Errata for Elementary Classical Analysis, Second Edition

Jerrold E. Marsden and Michael J. Hoffman
marsden@cds.caltech.edu, mhoffma@calstatela.edu
Version: October 16, 2000.

What follows are the errata known to the authors in the 2nd edition of Elementary Classical Analysis as of the above date. Please notify one of us if you know of errata not on this list. The book will be reprinted at the next available opportunity with these errata corrected.

We would like to thank several readers for taking the trouble to send in errata, especially our own students as well as Collin Bennett, Rob Pratt, Sean Bates and Faan Tone Liu.

In these errata, "line 3↑" means "line 3 from the bottom of the page".

PREFACE

page ix, line 3↑, "transformation" should be "transformations"

INTRODUCTION

Page 2, Line 7, the symbol $\cap$ should be $\cup$.

Page 4, in the middle of the page in exercise 1, "Any" should be "Each."

Page 7, note 2, Halmos' book was published in 1960

Page 15 line 8, should be union and not intersection

Page 15 line 4↑, "countable" should be "denumerable" or "countably infinite."

Page 16, line 3↑, "... for any set ..." should be "... for every set ..."

Page 21, Problem 6 should read: "by setting up a one-to-one correspondence between the set $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ and the set $\mathbb{N}$."

Page 22, Problem 10, line 2, $h \circ (f \circ g) = (h \circ f) \circ g$ should be $h \circ (g \circ f) = (h \circ g) \circ f$.

CHAPTER ONE

Page 33, the proof of 1.1.9 requires correction. Replace " One way to do this is:..., avoiding repetitions" with "One way to do this is to consider the points in the plane with integer coordinates, say (p, q) and to assign the fraction p/q (simplified to lowest terms) to this point (leave out assignments when $q = 0$). Now starting at the origin, spiral out, listing all the fractions as you go, omitting any fraction that you have already encountered; that is, avoiding repetitions. In this way, you will count all possible fractions (positive and negative) and thereby set up a one-to-one correspondence between $\mathbb{N}$ and $\mathbb{Q}$."

Page 35, exercise 4, the words in the parenthesis should read, "Do this without using a numerical approximation for $\sqrt{2}$."

Page 41, Example 1.2.13 Solution, the second to last fraction in the first set of equations should be $\frac{\sqrt{2}}{(n-1)!}$

Page 45, In exercise 5 put the word *strictly* in italics.

Page 45, In Definition 1.3.1 define $\sup(S) = +\infty$ if S is nonempty and is not bounded above. If S is empty, let $\sup(S) = -\infty$.

Page 47, In Proposition 1.3.3 add the condition that A and B be non-empty sets.

Page 53, Definition 1.5.3 has some ambiguities in the degenerate cases. We should set:

- $\limsup x_n = +\infty$ if x_n is unbounded above.
- $\limsup x_n = \sup\{\text{cluster points}\}$ if x_n is bounded above (thus, $\limsup x_n = -\infty$ if x_n is bounded above and has no cluster points.)
- $\liminf x_n = -\infty$ if x_n is unbounded below.
- $\liminf x_n = \inf\{\text{cluster points}\}$ if x_n is bounded below (thus, $\liminf x_n = -\infty$ if x_n is bounded below and has no cluster points.)

Page 61, first paragraph, the "Properties" should range from i–v.

Page 70, exercise 5, metric should be inner product.

Page 78, exercise 7c, "any" should be "each."

Page 82, line 5↑ should read "$1, 2, 3, \ldots$ is bounded above by x.

Page 84, The Proposition should read as on Page 47 (as corrected) and in particular, line 1↑, should be $\inf B \leq \inf A \leq \sup A \leq \sup B$.

Page 85, In sentence 2 of Method 1, the phrase should be "an amount $1/2^n$" instead of "an amount $1/2$."

Page 88, 1.4.6 Proof, the second to last $|x_n|$ should be $|x_N|$.

Page 89, line 3, should read "must be larger than N."

Page 89, in the proof of iii., the statement in the sixth sentence follows not from i., but from the negation of the definition of convergence.

Page 90, line 3↑, **1.5.5ib** should be **1.5.5iia**.

Page 94, Proof iii, $d(w - v)$ should be $d(w, v)$.

Page 96, lines 5 and 6, delete comma from x_i^2, z_i^2 (twice).

Page 97, exercise 3b, any should be each.

Page 99, In Problem 20, line 4, replace " where the condition" by "of nonzero orthogonal subspaces for which the condition".

Page 99, exercise 22a, = should be $\leq$.

Page 99, exercise 23, "... for any integer ..." should be "... for each integer ..."

Page 100, exercise 29, at the beginning, "For any" should be "For each."

Page 100, exercise 35, the last) should be omitted.

Page 102, exercise 46, the beginning of the question should read "Prove that each nonempty set S of $\mathbb{R}$ bounded ...".

Page 102, exercise 47. Delete this exercise; it will be fixed in the next edition.

CHAPTER TWO

Page 105, Figure 2.1-3 the picture on the right should be $\mathbb{R}^2$.

Page 109, second line of the solution to Example 2.2.3, insert a backslash (set theoretic difference) before the $\{1\}$

Page 112, in exercise 5, R should be $\mathbb{R}$.

Page 115, Figure 2.4-2 The top most disk should not be shaded.

Page 123, Definition 2.8.2 should read "...if there is a number B such that $||x_k|| \leq B$ for every k. In a metric space we require that there be a point x_0 such that $d(x_k, x_o) \leq B$ for all k."

Page 125, exercise 2 "at least one cluster point" should read "at least one cluster point in M".

Page 128, in the first displayed equation, the term $\leq \sum_{k=0}^{n} \frac{1}{2^k}$ should read $\leq \sum_{k=0}^{\infty} \frac{1}{2^k}$.

Page 134, 4th line, $D(x, r)$ should read $D(x, R)$.

Page 134, line 7↑, the word "Proposition" is missing an "o."

Page 137, in the fourth line of equations regarding s_{2^k-1}, the last term is $\frac{1}{(2p-1)^{p-1}}$ and should be $\frac{1}{(2^{k-1})^{p-1}}$.

Page 140, solution to Example 2.1, the first equation should have "1 or" after the $>$ sign.

Page 143, exercise 1, parts c., f., g. have typesetting errors placing the "in" too close to the $\mathbb{R}$.

Page 147, exercise 30, "... any open set ..." should be "... each open set ...."

Page 148, exercise 46 a, should be, "If $f = o(g)$ and if $g(x) \to \infty$ as $x \to \infty$, then show that $e^{f(x)} = o(e^{g(x)})$ as $x \to \infty$.

CHAPTER THREE

Page 156, line 7↑, should read $1/(N-1)$ rather than $1/(1-N)$.

Page 166, after Lemma 3, add "The number r is called a *Lebesgue number* for the covering. The infimum of all such r is called *the* Lebesque number for the covering."

Page 170, in the Proof of Theorem 3.5.2, (1) C should be defined as $C = \phi^{-1}(U \cap A)$, and D should be defined as $D = \phi^{-1}(V \cap A)$. (2) The statement that C and D are nonempty since $a \in C$ and $b \in D$ should read $\phi^{-1}(x) \in C$ and $\phi^{-1}(y) \in D$.

Page 174, exercise 20 should read, "Prove that a compact subset of a metric space must be closed as follows: Let x be in the complement of A. For each $y \in A$, choose disjoint neighborhoods U_y of y and V_y of x. Consider the open cover U_y $y \in A$ of A to show the complement of A is open."

Page 175, exercise 26 should say, "Show that the completeness property of $\mathbb{R}$ may be replaced by the *Nested Interval Property.* If $< F_n >_1^{\infty}$ is a sequence of closed bounded intervals in $\mathbb{R}$ such that $F_{n+1} \subseteq F_n$ for all $n = 1, 2, 3, \ldots$, then there is at least one point in $\bigcap_{n=1}^{\infty} F_n$."

Page 175, exercise 33, " ...for any ..." should be "... for each ..." and "... $\text{int}(\text{cl}(S)) \neq \varnothing$ ..." should be "... $\text{int}(\text{cl}(S)) = \varnothing$ ...."

CHAPTER FOUR

Page 184, exercise 5 should read, "Let A and B be subsets of $\mathbb{R}$ with B not empty. If $A \times B \subseteq \mathbb{R}^2$ is open, must A be open?"

Page 191, exercise 4 should specify that c is a continuous function.

Page 191, 3rd line down from the beginning of §4.5, replace "non-continuous" by "discontinuous."

Page 208, eighth line down, the integral should be underlined: "Hence $\underline{\int_0^1}$."

Page 217, the second sentence after equation (3) should read "Then the right-hand side of (3) is bounded by..."

Page 232, exercise 9, $[a, b[$ should be $[a, b]$.

Page 233, exercise 14 c hint should be, "Consider the function given in polar coordinates by $r \tan(\theta/4), 0 \leq r < \infty, 0 \leq \theta \leq 2\pi$."

Page 234, exercise 22, the d should be ρ.

Page 236, very top of the page The $k^2 =$ should come before the fraction, not in the numerator.

CHAPTER FIVE

Page 238, line 3↑, the second x should be $f(x)$.

Page 239, in the two paragraphs after 5.1.2, the large number N should be L. There are eight of them.

Page 242, in Solution (a), the last line should read $n > (\log(\epsilon|1-x|)/\log x) - 1$.

Page 242, in Solution (b), the error should be smaller for $-x$ than it is for x rather than the absolute values of these quantities, and the numerator of the RHS fraction should be $nx^n(1-x) + x^{n+1}$.

Page 250, the caption to figure 5.3-1 should read: This sequence converges to zero pointwise, but the corresponding sequence of integrals does not converge to zero.

Page 251, the integrals in the third line of b. should be $\sum_{n=0}^{N} \int_0^x t^n dt = ... - \log(1-x) - \int_0^x \frac{t^{N+1}}{1-t} dt$.

Page 286, exercise 4, x, y should be (x, y).

Page 289, exercise 3, the second $\leq$ should be $<$.

Page 296, the last formula should read $\int_a^b f_n(x)dx - \epsilon(b-a) \leq \underline{\int_a^b} f(x)dx$.

Page 300 diagrammatically is misspelled.

Page 301, line 9↑, $x_n + 1$ should be x_{n+1}.

Page 315, line 5↑, "... $\sum_{k=1}^{\infty} g_\sigma(k)$ ..." should be "... $\sum_{k=1}^{\infty} g_{\sigma(k)}$ ..."

Page 317, exercise 5, second line, $\mathbb{R}$ should be $\mathbb{R}^m$, and in the next line, $||f(x)||$ should just be $|f(x)|$.

Page 317, Problem 11b, "for all $x, y \in X$." should be "for all $x, y \in X, x \neq y$."

Page 318, exercise 13, omit the hint.

Page 319, exercise 25, g should be f (twice).

Page 319, exercise 27, after the comma, the sentence should be, "continuous, and that $x_n \in]a, b[$ with $x_n \to b \ldots$"

Page 320, exercise 29 d, $f(x)$ should be $x \sin(1/x)$.

Page 321, exercise 45 a, the word "equicontinuous" should come before "sequence" and f_n should be f_k. Same correction in exercise 47, page 322.

Page 324, exercise 59 c, the first c should be omitted from the equation.

Page 325, exercise 66, the last equation of the exercise should be "... that $t_n \leq Cn^{-1/\beta}$.

CHAPTER SIX

Page 329, figure 6.1-2, of the figure on the left, x_2 should be y, and x_1 should be x.

Page 332, line 8↑, $\mathbf{D}L = L$ should be $\mathbf{D}L(x_0) = L$.

Page 345, 6.5.1 Chain Rule, second line, $\mathbb{R}^n$ should be $\mathbb{R}^m$.

Page 370, line 7↑, ∂x_1 should be ∂x_i.

Page 379, line 9, $h = 2x$ should be $h = \lambda x$

CHAPTER SEVEN

Page 399, In Corollary 7.2.2, the partial derivatives in the second matrix, (the matrix whose inverse is taken) should be with respect to $y_i, i = 1, ..., m$, not x_i.

Page 403, and 427. In Theorem 7.4.1, replace the last line by "*class* C^r *and a neighborhood* W *of* x_0 *in* $\mathbb{R}^p$ *such that* $g \circ f(x_1, \ldots, x_p) = (x_1, \ldots, x_p, 0, \ldots, 0)$ *for all* $(x_1, \ldots, x_p) \in W$."

Page 439, line 8↑, replace $\cos\theta$ by $\cos\varphi$

CHAPTER EIGHT

Page 464, in Example 8.5.6, the first line of the solution should read: If f were absolutely integrable on $[1, \infty)$, then $|f|$ would be integrable...

Page 468, in line 3 of Example 8.6.6, $[a_{i+1}, a_i]$ should be $[a_{i+1}, a_i[$ and in the next line, the sum should be $\sum_{j=1}^{\infty} f_j(a_{j+1} - a_j)$ and in line 2↑ the sum should be $\sum_{j=1}^{n-1} f_j(a_{j+1} - a_j)$.

Page 477, in Step 1, in lines 5 and 6, change f to g, three times.

Page 478, in the last equation, omit the prime ' after $S \in C$ which is located under the summation sign.

Page 479, fifth line, $1_A(x) - 1_A(y)$ should be $|1_A(x) - 1_A(y)|$.

Page 479, 8.3.4ii, part of the sentence should be "... *for all* x *in* A *and*..."

Page 490, In Problem 11, $v(S)$ should be $v(A)$.

Page 494, In Problem 39, the last term in the sum should be $1/(2n)$, not $1/(2^n)$.

CHAPTER NINE

Page 500, In line 2↑ $dydy$ should be dy.

Page 513, line 4↑, the beginning of the second sentence should be,"Let $0 = y_0 < \ldots$"

Page 513, line 3↑, part of the sentence should read, " $\ldots$ partition of an interval $[0, y_n]$ with $y_n > \sup\{f(x) | x \in [a, b]\}$ containing the range of $f \ldots$"

Page 536 In Problem 8, all the ν should be v. Also, $f(u, v)$ denotes the function f after the change of variable is made. Part of the exercise is to figure out the notation.

CHAPTER TEN

Page 547, in the last line of definition 10.1.4 replace ∞ by n.

Page 554, in the formula for $P_n(x)$, $2n + 1$ should be $\sqrt{2n + 1}$

APPENDIX A

Page 664, Problem 9b The integral of f over B should be the integral of g over A

Page 665, line 4↑, "coordinates" is missing an r.

APPENDIX B

Page 676, Miscellaneous Exercise 66.(b), "linear" is missing the "e"

APPENDIX C

Page 685, the hint for 1.5 (3) should be "Use **1.5.5** to show that there are points $x_{N(n)}$ within $\frac{1}{n}$ of a (or b).. . . "

Page 686, 1.7, exercise 1, "The sup norm is . . ." should be "The distance given by the sup norm . . ."

Page 686, 1.8 Complex Numbers, exercise 1 b should be $(11/5) + 2i$.

Page 688, exercise 39 a, the denominators $(x^2 + y^2)$ should be $(x^2 + y^2)^2$.

Page 688, 2.1, exercise 5 should be "No in general; yes if B is open or $0 \notin B$.

Page 689, 2.5 (1), instead of $\mathrm{cl}(S)$, the answer should be $\mathrm{cl}(A)$.

Page 689, 2.5 (5), capital D should be d, and a lower case b should be capital B.

Page 691, 3, omit "B is closed and $B \subset A$, then $B \subset cl(A)$."

Page 691, 17, the beginning should say, "The series $\sum(\sin m)x_m$ converges . . . "

Page 693, exercises for Chapter 3, (1 c) should be $n = 1$: compact and not connected and $n \geq 2$: compact and connected.

Page 694, exercise 11b, The suggestion given is actually for part c.

Page 694, Problem 15a In first line, "form" should be "from."

Page 695, exercise 31 should read, "The set A must be either not closed or not bounded or both. Treat these cases separately. If A is not closed, there must be an accumulation point of A which is not in A.

Page 696, 4.1 (5a) should read $f(x) = 1$, $U =$ any non-empty open set.

Page 696, 4.1 (5b) should read "1, if $x \geq 1$."

Page 696, 4.2 (3), B_n should be B.

Page 698, the last sentence of 4.8 (7) should read "Show that inf $U(f, P) \leq$" rather than "Show that $U(f, P) \leq$."

Page 698, Ch.4 (3 a), k should be K.

Page 698, Ch.4 (5), replace the last $=$ by $\geq$.

Page 698, Ch. 4 (9), $f^{-1}(F) \cap f^{-1}(F)$ should be replaced by $f^{-1}(F) \cup g^{-1}(F)$.

Page 698, Ch. 4 (13), $\sup(F(V))$ should be $\sup(f(V))$

Page 701, 5.4 (7), the three capital E's should be replaced by e's.

Page 701, 5.6.3 (a), the reference should be to **5.6.6** instead of **5.6.4**.

Page 701, 5.6 (5), The reference to Example 1 should be **5.6.4**

Page 702, Ch. 5 (3 a), The answer should be "Converges pointwise on $\mathbb{R}$. Uniformally on $]-\infty, b]$. Discontinuities at the positive integers."

Page 702, Ch. 5 (3 d)The answer should be "Converges uniformally on any set bounded away from odd multiples of π where the sum has discontinuities."

Page 702, Ch. 5 (5), The answer should be "The key inequality is

$$\|f_k(x)g_k(x) - f(x)g(x)\| \leq |f_k(x)|\|g_k(x) - g(x)\| + |f_k(x) - f(x)|\|g(x)\|.$$

Page 702, Ch. 5 (7), there is a period missing before "So ..."

Page 703, Ch. 5 (29 b), interval $[1, \infty]$ should be $[1, \infty[$, and $[0, \infty]$ should be $[0, \infty[$.

Page 703, Ch. 5 (31), "$|(a_1) + \ldots$" should be "$|(a_1 - a) + \ldots$" in the fourth and fifth lines of the solution.

Page 704, exercise 47, "countably" should be "countable."

Page 704, exercise 53 b, 2 should be e^2.

Page 704, exercise 57, In the third term of the last line, "f_n" should be "f."

Page 705, exercise 63, The beginning of the solution should say, "Let $u_k = (\alpha(\alpha - 1)\ldots$"

Page 706, 6.4, exercise 5, "Study Example 2 of §**6.3**." should be "Study Example **6.3.4**."

Index

Page 734, add "Lebesgue number, 166" in the index.