CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110
Homework Set #7

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All students should complete the following problems:

1. For the control systems below, determine that steady state error, the maximum frequency for which the closed loop system can track with less than 5% error, and the approximate closed loop bandwidth of the system.

   (a) Disk drive read head positioning system, using "lead" compensator:

   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10} \qquad C(s) = 1000\frac{s+1}{s+10}$$

   (b) Second order system with PD compensator:

   $$P(s) = \frac{100}{(100s+1)(s+1)} \qquad C(s) = s + 10$$

2. For the control systems below, design a PID control law that stabilizes the sytem and gives at least 30° phase margin. You may you any method (loop shaping, Ziegler-Nichols, sisotool, etc). For the closed loop system, determine that steady state error and the maximum frequency for which the closed loop system can track with less than 5% error.

   (a) Disk drive read head positioning system:

   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}$$

   (b) Second order system:

   $$P(s) = \frac{100}{(100s+1)(s+1)}$$

Only CDS 110 students need to complete the following additional problems:

3. Consider the cart pendulum system with the pendulum hanging *down* (you can think of this as the problem of moving the cart without exciting the pendulum too much; similar to walking without sloshing your coffee). The dynamics describing how the position of the cart depends on the applied force is given by the transfer function

   $$P(s) = \frac{ls^2 + g}{Mls^4 + +bls^3 + (M+m)gs^2 + bgs}$$

   $M = 0.5$ kg, $m = 0.2$ kg, $l = 0.3$ m, $b = 0.1$ N/m/sec, $g = 9.8$ kg m/sec^2

   (you can verify this from the equations given on a previous homework set). In this problem you will design a control law that satisfies the following specifications:

   - 0.1% steady state error
   - Position (x) tracking within 10% up to 0.05 Hz
   - Overshoot of less than 10% to step changes in x position
   - Disturbance rejection of 10X for all disturbances above 10 Hz (for inputs with frequency above 10 Hz, the output of the system should be *smaller* than 0.1X the size of the input)

(a) Write the frequency domain portions of the specification as constraints on the loop transfer function in the appropriate frequency ranges. Estimate the phase margin requirement imposed by the step response using a second order approximation. Show your results by sketching them on a Bode plot.

(b) Design a PID control law that satisfies the specification. Make sure to discuss how you determined the form of the control law and demonstrate that your control law satisfies all specifications.