CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

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Homework Set #8

Issued: 20 Nov 2019
Due: 27 Nov 2019

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [DFT 5.1] Compute a coprime factorization over $\mathcal{S}$ of

$$G(s) = \frac{s^3}{s^2 - s + 1}.$$

2. [DFT 5.4] Suppose that $P(s) = 1/s$ and $C = Q/(1 - PQ)$, where Q is a proper real-rational function. Characterize those functions Q for which the feedback system is internally stable.

3. [DFT 5.5] Suppose that N, M are coprime functions in $\mathcal{S}$. Prove that if $NM^{-1} \in \mathcal{S}$ then $M^{-1} \in \mathcal{S}$. Is this true without the coprimeness assumption?

4. [DFT 5.8] For formulas (5.4) to (5.7) in DFT, verify that $NX + MY = 1$.

5. [DFT 5.9] Consider the feedback system with plant P and controller C. Assume internal stability. Consider a coprime factorization of P over $\mathcal{S}$, $P = N/M$. Suppose that P is perturbed to

$$P = \frac{N + \Delta_1}{M + \Delta_2}$$

where

$$\Delta_1, \Delta_2 \in \mathcal{S}, \qquad \|\Delta_1\|_\infty, \|\Delta_2\|_\infty \leq \gamma.$$

Find a bound on γ for robust stability.