Caltech

CS/IDS 142: Lecture 1.2
Models of Computation

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Goals:

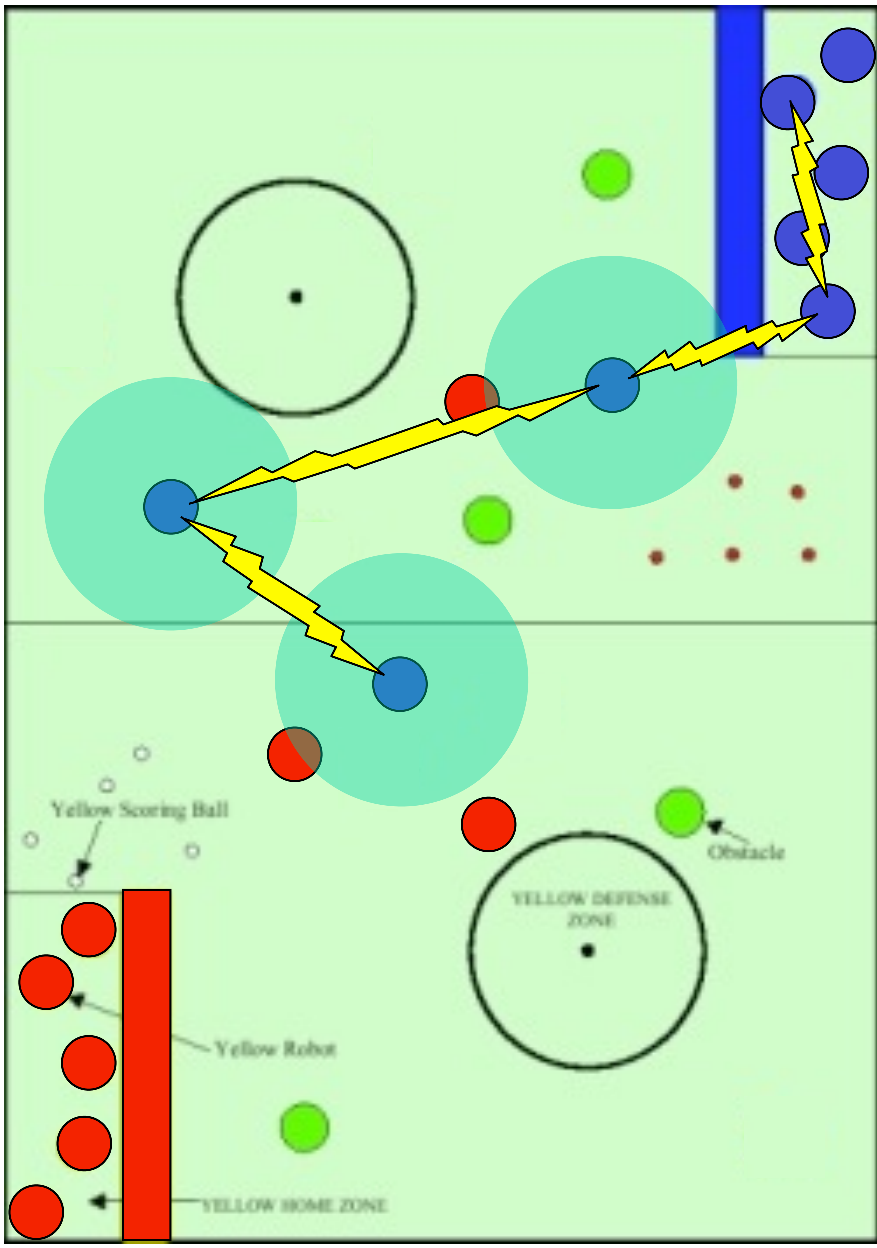
- Introduce state transition systems and the computational model (UNITY)
- Define weak and strong fairness assumptions for program execution

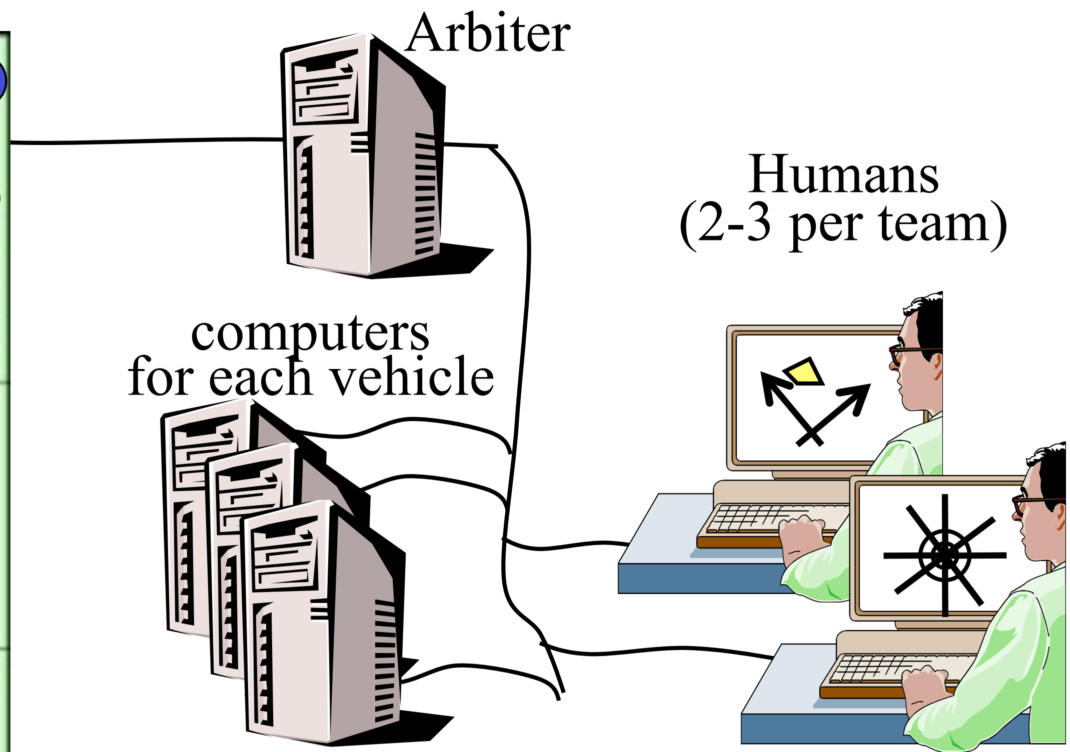
Reading:

- P. Sivilotti, *Introduction to Distributed Algorithms*, Chapter 2

D’Andrea & M
ACC 2003

Example: RoboFlag (D’Andrea, Cornell)





Robot version of “Capture the Flag”

- Teams try to capture flag of opposing team without getting tagged
- Mixed initiative system: two humans controlling up to 6-10 robots
- Limited BW comms + limited sensing

Klavins
CDC, 03

Distributed Decision Making: "RoboFlag Drill"

Task description

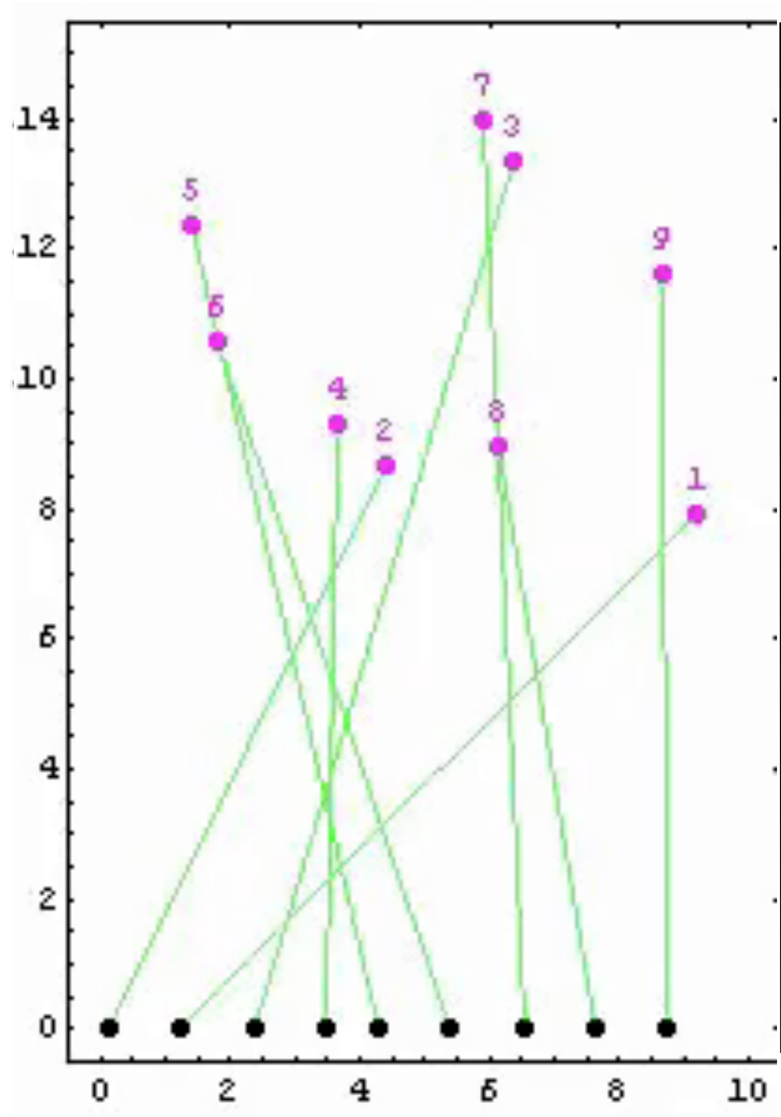
- Incoming robots should be blocked by defending robots
- Incoming robots are assigned randomly to whoever is free
- Defending robots must move to block, but cannot run into or cross over others
- Allow robots to communicate with left and right neighbors and switch assignments

Goals

- Would like a provably correct, distributed protocol for solving this problem
- Should (eventually) allow for lost data, incomplete information

Questions

- How do we model a (distributed) protocol?
- Given a protocol, how do we prove specs?
- How do we design the protocol given specs?



Programs

Programs (also called "processes") consist of

- A set of typed variables, possibly with initial values
- Assignment statements (or "actions")
  - Fatbar ($[\!]$) separates assignments
  - Actions can be executed in any order (nondeterministic)

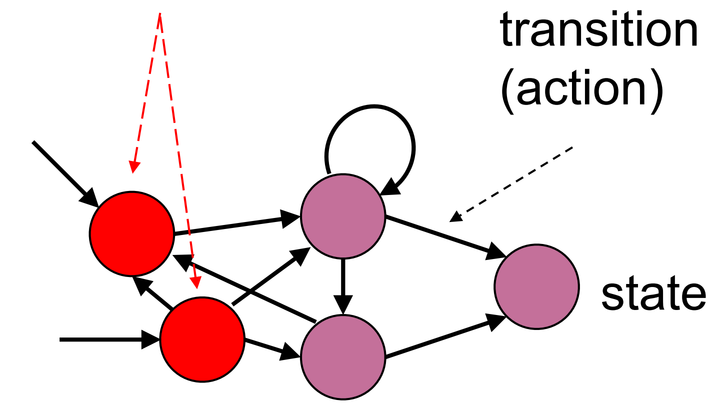
Visualization of programs as graphs

- Each state (possible value of variables) is a vertex
- (Directed) Edges represent assignments (actions) that change state

"Skip"

- All programs implicitly contain the **skip** assignment, which leaves the state of the program unchanged

$$\begin{array}{ll} \textbf{Program} & \textit{Trivial} \\ \textbf{var} & x, y : \text{number} \\ \textbf{initially} & x \neq 2 \\ \textbf{assign} & \\ & \quad x := 2 \\ & [\!] \quad y := f(7) \end{array}$$



Actions

Simple assignments: ***x := a***

- Value of the variable on the left hand side takes the value given on the right hand side
- Can also implement nondeterministic assignments: x := rand(1, 10)

Multiple assignments: ***x, y := a, b*** **or** ***x := a || y := b***

- Assign multiple variables at the same time (be careful not to confuse || with $[\!]$)

Guarded commands: ***g → a***

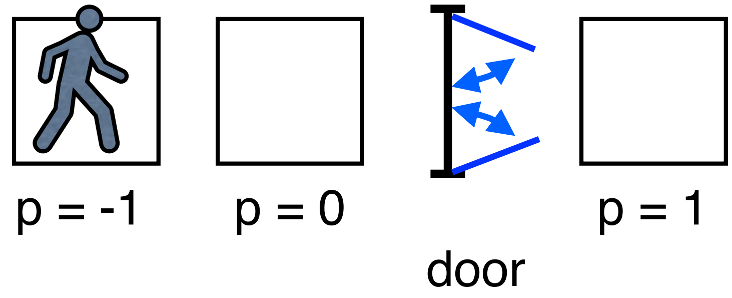
- Assignment (or "action") is predicated on "guard": only execute action if guard is true
- If the guard is true in a given state of the system, the guard is said to be "enabled"

Sequential composition: not formally implemented

- Unlike sequential programming languages, we will not assume sequential execution
- If you need to implement sequential computation, use a guarded commands + multiple assignments + a program counter (PC)

$$
\begin{array}{ll}
\textbf{Program} & \mathit{SequentialSwap} \\
\textbf{var} & x, y, temp : \text{int}, \\
 & pc : \text{nat} \\
\textbf{initially} & pc = 1 \\
\textbf{assign} & \\
\quad\;\; pc = 1 \longrightarrow temp, pc := x, 2 & \\
\quad [\!]\; pc = 2 \longrightarrow x, pc := y, 3 & \\
\quad [\!]\; pc = 3 \longrightarrow y, pc := temp, 4 &
\end{array}
$$

Example: Nondeterministic Door



Door dynamics: open and close at random

Person dynamics: move back and forth

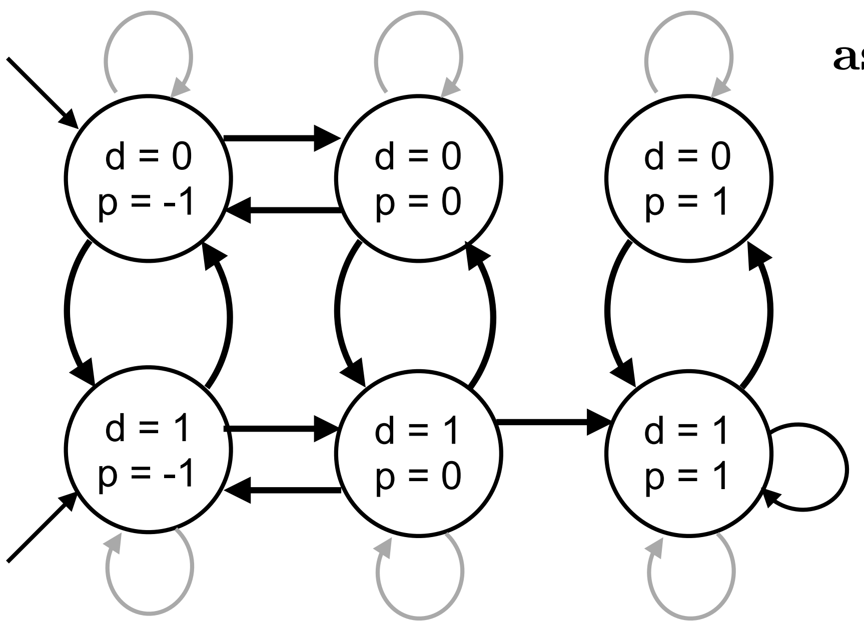
- Can move back and forth between positions (p)
- Can only move from p = 0 to p = 1 if door is open

States: all possible values of variables

- Initial value marked by arrows

Actions: all possible transitions

- For guarded commands, guard must be true in order to execute the assignment ⇒ only include transition if guard is true
- Skip actions allow state to remain unchanged



Program $AutoDoor$

var d : binary

$p : \{-1, 0, 1\}$

initially $p = -1$

assign

door:

$d := 0$

$\square \; d := 1$

person:

$\square \; p = -1 \rightarrow p := 0$

$\square \; p = 0 \rightarrow p := -1$

$\square \; (p = 0 \wedge d = 1) \rightarrow p := 1$

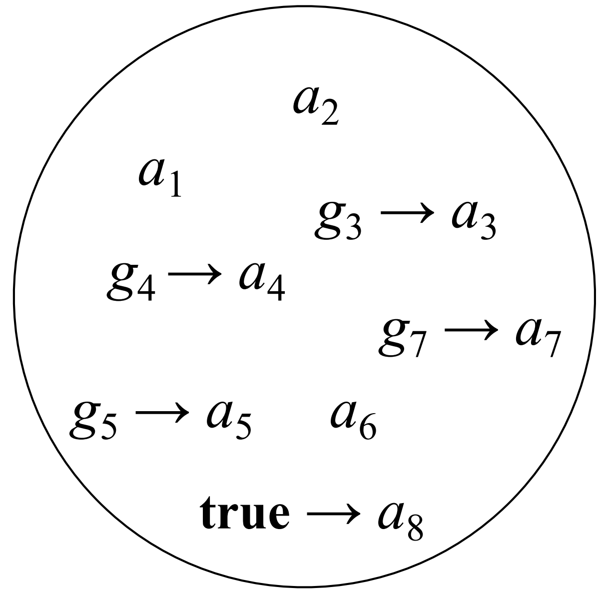
$\square \; p = 1 \rightarrow p := 1$

Program Execution: UNITY (Chandy and Misra)

UNITY = Unbounded Nondeterministic Iterative Transformations

Description

- Program consists of a set of (possibly guarded) variable assignments (or “actions”)
- Behaviors are generated by starting an an initial state, then choosing any assignment for which the guard is true
- Command (g → a) may be evaluated in any order, at any time
- Require that all assignments be applied infinitely often in any execution (built in fairness)
- Reason about “programs” using formal (temporal) logic



Properties

- Useful for reasoning about systems in which there is very asynchronous behavior
- Fairness constraint is a bit too loose for some applications; only assume that each command executes *eventually* (instead of once every iteration) [more on this in a few slides]

Program Termination and Fixed Points

Q: Under the UNITY execution model, when is a program done (terminated)?

- Scenario #1: system might continue to go back and forth in a cycle
- Scenario #2: since the **skip** action is always enabled, we never *really* stop

A: P *terminates* at state *v* if any enabled action from v leaves the state unchanged

- We call such a state a *Fixed Point (FP)*

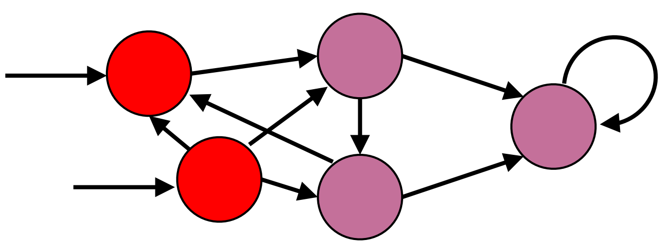
Simple example: what are the fixed points of the following programs?

$$\begin{array}{ll} \textbf{Program} & \textit{Trivial} \\ \textbf{var} & x, y : \text{number} \\ \textbf{assign} & \\ & \quad x := y \\ \| & \quad y := f(7) \end{array}$$

$$\begin{array}{ll} \textbf{Program} & \textit{Trivial} \\ \textbf{var} & x, y : \text{number} \\ \textbf{assign} & \\ & \quad x = y \rightarrow x := 2 \\ \| & \quad y := f(7) \end{array}$$

Looking for fixed points on a program graph

- Let *Reachable*(V) represent the set of all vertices that can be reached (eventually) from a set of vertices V = {v_1, v_2, …, v_n}
- A state v is a fixed point if *Reachable*({v}) = {v}
- A program may not terminate if the graph representing the program contains ___________



- For guarded program FP, all actions of the form g → x := E must satisfy ___________

Distributed Systems

Distributed systems

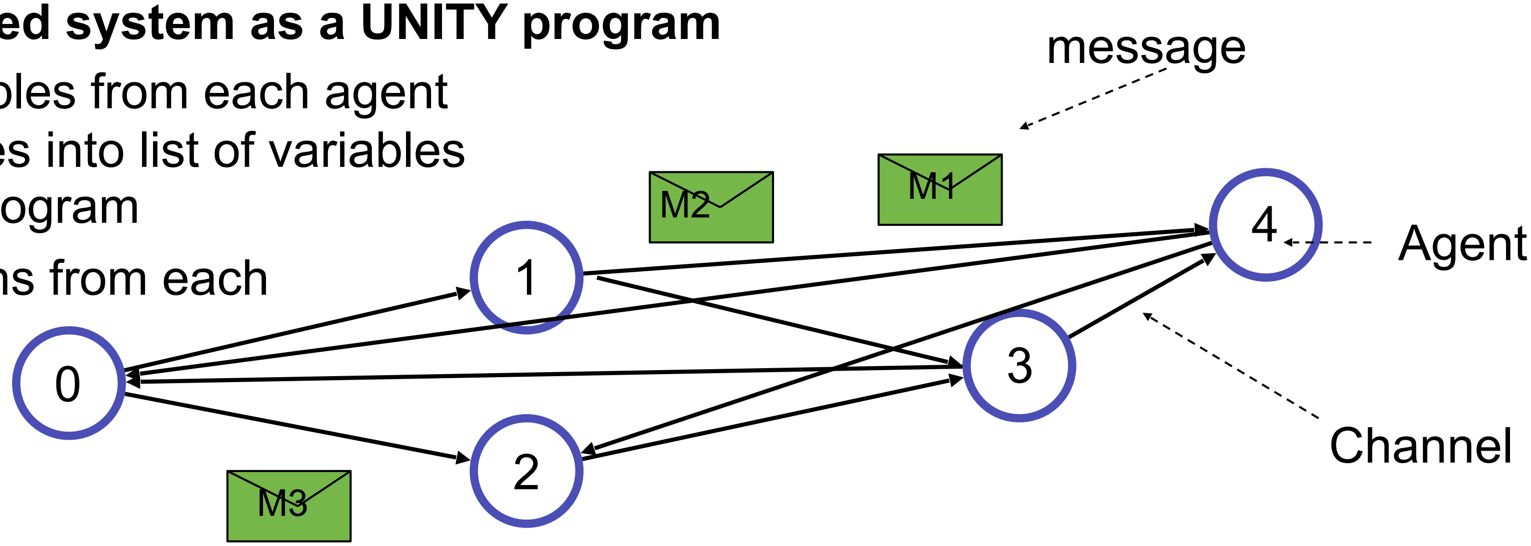
- A distributed system consists of a set of agents (also called processes) and a set of directed channels.
- A channel is directed from one agent to one agent. The system can be represented by a directed graph (separate from the program graph within each agent)

Definition of the “state” of a distributed system

- Minimum amount of information such that the future behavior can be predicted without any other information about the past
- Typically consists of the value of all variables that are part of any processes as well as messages that might be in transit

Modeling a distributed system as a UNITY program

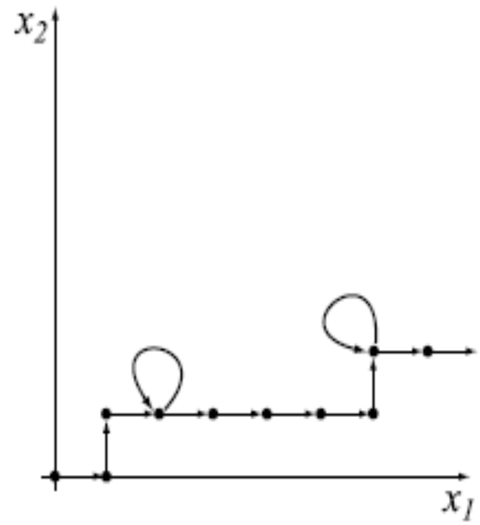
- Combine all variables from each agent + channel variables into list of variables for the (master) program
- Combine all actions from each agent into actions for the program
- Execute actions in arbitrary order



Fairness

Weak Fairness

- Every action is guaranteed to be selected infinitely often
- Implication: between any two selections of a particular action, there are a *finite* (but *unbounded*) number of selections of other actions.



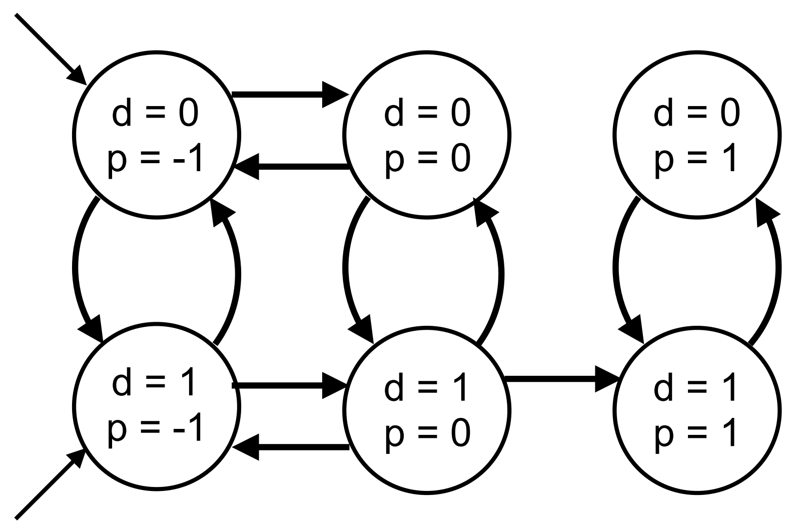
Strong Fairness

- Each action is selected infinitely often *and* if an action is enabled infinitely often then it is selected infinitely often
- Avoids situations where we get "unlucky" and never select an action at a time when it is enabled (mainly applies to *guarded* actions)

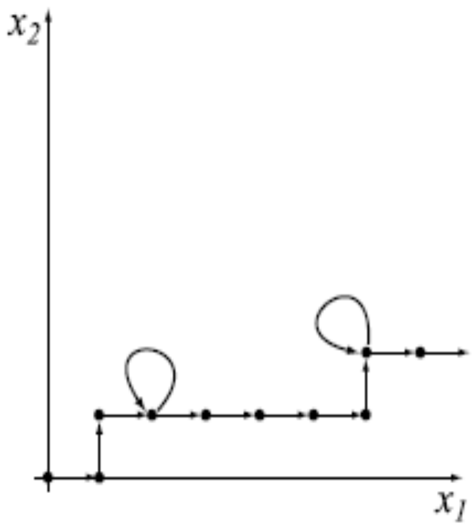
Door opening example

- Q: under weak fairness, does person always reach other side?
- Q: what about under strong fairness?
- Q: can you *prove* it?

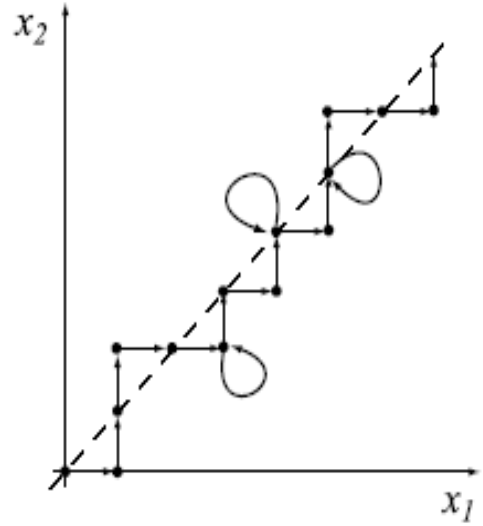
$$
\begin{array}{ll}
 & d := 0 \\
\square & d := 1 \\
\square & p = -1 \rightarrow p := 0 \\
\square & p = 0 \rightarrow p = -1 \\
\square & p = 0 \wedge d = 1 \rightarrow p = 1 \\
\square & p = 1 \rightarrow p := 1
\end{array}
$$



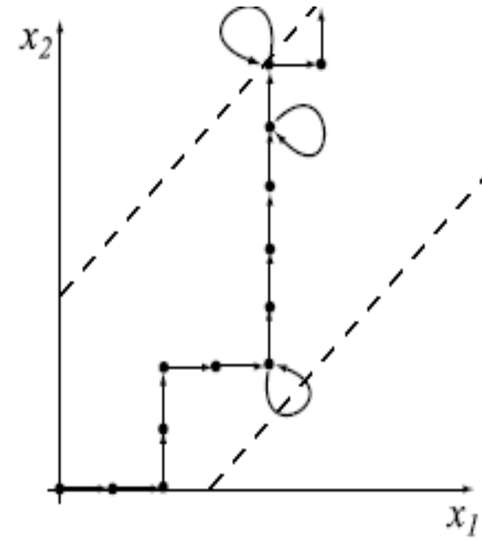
Other Models of Scheduling



UNITY
Each command must be executed infinitely often.



EPOCH
Each command is executed before any are again.



SYNCH(τ)
In any interval, the difference in the number of times any two commands are executed is ≤ τ.

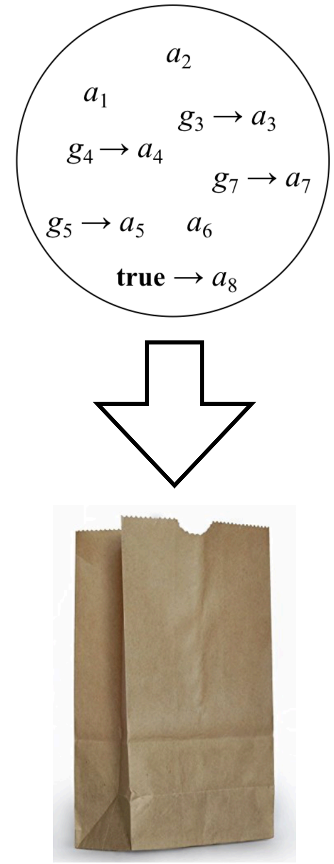
$$\begin{aligned} SYNCH(1) &\subseteq EPOCH \subseteq SYNCH(2) \\ &\subseteq SYNCH(3) \subseteq \cdots \\ &\subseteq UNITY \end{aligned}$$

If program is correct for UNITY, it is correct for the others

Summary: Models of Computation

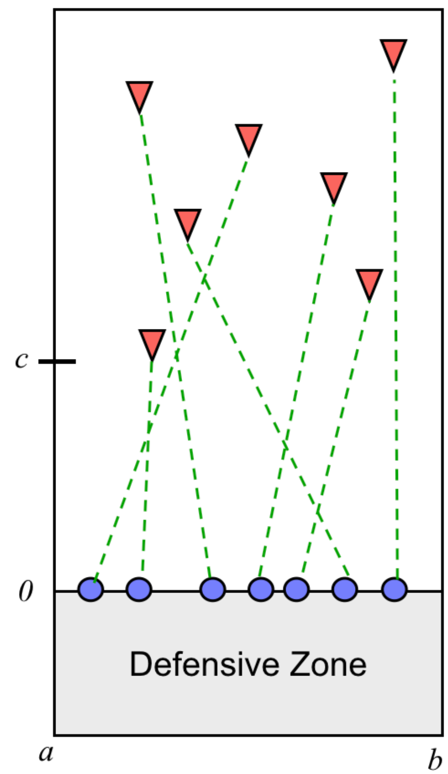
UNITY model provides (seemingly) simple description of programs

- Program = variables + actions [assignments] (that's it!)
- Guarded assignment (g → a) allows modeling of finite state automata
- Distributed programs captured by nondeterministic execution model
- Termination = reaching a *fixed point* (variables remain constant)



Next: how to we *prove* that specifications are satisfied?

- A1: exhaustive testing [remember ZA002!]
- A2: model checking [for specific instantiation]
- A3: formal proof [often generalizable]



Fri: how to prove things using predicate calculus and *quantification* (review + some new stuff)

Next week: invariants (safety) and metrics (liveness)