 279
    - point, 125
    - value, 125
  - regular Lagrangian, 186
  - neighborhood theorem, 193
  - related, 277
    - vector field, 277
  - relation
    - commutation, 378, 388
  - relative
    - Darboux theorem, 150, 326
    - equilibrium, 40, 263
    - Poincaré lemma, 136
    - relativistic field theory, 15
    - relativity, 59
    - reparametrization, 204
    - representation, 309
    - adjoint, 310, 443
    - coadjoint, 311
    - contragredient, 311
    - coordinates, 341
    - projective, 323
    - projective unitary, 409
    - spatial, 489
    - theory, 357
    - unitary, 322, 394

- resonance, 32, 49
- bifurcation, 45
  - condition, 98
  - example, 33
  - one-to-one, 50
- problem, 57
- restricted four-vortex problem, 99
- restriction
  - Lie–Poisson bracket, 459
  - method, 460
  - retraction, 136
- revolution
  - ellipsoid, 411
  - surface, 408
- right
  - action, 173, 309
  - extension, 271
  - invariant, 270
  - lift, 173
  - reduction, 422
  - translation map, 267
- right-invariant
  - extension, 417
- rigid body, 6, 265
  - fluid, 58
  - free, 265
    - in fluid, 41
    - kinetic energy, 7
  - pendulum, 512
- rigid body with internal rotors, 58
  - rigid-body
    - bracket, 329, 352, 487
    - Casimir, 35, 335
    - dynamics, 432, 515
    - Hamiltonian, 8
    - Lagrangian, 7, 433, 485
    - map, 13
    - phase formula, 56
    - Poisson bracket, 8
    - stability, 31, 503
    - variational principle, 7
  - rigid-body dynamics
    - phase, 54

- rigid-body equation
  - Euler's, 6
- rigid-body stability, 31
  - theorem, 505
- rigidity, 236, 482
- robotic locomotion, 54
- Rodrigues' formula, 291, 292
- roller racer, 42
- rolling
  - constraint, 208
    - disk, 42
    - slipping, 491
  - rotated potential, 250
- rotating
  - fluid system, 338
    - frame, 249
    - hoop, 43
    - liquid drop, 44
    - system, 253
  - rotating frame, 249
    - Newton's law, 249
  - rotating hoop
    - ball, 210
    - particle, 87
  - rotating system
    - actively, 219
    - passively, 219
  - rotation, 73, 265, 290, 340, 370
    - group, 366, 444, 452, 457
    - in plane, 289
    - in space, 289
      - infinitesimal, 368
      - phase space, 405
      - stationary, 504
    - rotation group, 51
      - proper, 265
    - rotation rate
      - critical, 90
    - rotation vector
      - instantaneous, 249
  - rotational
    - dynamics, 6
    - motion, 361
  - rotor, 58
  - Routh
    - problem, 42
    - reduction, 260
    - stability criterion, 263
  - Routh–Hurwitz criterion, 43
  - Routhian, 442
    - classical, 261
    - construction, 237
  - rule
    - Leibniz, 328
    - product, 132

## S

- saddle node bifurcation
  - Hamiltonian, 101
- Sard's theorem, 125
- satellite dynamics, 60
- satellite with rotors, 57
- scalar part, 304
- Schouten bracket, 353
  - theorem, 354
- Schrödinger
  - bracket, 116
  - equation, 78, 82, 109
  - nonlinear equation, 114, 115
- Schnur–Horn theorem, 15
- second
  - cohomology, 397
  - fundamental form, 241
  - variation, 37, 504, 509
  - Whitehead lemma, 399
- second law
  - Newton's, 3, 62
- second variation criterion, 112
  - equation, 186
  - submanifold, 228
  - vector field, 207
- self-adjointness, 109
  - semidirect product, 18, 22, 325, 444, 473
  - group, 388
- separation of variables
  - method, 215
- separatrix
  - splitting, 100
- space
  - angular momentum, 487
  - configuration, 148, 265, 482
  - coordinates, 13
  - ellipsoid of inertia, 491
  - functions, 83
  - Hilbert, 152
  - homogeneous, 319
  - inner product, 67
  - map, 220
  - orbit, 311
  - path, 220
- splitting, 100
- stability criterion, 263
- stability of permutation, 130
- similarity class, 310
- simple pendulum, 237
- sinh–Gordon equation, 113, 114
- single species, 23
- skew, 129
- skew-Hermitian matrix, 301
- skew-symmetric bilinear form, 77
- slipping
  - rolling, 491
  - smoothness
  - autonomous, 278
- solution, 59
  - solution, 112
  - stability, 29, 41
- solution
  - chaotic, 506
  - equilibrium, 90
  - nonconjugate, 193
  - solution, 112

- phase, 63, 148, 165
- projective, 154, 394
- quotient, 311
- shape, 260
- tangent, 123
- spatial
  - angular momentum, 488
  - angular velocity, 485
  - angular velocity vector, 432
  - coordinate system, 483
  - coordinates, 481
  - mass density, 487
  - point, 481
  - representation, 489
  - velocity, 483
  - velocity field, 19
- special
  - Euclidean group, 377
  - linear group, 285
  - orthogonal group, 288
  - unitary group, 301
- special linear group
  - plane, 465
- spectral stability, 39
- spectrally stable, 32
- sphere
  - Poincaré, 9
- spherical
  - coordinates, 122, 128
  - pendulum, 100, 195, 202, 242, 260, 263
- spin, 302
- spin matrix
  - Pauli, 302
- splitting
  - eigenvalue, 46
  - separatrix, 100
- spray
  - geodesic, 196, 239
- stability, 511
  - heavy top, 507
  - nonlinear, 29
  - rigid-body, 31, 503
  - soliton, 29, 41
- stability criterion

- Routh's, 263
- stability theorem
  - heavy top, 510
  - rigid-body, 488
- stabilization, 60
- stabilizer, 310
- algebra, 313
- stable, 30
  - formally, 37
  - linearly, 32
  - manifold, 94
  - spectrally, 32
- state, 30
  - equilibrium, 30
- stationary rotation, 504
- steady motion, 263
- stellar dynamics, 24
- Stokes' theorem, 141, 163
- Stone's theorem, 120
- stratification
  - symplectic, 345
- stratified shear flow, 29
- strong
  - Kähler manifold, 154
  - symplectic manifold, 147
- strongly nondegenerate, 66
- structure
  - complex, 152, 246
  - constant, 10
  - differentiable, 122
  - Poisson, 64, 328, 341
  - symplectic, 65
  - structure equation
    - Maurer–Cartan, 280
  - structure theorem
    - Abelian Lie group, 279
    - subalgebra
      - momentum map, 376
    - subgroup
      - closed, 280
      - Lie, 279
      - normal, 319
      - one-parameter, 273
    - subharmonic bifurcation, 100

- submanifold, 125
  - injectively immersed, 126
  - isotropic, 175
  - Lagrangian, 216
  - Poisson, 345
  - property, 125
  - second-order, 228
  - symplectic, 242
- submersion, 125
  - theorem, 125
- subspace
  - Lagrangian, 218
  - summation convention, 128
  - superconductor, 23
  - superfluid, 23
  - $^3\text{He}$ , 97
- surface of revolution, 408
- suspended system, 94
- symbol
  - Christoffel, 196
  - symmetric, 284
  - free rigid body, 57
  - symmetry, 109
  - algebra, 313
  - breaking, 43
  - broken, 18, 91
  - group, 265, 310
  - particle relabeling, 19
  - phase, 117
  - translational, 233
- symplectic
  - algebra, 294
  - bracket, 328
  - form, 66
  - geometry, 5
  - group, 72, 294
  - integrator, 176
  - leaf, 345, 347
  - manifold, 147
  - map, 69, 71, 150
  - matrix, 64, 72
  - orthogonal complement, 72, 204
  - stratification, 345
  - structure, 65

- submanifold, 242
- transformation, 70
- vector space, 66, 148
- symplectic case, 335
- symplectic eigenvalue theorem, 295
- symplectic form
  - canonical, 66, 165
  - invariance, 232, 234
  - orbit, 476
  - quantum mechanical, 68
- symplectic manifold
  - strong, 147
- symplectic stratification theorem, 345
- symplectic structure
  - coadjoint orbit, 453
  - KdV, 109
- symplectic transformation, 150
- infinitesimal, 78
- symplectic vector space
  - strong, 66
- system
  - dynamical, 12, 30
  - gyroscopic, 39
  - integrable, 15, 59
  - Kaluza–Klein, 200
  - Lagrangian, 186
  - Maxwell–Bloch, 516
  - Maxwell–Vlasov, 22
  - moving, 257
  - $N$ -particle, 372
  - nonholonomic, 60, 338
  - rotating, 253
  - suspended, 94

**T**

- tangent, 446
- bundle, 3, 123
- group, 282
- space, 123
- vector, 123
- tangent space of orbit, 313
- tangent vector
  - coadjoint orbit, 451
  - example, 452

- technicality for infinite-dimensional system, 120
- tensor, 129
  - elasticity, 113
  - inertia, 486
  - Poisson, 340
  - product, 130
- term
  - Coriolis, 262, 263
  - magnetic, 176, 177, 262, 406, 478
- theorem
  - averaging, 255
  - Chabaut's, 407
  - classical Stokes', 142
  - coadjoint orbit, 453
  - coadjoint orbit covering, 369, 463
  - collective Hamiltonian, 404
  - complex polar decomposition, 326
  - cotangent bundle reduction, 177
  - Darboux', 148
  - divergence, 142
  - equivariant Darboux, 325
  - Euler's, 290
  - existence and uniqueness, 127
  - Frobenius', 140
  - Green's, 141, 145
  - Hamilton–Jacobi, 213
  - Hodge decomposition, 446
  - Hopf–Rinow, 198
  - immersion, 126
  - KAM, 36
  - Kelvin's circulation, 21, 35
  - Lagrange multiplier, 234
  - Lagrange–Dirichlet, 36
  - Liapunov, 32
  - Le derivative, 137
  - Le–Poisson reduction, 415
  - Le–Weinstein, 348
  - Lionville–Arnold, 214
  - local 1-to-1, 125
  - local onto, 125

- Noether's, 372
- Pauli–Jost, 344
- Poincaré–Melnikov, 95
- Poincaré's, 490
- Poisson reduction, 349
- Poisson–Darboux, 348
- polar decomposition, 284
- reconstruction, 425
- relative Darboux, 326
- rigid-body stability, 505
- Sard's, 125
- Schouten bracket, 354
- Schur–Horn, 15
- Stokes', 141
- submanifold, 125
- symplectic eigenvalue, 295
- symplectic stratification, 345
- theory
  - KAM, 511
  - third fundamental theorem
    - Lie's, 280, 337
  - three-body problem, 47, 100
  - three-manifold theory, 54
  - three-sphere, 302
  - three-wave interaction, 57
  - time-dependent, 183
  - How property, 127
  - vector field, 127
- Toda
  - lattice, 453, 459
  - orbit, 448
- Toda lattice
  - Drucker, 330
  - non-periodic, 331
- top
  - heavy, 16, 41, 45, 266
  - Lagrange, 507
  - torus, 148, 158
  - KAM, 99
  - $n$ -dimensional, 278
- total
  - energy, 63, 107
  - linear momentum, 373
  - total angular momentum conservation, 8
- totally geodesic, 242
- transform
  - Legendre, 183
  - transform to equilibrium, 212
- transformation
  - canonical, 69, 70, 210
  - Cayley, 79, 80
  - group, 336
- Hamiltonian system, 158
- Legendre, 183, 227, 437
- linear, 63
- point, 170
- Poisson, 70
- symplectic, 70
- transitive, 310
- action, 310
- Hamiltonian action, 463
- translation, 393
  - fiber, 176
  - momentum, 393
  - phase space, 406
- translation map
  - left, 267
  - right, 267
- translation-invariant, 373
- translational
  - symmetry, 233
- transport equation, 111
- transported
  - parallel, 199
- transversal intersecting separatrix, 47
- traveling wave, 111, 115
- solution, 111
- trivial Poisson structure, 328
- twist
  - half, 506
- two-body problem, 263
- two-co-cycle, 397
- two-co-cycle, 378
- two-form, 129
- canonical, 165
- Lagrangian, 184, 227
- two-sphere, 128, 148
- two-tensor, 340

## U

- underwater vehicle, 41, 57
- dynamics, 60
- uniqueness of solution, 120
- unit circle, 278
  - bundle, 9
- unitary, 82, 300
  - group, 301
  - representation, 322, 394
- unitary group
  - Hilbert space, 322
- projective, 323
- unstable, 505
- formally, 39
- manifold, 94
- upper and lower estimates, 102
- upper triangular matrix, 448

## V

- value
  - critical, 125
  - regular, 125
- van der Waals fluid, 98
- variable
  - change of, 142
  - complex, 64
  - conjugate, 68
  - cyclic, 215, 260
  - external, 53
  - group, 53
  - internal, 53
  - shape, 53
- variation
  - first, 37
  - infinitesimal, 220
  - second, 37, 504, 509
  - variation of constants, 233
- variational
  - derivative, 5
  - principle, 62, 105
- variational principle, 219
  - fund, 60
  - Hamilton, 2, 221, 434
  - reduced, 434
  - rigid-body, 7

- vector
    - field, 127
    - group, 267, 274
    - Melnikov, 99
    - position, 173
    - tangent, 123
  - vector bundle map, 341
  - vector calculus and differential form, 133
  - vector field, 30
    - Hamiltonian, 12, 64, 74, 85, 162, 168, 333, 361, 412
    - invariant, 269
    - Lagrangian, 186, 227
    - left-invariant, 269
    - locally Hamiltonian, 158, 412
    - pull-back, 71
    - push-forward, 71
    - related, 277
    - second-order, 207
    - time-dependent, 127
  - vector space
    - duality, 67
    - symplectic, 66, 148
  - vectorial part, 304
  - vehicle
    - underwater, 57
  - velocity
    - angular, 250, 484
    - body, 483
    - convective, 483
    - Eulerian, 483
    - Lagrangian, 483
    - phase space, 3, 482
    - spatial, 483
  - velocity constraint
    - nonholonomic, 442
  - velocity field
    - material, 19
    - spatial, 19
  - velocity vector, 124
    - spatial angular, 432
  - vertical, 203
    - Hf, 185, 203, 240
  - vibrating antenna, 39
- 
- Virasoro algebra, 437
  - virtual work, 394
  - volume
    - form, 135, 139
  - Lionville, 149, 163
  - manifold, 149
  - preserving, 140, 151
  - volume-preserving diffeomorphism
    - group, 266
  - volume-preserving transformation
    - group, 18
  - vortex dynamics, 59, 99
  - vortex stretching, 39
  - vorticity, 21
- 
- W**
- wave
    - equation, 107, 113
    - function, 68
    - nonlinear, 59
    - traveling, 111
  - weak form
    - equation of motion, 106
  - weakly
    - dissipative, 209
    - nongenerate, 66
  - wedge product, 130, 281
  - whirling pendulum, 89
  - Whithead lemma
    - first, 371
    - second, 399
  - work
    - virtual, 394
- 
- Y**
- Yang–Mills
    - field, 179, 201
    - particle, 53
    - theory, 59