RMM 12 Nov 02

0

Lecture 7.2

Today

1. Nyquist analysis details
2. Control of second order systems
3. Time delays

Reading

1. Astrom, Sec 3.5 & Sec 4.5
2. Option: APH, Ch 30-31 & Lewis Ch 7

Outline

I. Nyquist analysis
   - A. Principle of the argument
   - B. Adding loop gain (encirclements of $-1/K$)
   - C. $j\omega$ axis poles + examples
   - D. Preview: robust stability & performance

II. Second order systems
   - A. Gain & phase margin for second order systems
   - B. Proportional, PD, PI & PID control
   - C. High frequency roll-off

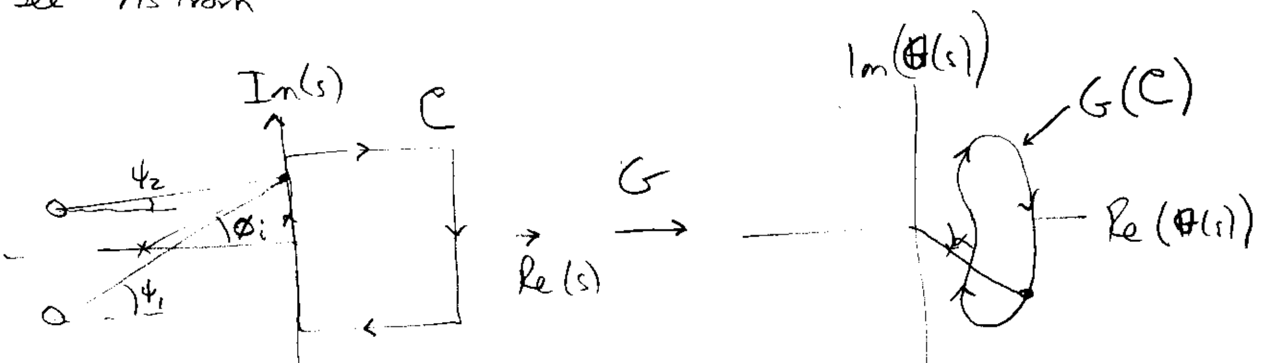
III. Time-delay
   - A. Transfer fcn for time delay (derive)
   - B. Pade approximation

Nyquist Analysis

Principle of the argument: Let $G: \mathbb{C} \to \mathbb{C}$ be a complex function, not necessarily analytic. Let C be a closed clockwise contour in $\mathbb{C}$ that contains Z zeros and P poles of $G(s)$. Then, the number of clockwise (CW) encirclements of the origin by $G(C)$ is given by

$$N = Z - P$$

Pf See Astrom

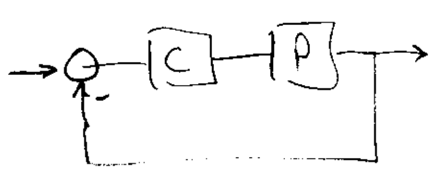


$$G(s) = \frac{s^2+2s+4}{s+1}$$

$$G(s) = \prod_{i=1}^{m} (s-z_i) \cdot \prod_{i=1}^{n} \left(\frac{1}{s-p_i}\right) \longrightarrow \alpha = \Sigma \psi_i - \Sigma \phi_i$$

argument; angle to zeros; angle to poles

Application to Nyquist



$$H_{yr}(s) = \frac{PC}{1+PC}$$ ← interested in poles

$$G(s) = 1 + PC$$

RMM 12 Nov 02

(2)

PoA: Apply to $G(s) = 1 + PC$

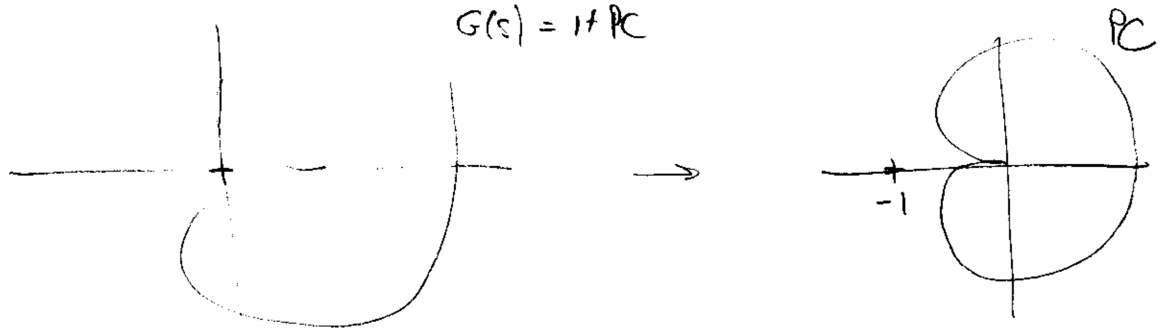
Choose C = right half plane

Z = # zeros in RHP (determines stability)

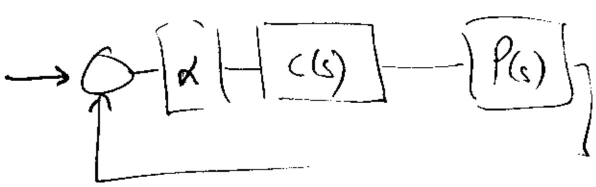
P = # poles in RHP = # RHP poles of PC

N = # CW encirclements of 0

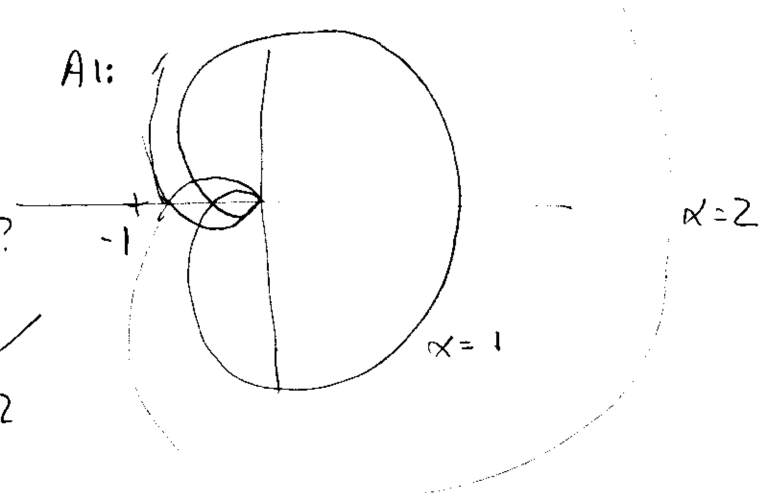
Now, shift origin to -1

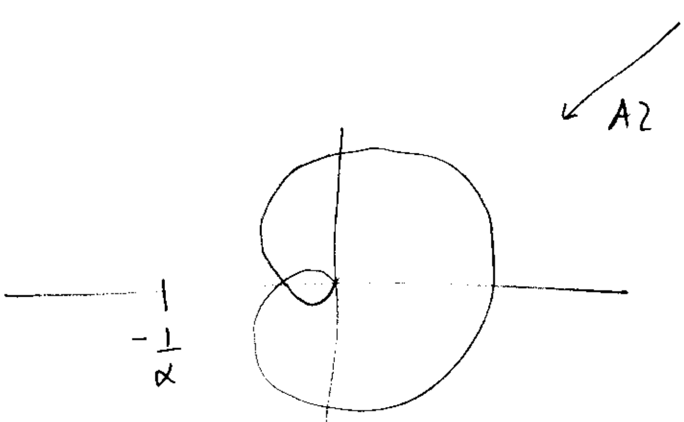


Using Nyquist plots to determine stable loop gain



Q: How does α affect stability?





Can scale -1 point to see range of gains that work

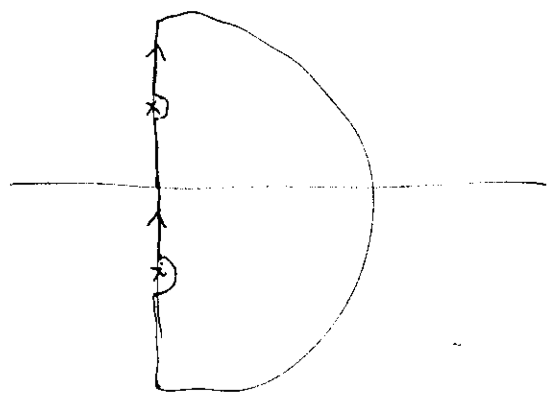
RMM ~~19~~ 12 Nov ~~97~~ 03

~~8~~ 3

Special cases & tricks

0. ~~Poles on the imaginary axis~~

1. Poles on $j\omega$ axis



Use small circle to avoid pole

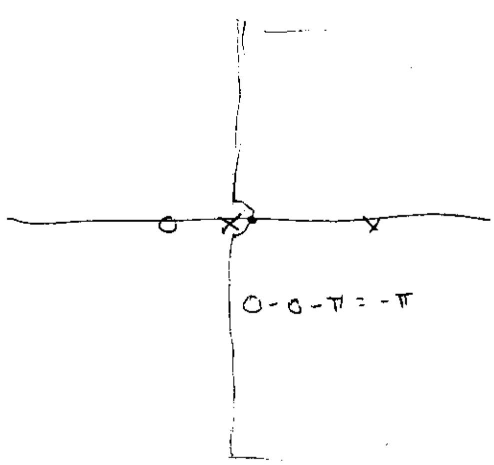
Easier to make circle on right

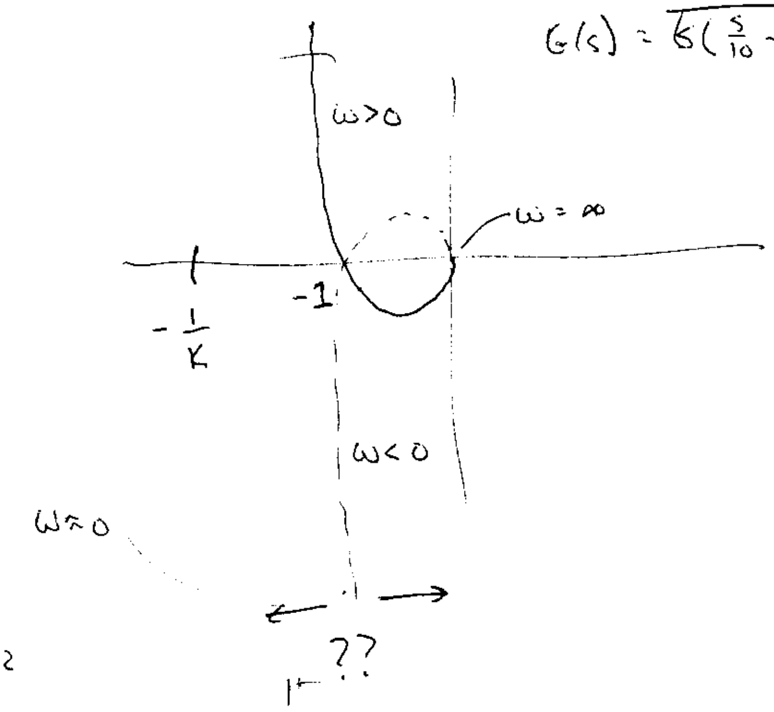
$\Rightarrow$ ~~avoid~~ leave poles out of D contour

~~Matlab does this automatically~~

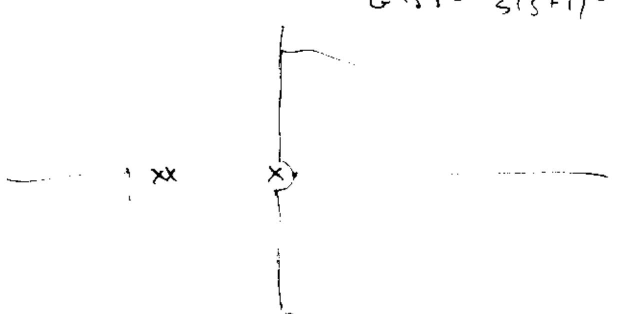
Note that near these pts, $H(j\omega) \to \infty$ $\Rightarrow$ Matlab has problems $\Rightarrow$ need to figure out how to close contour

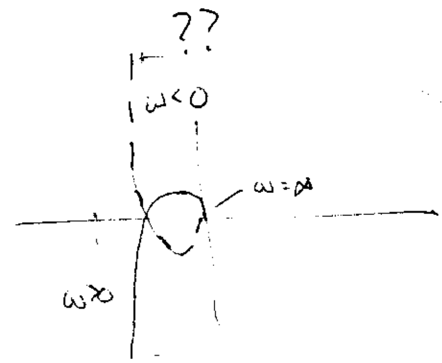
$$G(s) = \frac{s+1}{s\left(\frac{s}{10}-1\right)}$$



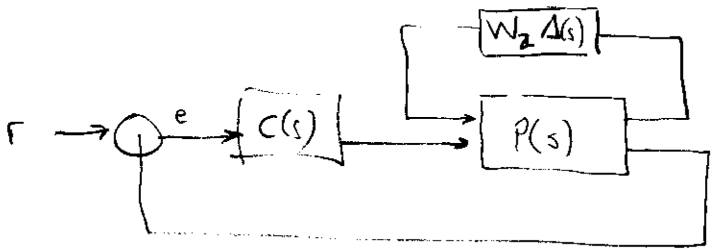


$$G(s) = \frac{1}{s(s+1)^2}$$





Preview: robust stability and performance



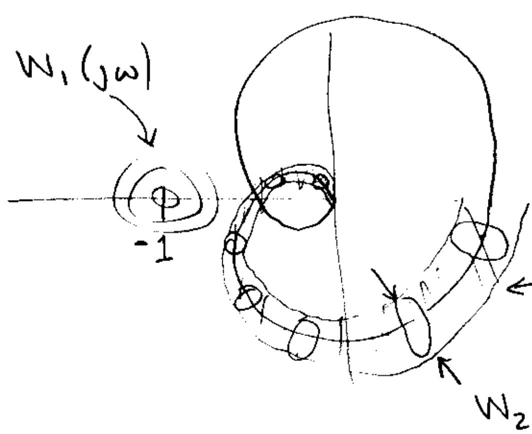
← Uncertainty model

$$\max_{\omega \in [0,\infty)} |\Delta(s)| < 1$$

Robust stability: make sure system is stable for all Δ

Robust performance: make sure $W_1 \cdot \frac{1}{1+PC}$ is small for all Δ

Perf weight ↑ ↑ Her



← Set of possible plants

Performance specified by distance from -1 at each frequency (combines gain and phase margin)

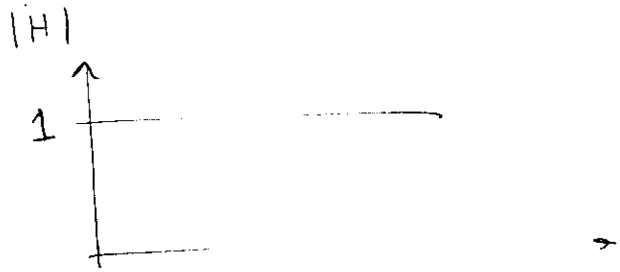
Plant dynamics uncertain ⇒ can insure robust stability & performance

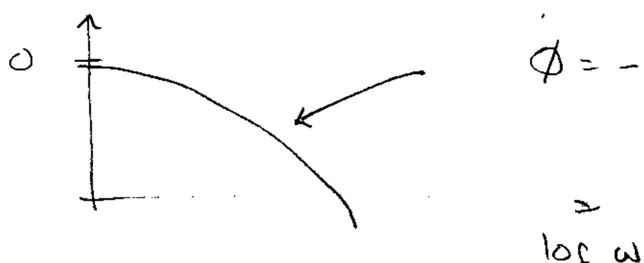
Time delay

$$u(t) \longrightarrow \boxed{\text{delay}} \longrightarrow y(t) = u(t-\tau)$$

Frequency response: $u = A\sin(\omega t)$

$$y = A\sin(\omega(t-\tau)) = A\sin(\omega t - \omega\tau)$$





$\phi = -\omega\tau = -e^{\log\omega}\tau$

↑ exponential decrease in phase

Transfer function: $H(s) = e^{-s\tau}$

Check: $|H(j\omega)| = \cancel{e^{-j\omega t}}\, e^{-j\omega\tau} = 1$ ✓

$\angle H(j\omega) = -\omega\tau$ ✓

Remarks

1. Time delay is very common
2. Time delay is very destabilizing

II. Padé approximation

Q: How do we account for time delays.

A: Approximate by rational polynomial

$$e^{-\tau s} \approx \frac{b_0 s + b_1}{a_0 s + 1} + \text{h.o.t}$$

$$(a_0 s + 1)\left(1 - \tau s + \frac{\tau^2}{2}s^2 + \dots\right) = b_0 s + b_1$$

$$\left.\begin{aligned} 1 &= b_1 \\ (a_0 - \tau) &= b_0 \\ -a_0\tau + \frac{\tau^2}{2} &= 0 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} b_1 &= 1 \\ a_0 &= \tau/2 \\ b_0 &= -\tau/2 \end{aligned}$$

$$e^{-\tau s} \approx \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}$$

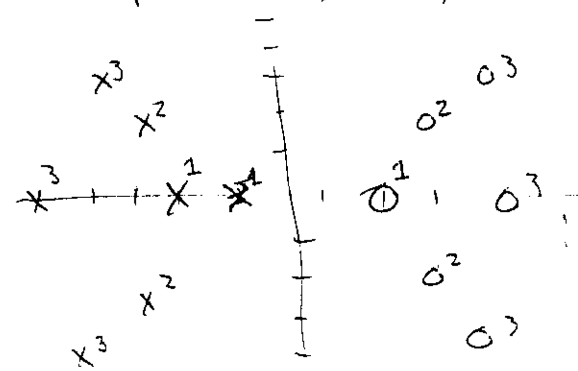
Higher expansions work similarly

$$e^{-\tau s} \approx \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}$$

Matlab: <u>pade</u>

$$e^{-\tau s} \approx \frac{1 - \frac{\tau}{2}s + \frac{\tau^2}{12}s^2}{1 + \frac{\tau}{2}s + \frac{\tau^2}{12}s^2}$$

$$e^{-\tau s} \approx \frac{1 - \frac{\tau}{2}s + \frac{\tau^2}{10}s^2 - \frac{\tau^3}{120}s^3}{1 + \quad + \quad +}$$



$\Rightarrow$ can give spurious behavior