Control of Compression Systems with Magnitude/Rate Limits

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Goals
• Describe how bifurcations & limit cycles arise in engineering applications
• Review some tools for characterizing bifurcations and limit cycles
• Show how feedback can be used for design of (nonlinear) dynamics

Outline
• Lecture 1: Introduction and background
• Lecture 2: Analysis and control of bifurcations
• Lecture 3: Modeling and control of limit cycles
• Lecture 4: Describing function analysis

http://www.cds.caltech.edu/~murray/wiki/cds140-bifctrl

Bifurcation Control Using 1D Bleed Valves

\[
\Psi = \frac{1}{4B^2 I_c} \left( \Phi - (\gamma + u) \sqrt{\Psi} \right)
\]

\[
u = kJ
\]

(Liaw and Abed, 1992)

Remarks:
• Can show system is not stabilizable ⇔ can only achieve operability enhancement
• Achieve performance benefit by engine redesign; operate closer to peak pressure
• 2D actuation (IGV, BV, or AI) gives stability extension, but more complex (?)
Implementation on Caltech Compressor Rig

Bleed valve control

$P_c$ vs $\phi$

- Open loop
- Closed loop

Inlet
Nozzle
Axial-flow fan
Control bleed valve
Disturbance valve
Throttle

Bifurcation Control w/ Magnitude and Rate Limits

Ideal control
Effect of magnitude limit
Effect of magn + rate limit

Noise destabilizes system
Control action: increase domain of attraction
Analysis of Effects of Magnitude and Rate Saturations

Approximate MG3 model using center manifold near bifurcation point

\[
\frac{dJ}{d\xi} = a_1 (\gamma - \gamma_0 + u) J + a_2 J^2 + O(\delta J^3), \quad a_1 = \frac{2\sqrt{\epsilon_C \mu}}{m + \mu}, \quad a_2 = \frac{1}{4(m + \mu)} \left( \psi_C^m + \frac{\psi_C^m}{\sqrt{\epsilon_C}} \right)
\]

Compute operability enhancement using piecewise linear approximations

\[
\Delta = u_{mag} \left( 1 - \frac{2}{\sqrt{\sigma \eta}} \arctan \left( \frac{\psi_C^m}{2 \sqrt{\epsilon_C}} \right) \right), \quad \sigma = \frac{\alpha_1 u_{mag}}{\alpha_2 \epsilon}, \quad \eta = \frac{\alpha_2 \epsilon u_{mag}}{u_{rate}}
\]

Wang & M, Conference on Decision and Control, 1997

Idea: Change the Compressor Characteristic

Adjust angles of flow of air injection to move between compressor characteristic curves

- Shape of the compressor characteristic affects \( \Psi'^', \Psi''', \) etc.
Experimental Results Using Modified $\Psi_c$
Yeung & Murray, JPC 97

Use steady air injection to shift $\Psi_c$
- Changes shape of $\Psi_c$ to give lower rate requirements
- Implication: actuator requirements strongly affected by system design

Other mechanisms possible
- Blade redesign
- Casing treatments

Control-Configured Design
required to minimize actuation requirements

Bifurcation Control in the Presence of Magnitude Limits

Analytical study of effects of magn limits
- Rely on normal forms around center mfd
- Different actuation mechanisms possible
- Gives qualitative information, insight
- SS-2: magn constraints don’t limit operability
- SS-5: can’t make equil pt arbitrarily small
Control of Rotating Stall Using Air Injection

Use air injection (AI) to modulate compressor characteristic

- Air injectors modify local angle of attack ⇒ affect $\Psi_c$
- MG3 model indicates operability enhancement should be possible

$$\Psi_c = \Psi_{c,\text{nom}} + \Psi_{c,\text{ai}}$$

Caltech Low Speed, Axial Flow Compressor Experiment

Rig parameters
- Operating point: 6,000 rpm (100 Hz)
- Stall frequency: 3925 rpm (65.4 Hz)
- Jet flow: 0.1% to 1% of mean flow
- On/off actuators, approx 200 Hz BW
Control using Axisymmetric Air Injection: No Overlap
(Proportional feedback with 1D actuation)

Results are similar to control with bleed valves
- Change the bifurcation behavior of the system through active feedback
- Specifically: subcritical $\rightarrow$ supercritical bifurcation

Pros/cons versus bleed valves
- **Pro**: bandwidth requirements appear less severe (based on experience)
- **Con**: requires new actuators $\Rightarrow$ increased weight, cost, complexity

Control Using Non-Axisymmetric Air Injection

Basic idea: inject air only where we need it:

Many parameters to adjust for optimal operation
- Actuation: magnitude, angle, and span of the injected flow
- Control: length of pulse, timing of pulse
Experimental Results
D’Andrea, Behken, Murray, JTM 98

Control algorithm:
• Detect magnitude, phase of stall cell
• Turn on injector when cell is in window
• Leave injector on for fixed time

Detailed View of Control Action (B)
**Actuator Authority Tradeoff**

Yeung, PhD 1998

![Graph](image)

**Remarks:**
- With optimization, can reduce power flux to approx 0.15%
- Additional tradeoffs by varying number and width of injectors

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**Summary: Active Control of Rotating Stall**

**Multiple Mechanisms for Active Control of Compressor Stall/Surge**
- Modal control using 2D (spatial) actuation
- Bifurcation control using 1D and 2D actuation
- Experimental evaluation + extensions to realistic situations (compressible, multi-stage, etc)

**System Level Trades Indicate Areas for Improvement**
- Key issue is spatio-temporal bandwidth versus cost/weight/complexity
- Normal operation does not require control ⇒ operational cost is potentially high