CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110
Homework Set #5

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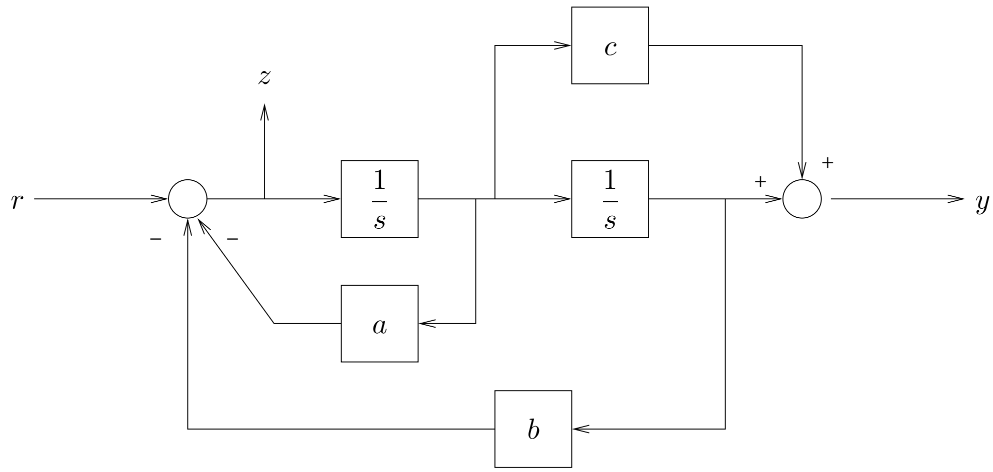
All students should complete the following problems:

1. Consider the block diagram for the following second order system



   (a) Compute the transfer function H_{yr} between the input r and the output y.

   (b) Show that the following state space system has the same transfer function, with the appropriate choice of parameters:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} b_2 & b_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + dr$$

   Give the values of a_i, b_i, and d that correspond to the transfer function you computed in (a).

   (c) Compute the transfer function H_{zr} between the input r and the output z. (Hint: it is *not* $H_{zr} = 1$.)

2. Consider the cruise control system from homework #2. The equations of motion for the system were given by

$$\begin{aligned} m\dot{v} &= -bv + K\tau + F_{\text{hill}} \\ \dot{\tau} &= -a\tau + u_e, \end{aligned} \tag{1}$$

   where $m = 1000$ kg is the mass of the vehicle, $b = 50$ N sec/m is the viscous damping coefficient, $K = 5$ is the conversion factor between engine torque and force applied to the vehicle, $a = 0.2$ is the lag coefficient, and F_{hill} represents the external effect of going up or down hills on the vehicle dynamics.

   The simplest controller for this system is a proportional control, $u_e = -K_p e$, where $e = (v - r)$ (r is the reference speed).

   (a) Draw a block diagram for the system, with the engine dynamics and the vehicle dynamics in separate blocks and represented by transfer functions. Label the reference input to the closed loop system as r, the disturbance due to the hill as d, and the output as y $(= v)$.

   (b) (MATLAB) Construct the transfer functions H_{er} and H_{yd} for the closed loop system and use MATLAB to generate the step response and frequency response for the each. Make sure to use the transfer function computation.

(c) Consider a more sophisticated control law of the form

$$\dot{x}_c = e$$
$$y_c = K_i x_c + K_p e.$$

This control law contains an "integral" term, which uses the controller state x_c to integrate the error. Compute the transfer functions for this control law and redraw your block diagram from part (a) with the default controller replaced by this one.

(d) (MATLAB) Using the default gains from previous homeworks ($K_i = 100$, $K_p = 500$), use MATLAB to compute the transfer function from r to y and plot the step response and frequency response for the system. This should match your answers in previous homework sets, but make sure to use the transfer function computation.

Only CDS 110a students need to complete the following additional problems:

3. In this exercise we will explore the use of state feedback to generate a cruise controller for the system (1) in problem 2.

   (a) Construct the linear state space system corresponding to the open loop system, in the form $\dot{x} = Ax + Bu$. Verify that the system is controllable.

   (b) Design a eigenvalue placement control law that places the poles of the closed loop system at -1 and -2. Plot the step response and frequency response of the closed loop system with this control law. *Hint: MATLAB's* `place` *command may be useful.*

4. Consider the following transfer function for a second order system:

$$P(s) = \frac{10}{s^2 + bs + 1},$$

   where b is a postive number.

   Sketch the Bode diagram for this system for the cases where the poles of $P(s)$ are real and complex (so you should have two sets of plots). For the complex case, compute and label the frequency where the maximum gain occurs and where the gain crosses 1 (0 dB), as a function of b. For the real-valued case, compute and label the frequency where the slope changes (may be multiple points) and where the gain crosses 1, as a function of b.

Supplemental problem: may be done in place of problem #4.

Consider the environmental unit shown in the figure. The purpose of this unit is to remove component A from a component-B rich phase. The transfer of A to the water medium occurs across a semipermeable membrane. In this process, the concentration of A is a function of a position along the unit and of time. In place of using PDE's, divide the unit into sections, or "pools," and assume each pool to be well mixed. The dotted lines show the divisions of pools. Using this method, we find the differential in length, *dL*, is approximated by ΔL.

The mass transfer rate of component A is

$$N_A(t) = S \cdot k_A[x_{An,1}(t) - x^*_{An,2}(t)]$$

where

$N_A(t)$ = moles of A transferred/s
S = surface area, of membrane, across which the transfer takes place, m^2
k_A = mass transfer coefficient, a constant, moles A/m^2-s
$x_{An,1}(t)$ = mole fraction of A in liquid phase 1 (component B-rich phase). The subscript n refers to the pool number.
$x^*_{An,2}(t)$ = mole fraction of A in liquid phase 2 (water-rich phase) that would be in equilibrium with $x_{An,1}(t)$

Assume the equilibrium line is straight with slope m. Then

$$x^*_{An,2}(t) = m \cdot x_{An,2}(t)$$

where $x_{An,2}(t)$ = mole fraction of A in liquid phase 2. Component B and water are not transferred across the membrane, and the process occurs isothermally. Assume constant volumes and densities. Only the first two pools are considered in this problem.

a) Develop the mathematical model that describes this process.
b) Determine the transfer functions relating the output variables $x_{A2,1}(t)$ and $x_{A2,2}(t)$ to the forcing functions $x_{Ai,1}(t)$, $f_{i,1}(t)$, and $f_{i,2}(t)$.
c) Draw the block diagram for this process.