Flow Instability
(and control)

Tim Colonius

CDS 101

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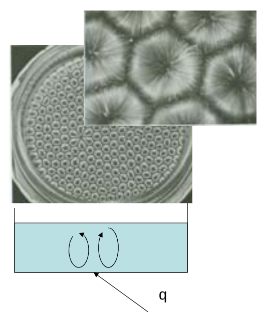
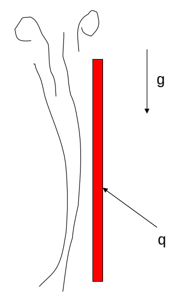
Flow control

- Many control problems contain fluid systems as components.
  - Dashpot in mass-spring-damper systems
  - HVAC system that thermostat controls
  - Aerodynamic forces for an auto-pilot
- Flow control (my definition)
  - *control the internal state (i.e. velocity, pressure, temperature, etc.) of the fluid to achieve a goal (i.e reduce drag)*
- Instabilities are important in three ways
  - Provide opportunity for control (i.e. stabilization)
  - Small energy input can lead to large change in output

Flow instability is all around us…

Thermal instabilities

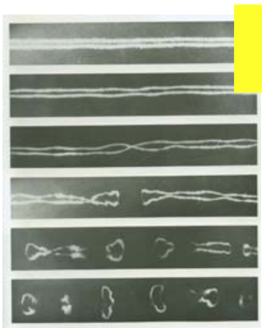




Free convection

Photograph: M. VanDyke, An Album of Fluid Motion, Parabolic Press, 1988

Vortex Instabilities



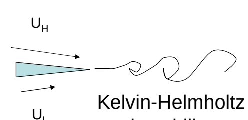
Instability and transition of a vortex ring (smoke ring)

Crow instability of aircraft trailing vortices

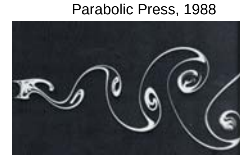
Photograph: M. VanDyke, An Album of Fluid Motion, Parabolic Press, 1988

Shear instabilities

Photograph: M. VanDyke, An Album of Fluid Motion, Parabolic Press, 1988



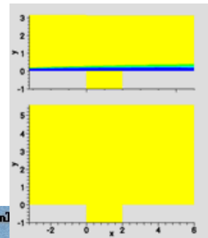
Kelvin-Helmholtz instability

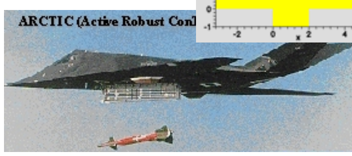


vortex shedding behind a cylinder

Flow/acoustic instabilities

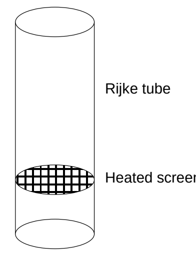
NASA's SOFIA





Thermo/acoustic (& combustion) instabilities

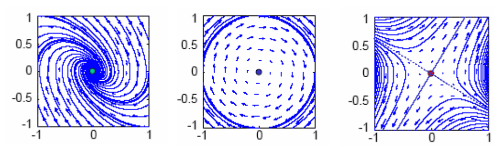
afterburner



and so on…

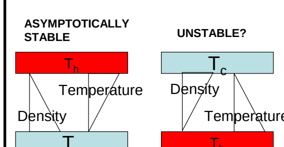
Flow instability

- Same definitions as any system. Given an *equilibrium* solution
  - If "small" disturbances die out in time, then asymptotically stable
  - If "small" disturbances remain small in time, then stable
  - Otherwise unstable



Sometimes the definitions are straightforward…

Thermal Instability



- Stability often dictated by a dimensionless parameter
  - Marginal stability
  - Critical parameter value

Analysis gives:

$$Ra_{crit} = g\alpha(T_2 - T_1)d^3/\kappa\nu > 1700$$

α = Coefficient of thermal expansion
κ = Thermal diffusivity
ν = Kinematic viscosity

Note for air near room temperature, this corresponds to no more than about1 degree C temperature diff. per centimeter of gap

Difficulties

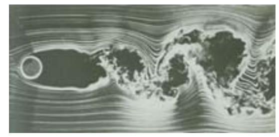
- Flows are infinite-dimensional systems
  - Need to solve PDE stability problems rather than ODE
  - Sometimes there are good low-dimensional models
- Equilibrium solutions are hard to obtain analytically
  - Equilibrium solutions are difficult or impossible to observe experimentally
- Understanding stability and control requires careful definition of the system

Equilibrium flow?

Re=U D / ν = 300

Re=4,300

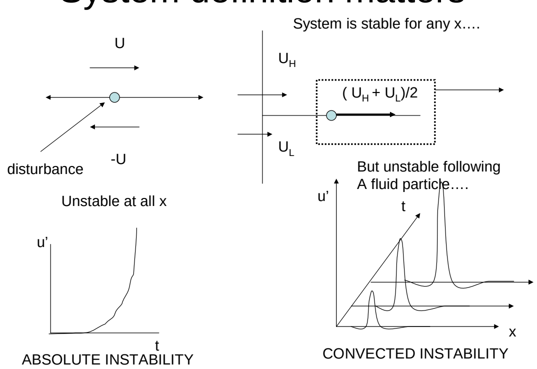
Re = 10,000



Re=10,000,000

Photographs: M. VanDyke, An Album of Fluid Motion, Parabolic Press, 1988

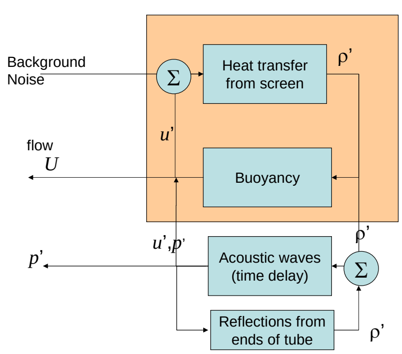
System definition matters



Natural feedback

- Many flows of interest are convectively unstable
  - Plumes, jets, shear layers, …
- Feedback of the convected disturbance can lead to absolute instability
  - Obstacles
  - Acoustic feedback
  - Nonlinearity (e.g. induced velocity)

Rijke Tube



Model (Prof. Culick)

- State
  - $q = q(U+u',T_s)$; the rate heat is supplied from the screen
  - P ; the amplitude of pressure fluctuation (averaged over pipe)
  - u'; the velocity fluctuation at the screen location
- Input
  - Maybe a little noise
- Output
  - P; the amplitude of pressure fluctuation (where?)
  - ω, α; the frequency and temporal growth rate
- Parameters
  - L_s; the location of the screen in the pipe (from bottom)
  - L; the length of the pipe
  - T_s; the initial temperature of the screen
  - a; the speed of sound
  - g; gravity
  - U; the bias flow in the pipe (really a state, but more simple to treat as static parameter)
  - screen properties (conductivity, porosity, etc.)
  - other fluid properties (ρ, γ, etc.)

Dynamics

Assume that instability mode is close to natural first mode of pipe,

$$p \approx P(t) \sin \frac{2\pi x}{L}$$

Then the acoustic dyanmics are described by:

$$\ddot{P}(t) + \epsilon \dot{P}(t) + \omega_0^2 (P(t) + BP(t-\tau)) = 0$$

where

1. ϵ is a small empirical damping coefficient
2. $\omega_0 = 2\pi a/L$, the natural frequency of the first mode when $Q = 0$.
3. $B = \frac{L(\gamma-1)}{\rho a^2} Q(U + u') \sin \frac{4\pi L_s}{L}$
4. τ is a delay associated with the time required for a velocity fluctuation to change the rate at which heat is transfered and buoyant fluid is converted into acoustic waves.

Instability

- Time delay in ODE → Infinite dimensional system
- These systems will be discussed later in this class
- Assume a constant frequency disturbance but with complex frequency (Laplace transform)

$$P = Ae^{i\Omega t} \qquad \Omega = \omega + i\alpha$$

- If $\alpha < 0$: instability

Inserting $P = e^{i\Omega t}$ into the model:

$$\left(-\Omega^2 + i\epsilon\Omega + \omega_0^2 \left(1 + Be^{-i\Omega t}\right)\right) P = 0$$

Thus we need to find the zeros of the nonlinear function of Ω. This can be done numerically, but we can see most important effects by linearizing for the case where Ω is not much different from ω_0. We obtain:

$$\omega = \omega_0(1 + \frac{B}{2}\cos\omega_0\tau)$$
$$\alpha = -\omega_0\frac{B}{2}\sin\omega_0\tau$$

And the Rijke tube is unstable if

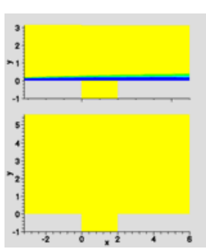
$$B \sin\omega_0\tau \sim \sin\omega_0\tau \sin\frac{4\pi L_s}{L} < 0$$

This is the case when:

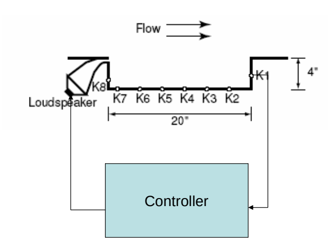
1. $0 < \tau < 2\frac{L}{a}$, the delay is not more than twice the traverse time of an acoustic wave, **and**
2. $0 < L_s < L/2$, the screen is in the lower half of the pipe

Cavity oscillations are similar…

- Thermal instability replaced by Kelvin-Helmholtz instability
- Inputs
  - Noise
- Output
  - *p'*
- Parameters
  - M ; mach number of incoming stream
  - L, D, H; cavity length, depth, breadth
  - θ ; boundary layer thickness



Cavity control



Ref: Rowley, C.W., et al [2002] AIAA paper 2002-0972

Dynamics

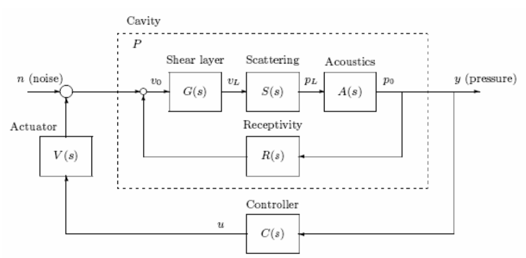
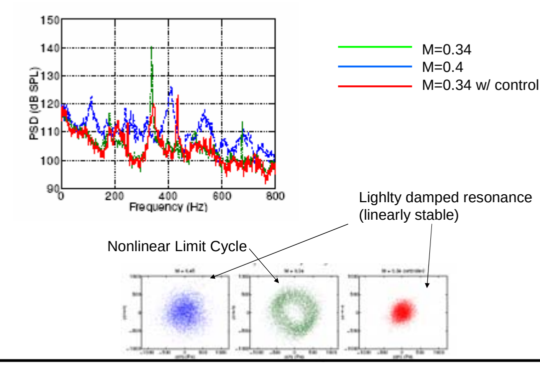


FIGURE 1. Block diagram of cavity model.

Experimental frequency response



Control

- The best controller was (nearly) as simple as feeding back the pressure with an appropriate phase lag (e.g. cancel natural acoustic feedback)
- The linear model was however very useful in assessing the limits of control
  - Maximum attenuation of tones
  - Actuator bandwidth
  - “Peak splitting”
- Similar control can be used to quiet a Rijke tube (and other combustion instabilities). Difficulty is in modeling flame in real situations.

Other topics in flow control

- *Many* other applications not covered here
- Hardware
  - Actuators & sensors (distributed sensing)
  - Cost, reliability in aeronautics applications
- Modeling
  - Systematic model reduction techniques
    - Extreme truncation of high-dimensional systems but preserving certain important features
  - PDE based control theory

Summary of main points

- Flow instabilities are everywhere
- Modeling is difficult
  - Flow governed by PDE (high-order systems)
  - Equilibrium states hard to define
- Standard definitions for stability depend on *how* system is modeled
  - Convective vs. absolute instability
- Natural feedback is often present and can induce absolute instability.
- Rijke tubes and cavity oscillations are example of acoustically driven instabilities that can be simply modeled and readily controlled