Dynamical Systems, Optimal Control, and Microsat Formation Flight

Jerrold E. Marsden

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Five Objectives of Microsat

- 1. Dynamics of multiple spacecraft in nearby orbits
- 2. Formation stabilization strategies
- 3. Formation reconfiguration algorithms
- 4. Mathematical modeling and simulation tools
- 5. Power requirements and limitations

Information

- ***URL's***
  - http://www.cds.caltech.edu/microsat/
  - http://www.cds.caltech.edu/~marsden/
  - http://www.cds.caltech.edu/~murray/

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Team

- ***Faculty:*** Jerry Marsden, Richard Murray, CIT, Meir Pachter, Air Force Institute of Technology
- ***Postdocs:*** Wang-Sang Koon, David Chichka,
- ***Graduate Students:*** Dong-Eui Chang, Alex Fax, Mark Milam, Bill Dunbar, plus other student help

Related Activities with JPL

- ***Genesis Discovery Mission***
  - Uses 3-body problem libration point dynamics and heteroclinic connections
- ***Jovian moon missions***
  - Uses transfers between different 3-body problem libration point dynamics
- ***Terrestrial Planet Finder***
  - Uses coordinated control and 3-body dynamics
- ***Development of LTool***
  - software for libration point missions

Posters

- ***Nonlinear Dynamics & Formation Flight***
  - Study of candidate reference orbits whose nearby orbits may support formation flight
  - Development of a geometric mechanics framework for the Kepler-J_2 dynamics

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- ***Nonlinear Dynamics & Formation Flight***
  - Study of candidate reference orbits whose nearby orbits may support formation flight
  - Development of a geometric mechanics framework for the Kepler-J_2 dynamics
- ***Control and Optimal Control of Formation Flight***
  - Optimal control for formation reconfiguration
  - Cooperative control
  - Lyapunov-based global orbit transfer

Three Topics

- Dynamics of satellites in Earth orbit
- Cluster reconfiguration
- Orbital transfer.

A Few Key References

- Broucke, R. A. [1994] *Numerical integration of periodic orbits in the main problem of artificial satellite theory*. **Celestial Mech. Dynam. Astronom. 58 (1994), 99–123.**
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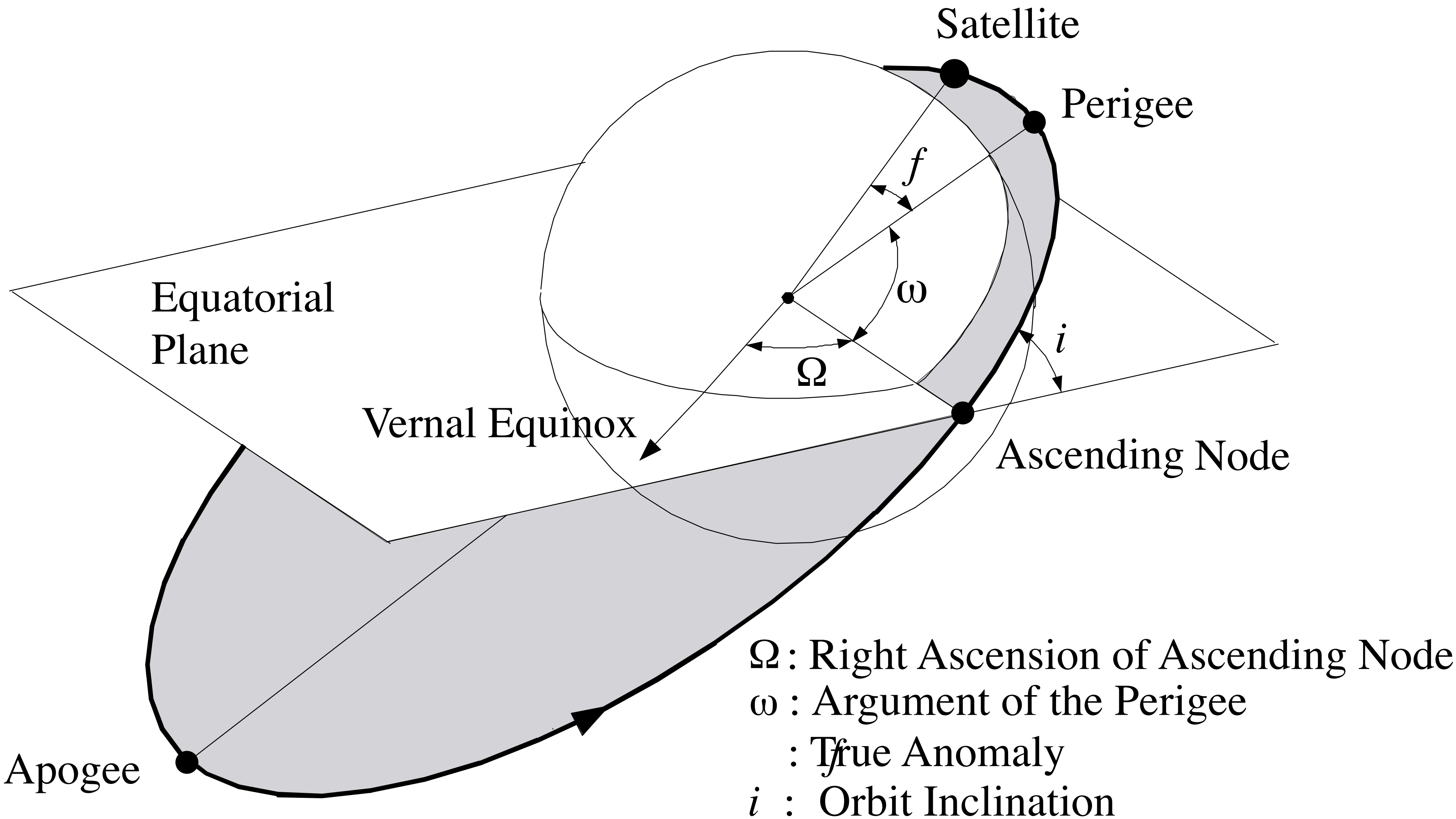
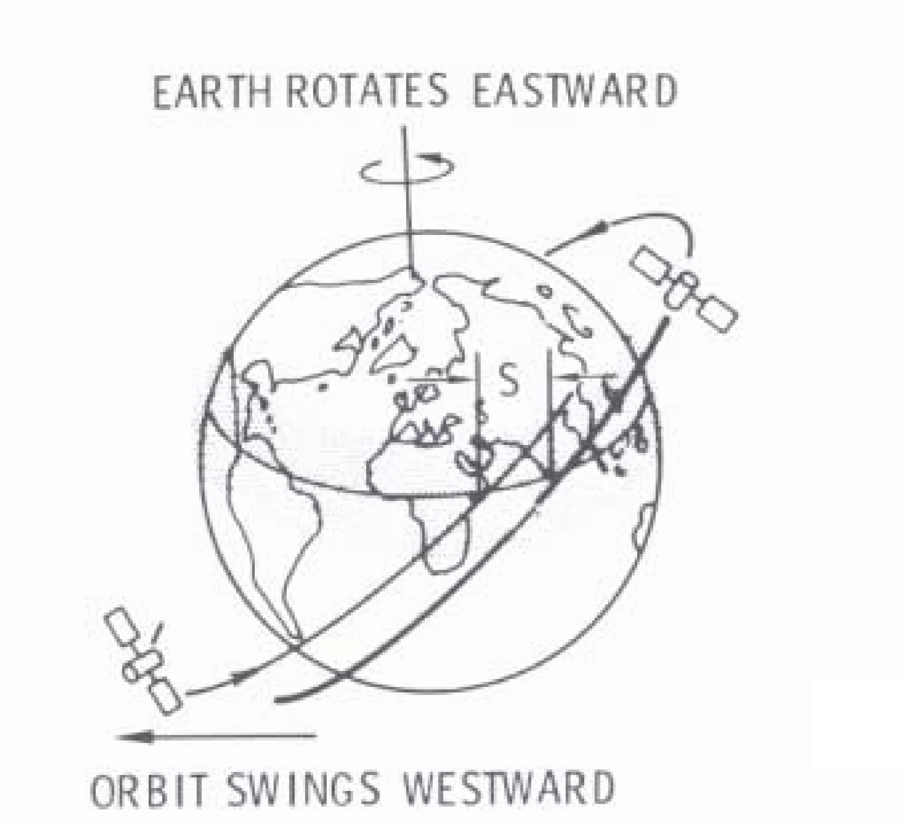
Basic J_2 dynamics

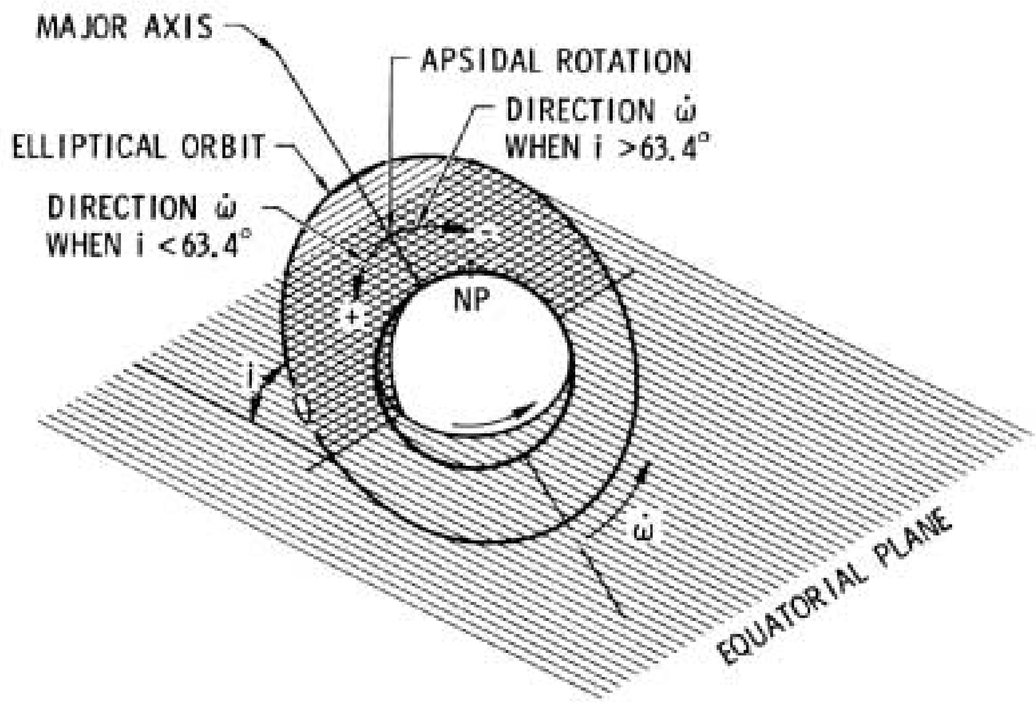
- ***Satellite in Earth orbit***
  - motion in Kepler potential plus J_2 perturbation
  - J_2 = ***bulge of the Earth***
- J_2 ***causes***
  - a drift in the orbit plane (geometric phase)
  - a drift in the major axis of the ellipse within the (approximate) orbital plane (direction of drift depends on the angle of inclination)

Basic J_2 dynamics



Basic J_2 dynamics



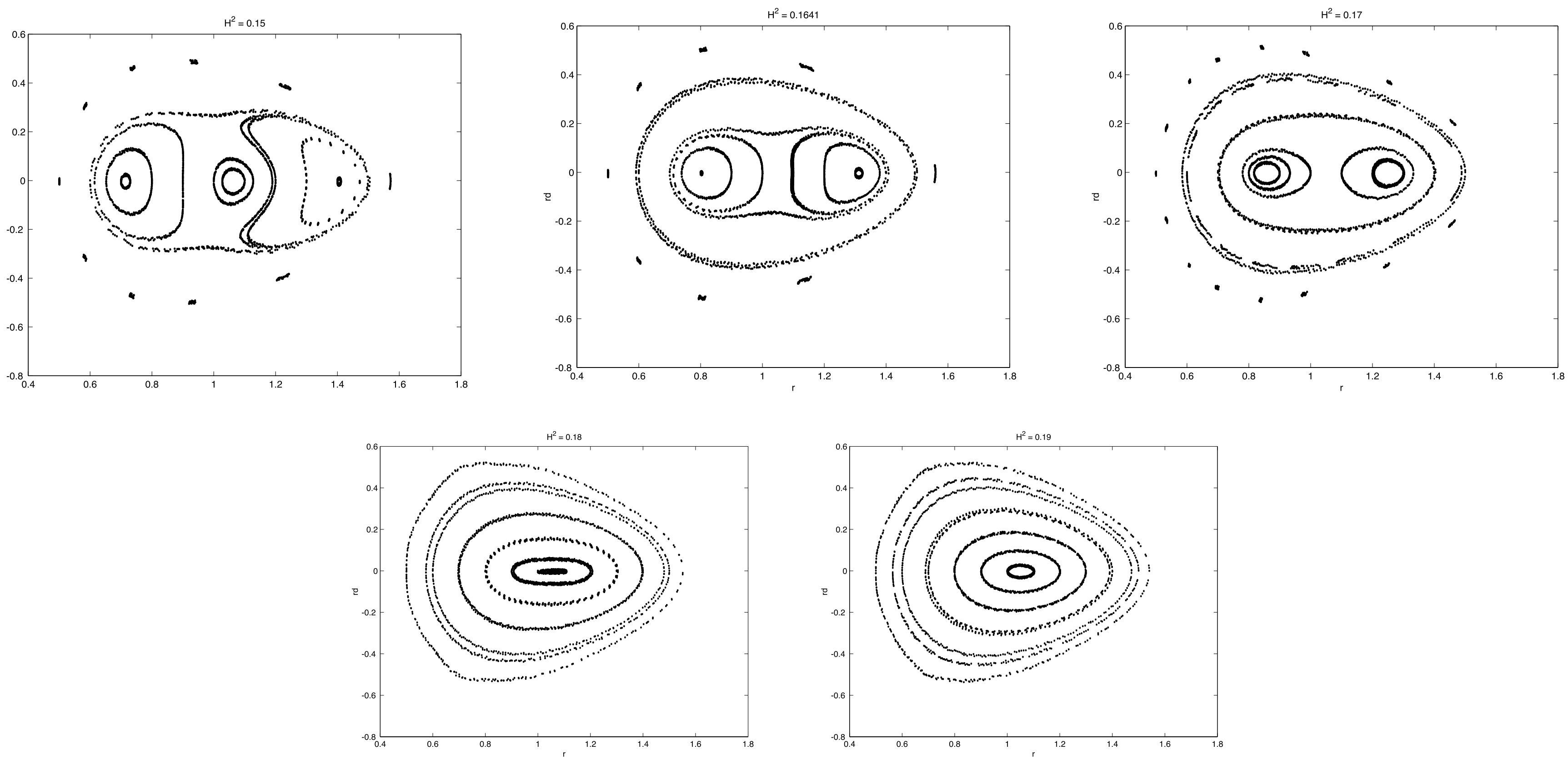


Example: **inclination = 28 degrees, altitude 300km**
period of Ω = roughly 50 days (left)
period of ω = roughly 30 days (right).

Poincaré Sections

- ***Shows rich nonlinear dynamics***
- ***Relative to rotations around the z-axis***
  - Involves Routh reduction; one studies orbits for various energies h and angular momenta ν about the z-axis.
  - Original problem 6 dimensional: constant h and ν and remove angular variable gives 3 and a Poincaré section in that is 2-dimensional.

Poincaré Sections

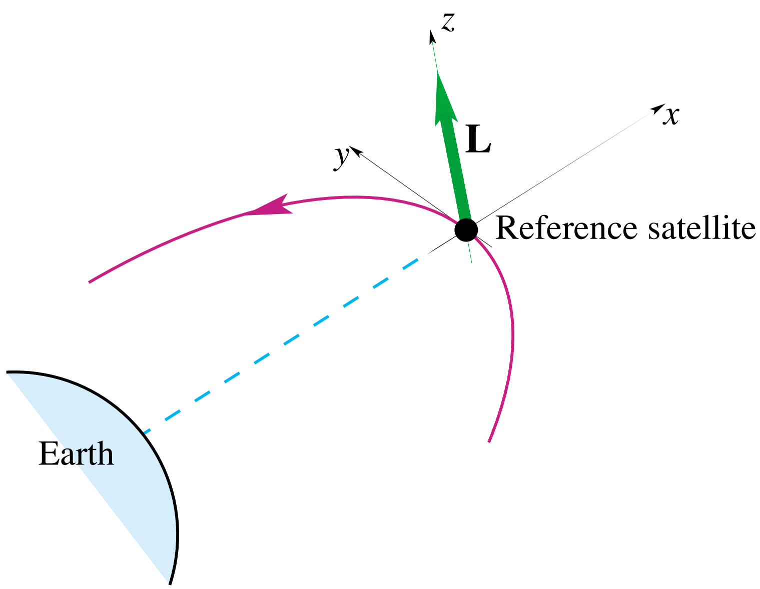


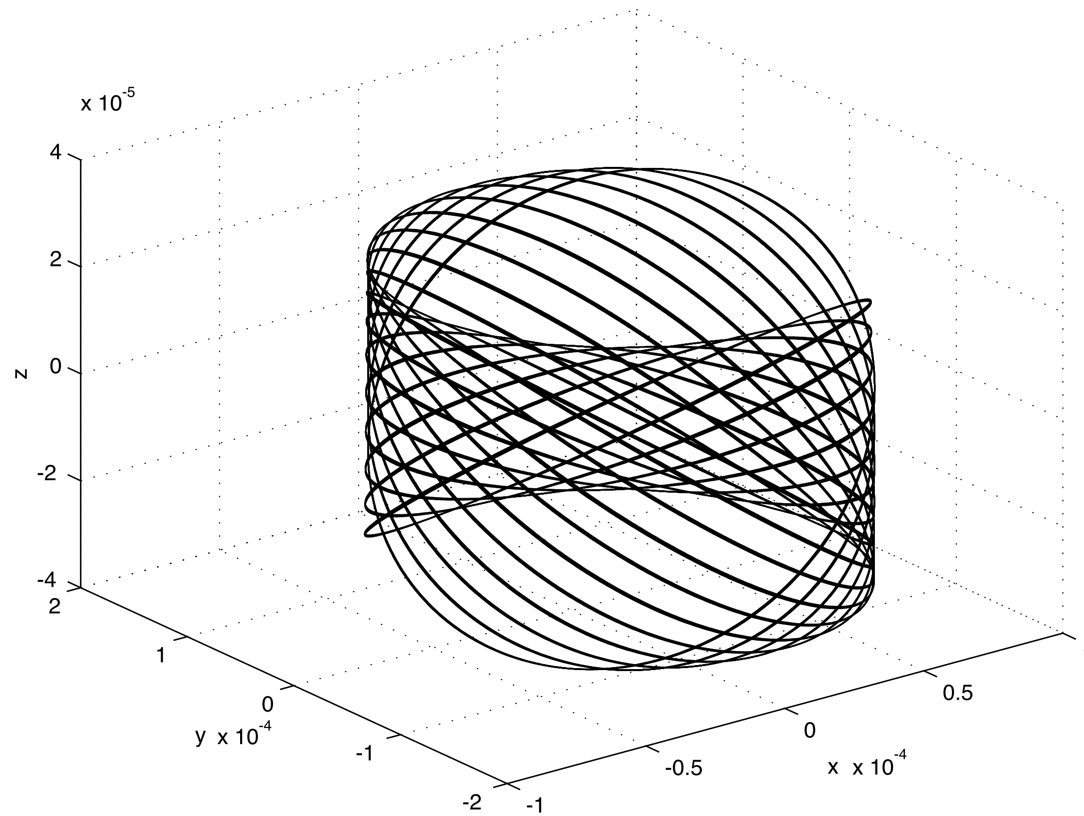
Poincaré maps for the J_2 problem (Broucke, 1992)

Relative Motions

- Poincaré sections useful for detecting bifurcations.
- What about relative motions?
  - Are there relative motions that remain tied together?
  - Not obvious because of phase drifts, etc.
- To answer this we need to have a closer look at the dynamics and the role of reduction theory.
- Indeed, there are such interesting orbits (see the posters for details)

Relative Motions





Relative dynamics (100 days) of a satellite in a frame moving with a reference satellite

Geometric Mechanics and J_2

Reduction theory

- all the crucial features (Ω-drift as a geometric phase etc) come out naturally
- gives a global picture of the dynamics.

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Symmetries

- rotational S^1 symmetry about the vertical axis.
- A $\mathbb{Z}_2$ symmetry—reflection in the equatorial plane
- Another $\mathbb{Z}_2$ symmetry—reflection in vertical planes + time reversal—an *antisymplectic* symmetry.

Moser Regularization

- Kepler Hamiltonian H_0 and dynamics transfered to geodesics on S^3.
- Symmetry group is now SO(4)
- Momentum map includes the Laplace-Runge-Lenz vector (a vector pointing to the periapsis):

$$(\mathbf{r}, \dot{\mathbf{r}}) \mapsto (\mathbf{L}, \mathbf{A}) = \left(\mathbf{r} \times \dot{\mathbf{r}},\ \dot{\mathbf{r}} \times (\mathbf{r} \times \dot{\mathbf{r}}) - \mu \frac{\mathbf{r}}{\|\mathbf{r}\|} \right).$$

Symmetry & Reduction

- On an energy surface, Keplerian orbits are closed with the same period—Kepler flow gives an S^1 action
- Total Hamiltonian is a sum:

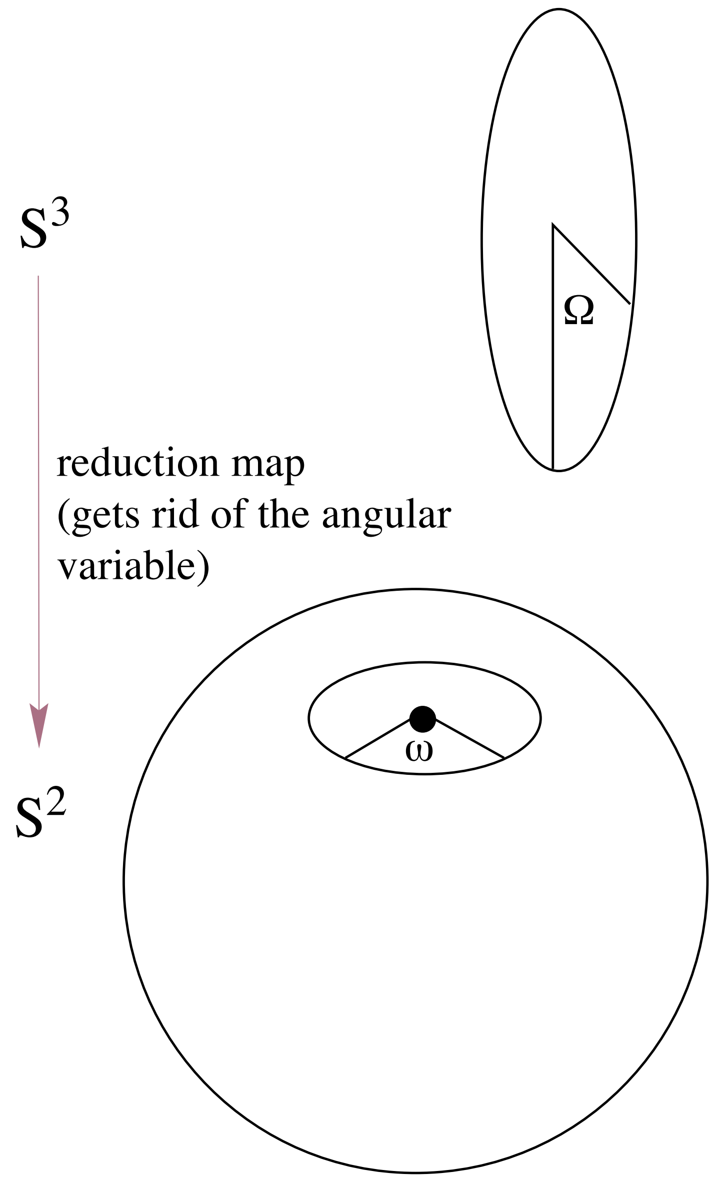
$$H = H_0 + \epsilon H_1; \quad \epsilon = J_2 \text{ size}$$

- Average with respect to the Keplerian S^1 action.
- Averaged Hamiltonian has symmetry

$$S^1 \times S^1 \times \mathbb{Z}_2 \times \mathbb{Z}_2.$$

- *Double* reduction–a Keplerian S^1 symmetry and an axial S^1 symmetry
- Discrete symmetries pass to the reduced space

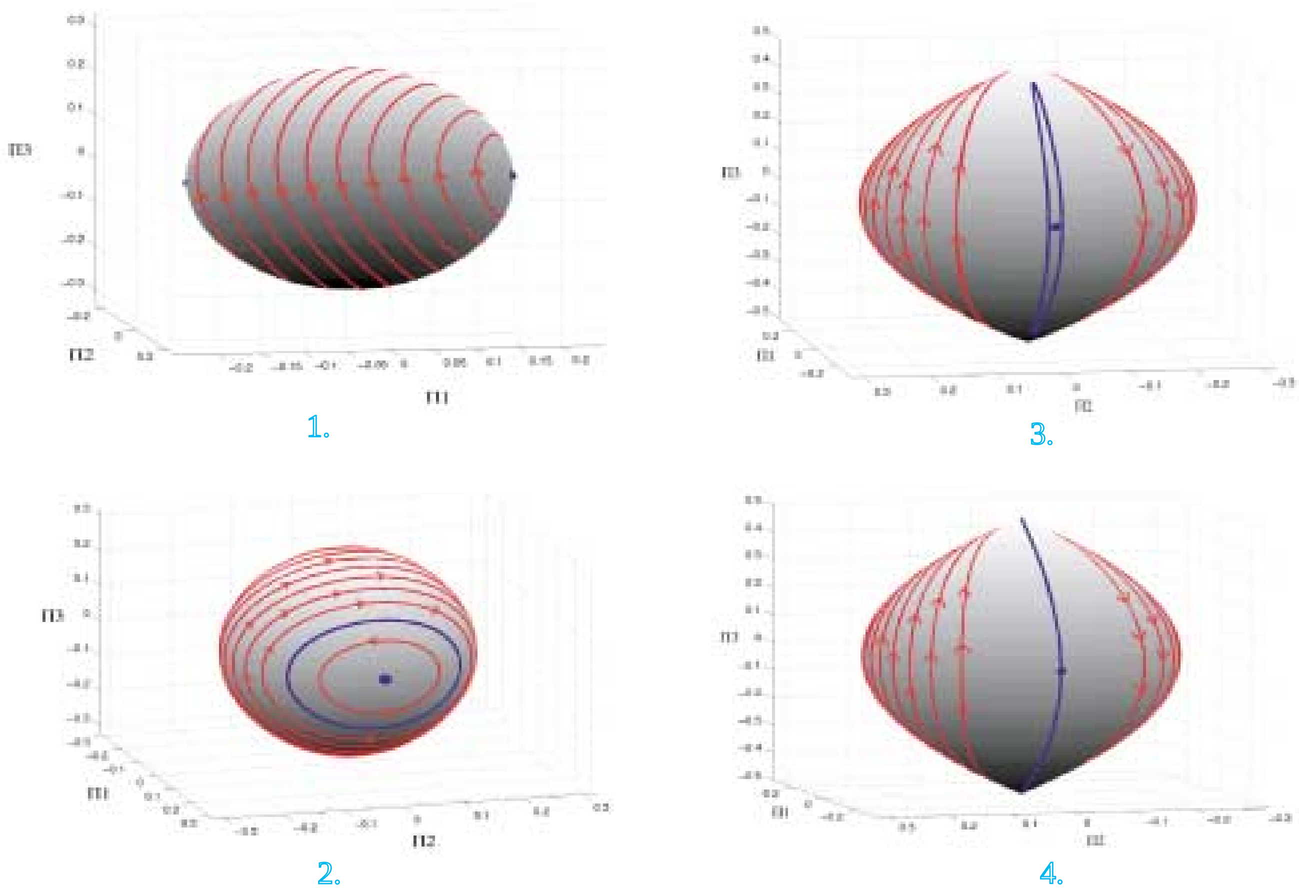
Symmetry & Reduction



Final Reduced Space–S^2

- flow parametrized by (Keplerian) energy h and ν
- *bifurcations* as h and ν are varied; bifurcation occurs at the *critical inclination*: $h^2 - 5\nu^2 = 0$
- fixed points on S^2 = periodic orbits on S^3
- *singularity* if angular momentum $\nu = 0$; *singular points* are orbits that head directly into poles
- can use *energy-momentum* methods for stability and bifurcation
- figures show a series of reduced phase portraits for fixed h as ν decreases to zero.

Final Reduced Space–S^2



1. $h/\sqrt{5} < \nu < h$
2. $0 < \nu < h/\sqrt{5}$, **(bifurcation at critical inclination)**
3. ν **near zero, 4.** $\nu = 0$ **(singular case).**

Variational integrators

- variational (symplectic) algorithms are remarkably good in many problems, especially for long term, sensitive integrations
- need to use symmetry properly
- We have developed a reduction theory for discrete mechanics and have applied the associated variational integrators to the J_2 problem.

Spacecraft Clusters

- ***Loose control***
  - For example, the simultaneous in situ measurement of the magneto-sphere may require a loose constellation scattered all over the magneto-sphere.

Spacecraft Clusters

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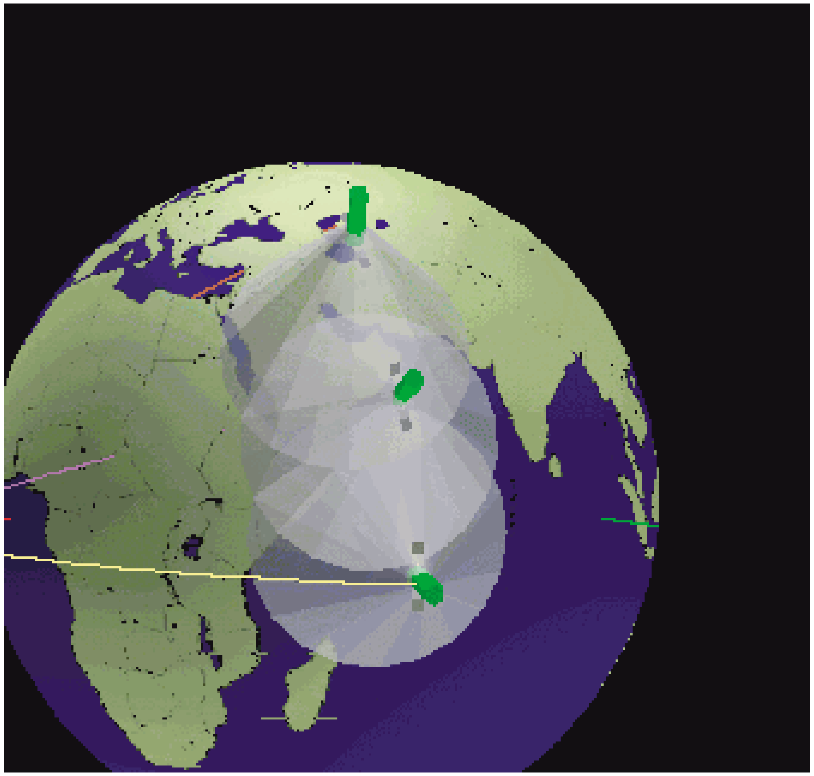
Precision control

- For interferometry, the shape and orientation of the formation must be maintained to some degree.
- Either
  - one has to maintain it very precisely or
  - one has to ***know*** the relative positions precisely so that this information can be managed in software.

Spacecraft Clusters

Performance Metrics

- 1. ***Coverage analysis***–estimate of the amount of data collected. Based on this metric, one could then make quantitative tradeoff studies and compare different designs.



Spacecraft Clusters

- 2. ***Propulsion requirements***–the use of natural dynamics greatly affects this.
- 3. ***Power requirements***–eg, battery size contributes significantly to the mass of the spacecraft.
- 4. ***Spacecraft mass***–depends not only on the trajectory design, control algorithm, propulsion and power subsystems, but also on the launch vehicle capability and launch deployment strategy.

Formation Maintenance

- Natural dynamics plays a critical role.
- since only small changes are presumably needed, standard *linear control techniques* should be adequate for maintenance.
- one still needs to deal with cooperative techniques as well as possible combinatorial explosions. Graph theoretic methods probably useful.
- Utilize the fact that this is a problem with two greatly differing scales: the relative distances between the satellites (the *shape dynamics*) and the absolute position of the cluster as a whole. Geometric mechanics can help in the separation of these effects.

Formation Reconfiguration

- ***More challenging***
  - Larger dynamic motions are involved
  - **Example:** reorient the whole formation
  - Fuel useage is potentially critical
- ***Optimal control***
  - Requires a good first guess
  - Works best when used with the natural dynamics
- ***Two examples***
  - *Trajectory correction maneuvers (TCM)* for halo orbit insertion
  - Earthbound satellite reconfiguration

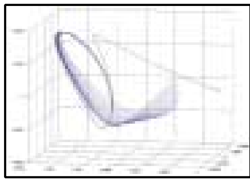
TCM for Halo Orbit Insertion

- Optimization software used is COOPT: (COntrol–OPTimization)
- Optimizes: *cost function* = ΔV subject to the *constraint of the equations of motion*.
- We vary the number of impulses and also consider the effect of *delaying the first impulse and the launch uncertainty*.
- Makes use of the dynamical systems structure of the three body problem (especially the invariant manifolds of halo orbits).

Movie-Optimal Insertion

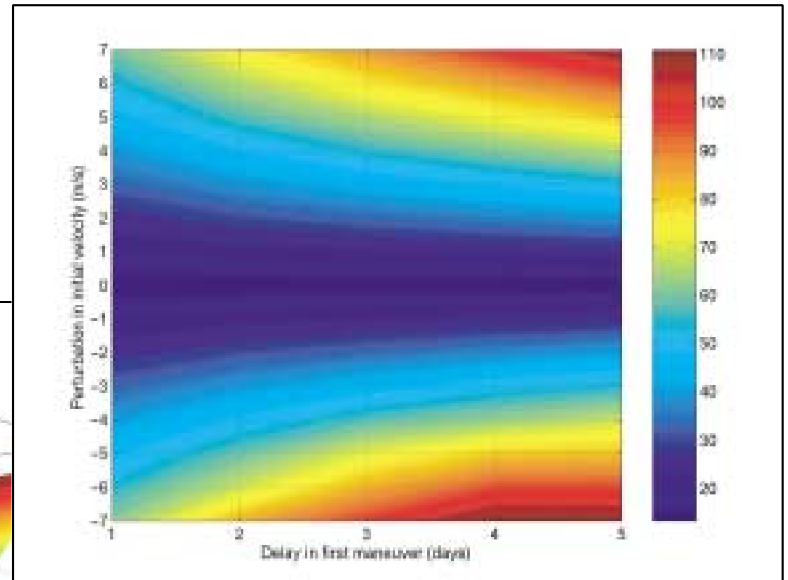
movie insert

Sensitivity Analysis



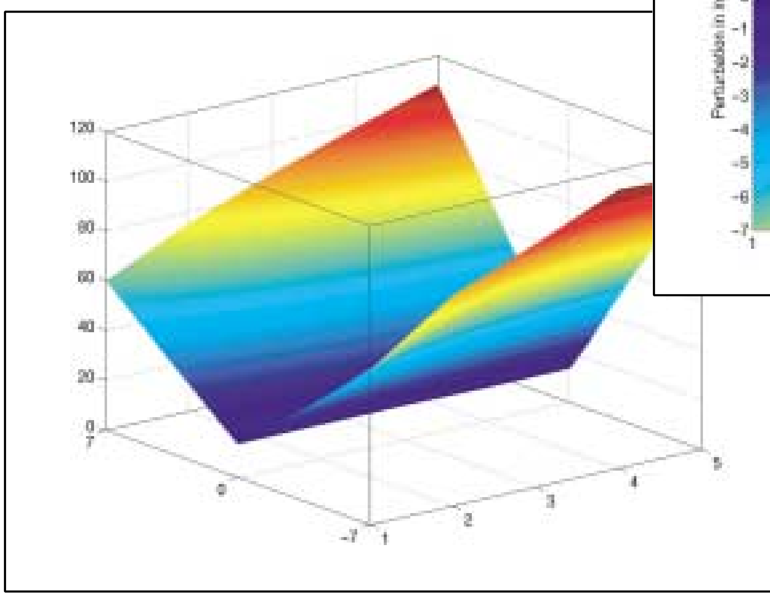
Parametric Study of the Optimal Solution

Computational Science and Engineering



Number of maneuvers:

- **Unperturbed injection velocity: 1**
- **Perturbed injection velocity: 2**



Influence of:

- **Delay in TCM1**
- **Perturbation in launching velocity**

Optimal solutions found for all cases

Varying launch velocity and first manouver delay

Earthbound Satellites

- ***Alternative software for the reconfiguration problem***
  - NTG (nonlinear trajectory generation)
  - Also uses direct (brute force) optimization
  - key difference with COOPT: substitutes the forces using the equations of motion, so one gets a ***higher order cost function***
  - avoids treating the equations of motion as constraints (horrors!)

Earthbound Satellites

Sample problem

- Consider the toy problem of the equations linearized about a circular orbit ***without any J_2 effect***.
- This is ***not realistic***, but it is a demonstration that ***the software is effective***.
- More generally, one would have to use the full non-linear equations about a nominal trajectory family.
- Linearization methods here may not be adequate, but the sofware treats the nonlinear problem with no difficulty.

Earthbound Satellites

- **Specific Objective**: Find a trajectory that minimizes the control energy, over a fixed time, for three microsats such that at the final configuration the the microsats will remain on a circle (projection onto the yz plane) of 100 m radius, 120 degrees apart with no control force, indefinitely.
- Use a ***rotating frame*** with coordinate axes so that the x-axis is along the line of sight from the earth, while the y-axis is along the orbit and z is perpendicular to these two. The yz plane is what you see looking up from the Earth.

Earthbound Satellites

- **Cost Function**: minimize

$$J = \int_0^T \sum_{i=1}^{3} \left(|a_{x_i}|^2 + |a_{y_i}|^2 + |a_{z_i}|^2 \right) \, dt$$

- **Equations of Motion** in normalized coordinates:

$$\begin{aligned} \ddot{x}_i &= 2\dot{y}_i + 3x_i + a_{x_i} \\ \ddot{y}_i &= -2\dot{x}_i + a_{y_i} \\ \ddot{z}_i &= z + a_{z_i}, \end{aligned}$$

where (x_i, y_i, z_i) is the position of the ith satellite, $i = 1, \ldots 3$ relative to the reference circular orbit.

Earthbound Satellites

- Note that if one substitutes from the equations of motion, then one gets a cost function that depends on second order derivatives.
- **Final Time Constraints**: Satellites should move in the known circular solutions of the equations that lie in the plane $2x + z = 0$ with line of sight radius chosen to be $R = 100$m.

Optimal Reconfiguration

3D movie insert

Optimal Reconfiguration

`Earth movie insert`

Information Exchange

- The optimal reconfigurations above assume, implicitly, a centralized controller with full state information.
- Can formations (nearly) recover optimal trajectories through minimum transfer of information between satellities (or vehicles) when only certain avenues of communication and sensing are available?
- The communication topology and information flow design are important questions.
- *Example*: six cars asked to acquire points on a regular hexagon (relative to one another) where each car can only sense its position relative to another car and can only communicate with one other car.

Information Exchange

- Movie 1 shows the solution for the hexagon problem when the only sensing information available to each car is the relative position of the car behind.

Information Exchange

- Movie 2 shows the solution when, in addition, some message passing from cars to cars ahead of them is available. (Reasonably close to the optimal solution).

Lyapunov Orbit Transfer

Key Features

- Method of orbit transfer for the Kepler problem with thrust control, based on Lyapunov stability theory.
- Makes use of the conserved quantities of the Kepler motion; the angular momentum $\mathbf{L}$ and the Laplace-Runge-Lenz vector $\mathbf{A}$.
- Traditionally, orbit transfer is designed using conic-section geometry and orbital elements. No guarantee of convergence.
- The controller we design makes it possible to use continuous thrust so that we can do orbit transfer without necessarily using a large impulsive thrust.

Lyapunov Orbit Transfer

- Lyapunov-based methods are efficient, guarantees convergence and reduces to many well known cases such as transfer between circular orbits and a one-pulse inclination change.
- Basic: $\mathbf{L}$ and $\mathbf{A}$ uniquely specify a Keplerian orbit.

Some key points

- Equation of the motion with control

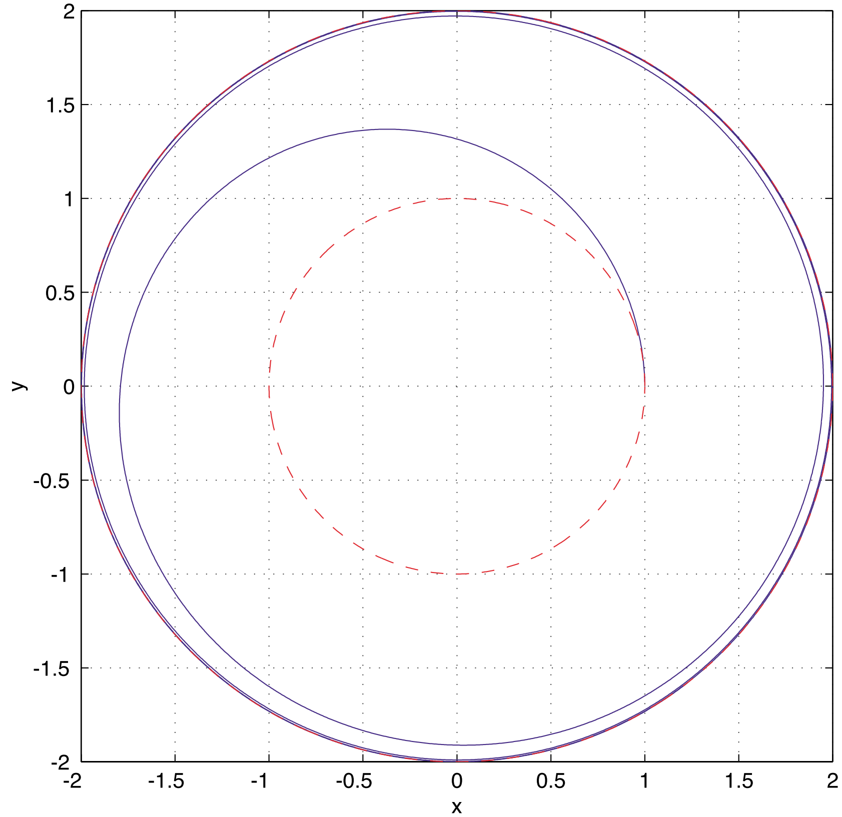
$$\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{\|\mathbf{r}\|^3} + \mathbf{F}$$

  where $\mathbf{F}$ is the control force.
- Fix a Kepler orbit with a given target value $(\mathbf{L}_T, \mathbf{A}_T)$.

Lyapunov Orbit Transfer

- Define $V : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ by
$$V(\mathbf{L}, \mathbf{A}) = \frac{1}{2} a \left\| \Delta \mathbf{L} \right\|^2 + \frac{1}{2} \left\| \Delta \mathbf{A} \right\|^2,$$
where $\Delta \mathbf{L} = \mathbf{L} - \mathbf{L}_T$, $\Delta \mathbf{A} = \mathbf{A} - \mathbf{A}_T$ and $a > 0$.
- Use the equations of motion and
$$\dot{\mathbf{L}} = \mathbf{r} \times \mathbf{F}, \qquad \dot{\mathbf{A}} = \mathbf{F} \times \mathbf{L} + \dot{\mathbf{r}} \times (\mathbf{r} \times \mathbf{F}).$$
to compute the time derivative of V
- Design the controller accordingly:
$$\mathbf{F}(\mathbf{r}, \dot{\mathbf{r}}, t; \mathbf{L}_T, \mathbf{A}_T) =$$
$$- f(\mathbf{r}, \dot{\mathbf{r}}, t) \left(a \Delta \mathbf{L} \times \mathbf{r} + \mathbf{L} \times \Delta \mathbf{A} + (\Delta \mathbf{A} \times \dot{\mathbf{r}}) \times \mathbf{r} \right)$$
with $f(\mathbf{r}, \dot{\mathbf{r}}, t) > 0$ suitably chosen.

Lyapunov Orbit Transfer



Planar transfer between two coplanar circular orbits of radii 1 and 2.

Lyapunov Orbit Transfer

`Orbit Transfer Movie`

The End

The End

TYPESETTING SOFTWARE: TEX, *Textures*, LATEX, `hyperref`, `texpower`, Adobe Acrobat 4.05
ILLUSTRATIONS & MOVIES: *Adobe Illustrator 8.0*, *Mathematica*, MATLAB, QuickTime, and other tools
LATEX SLIDE MACRO PACKAGES: Wendy McKay, Ross Moore