

CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110
Homework Set #2

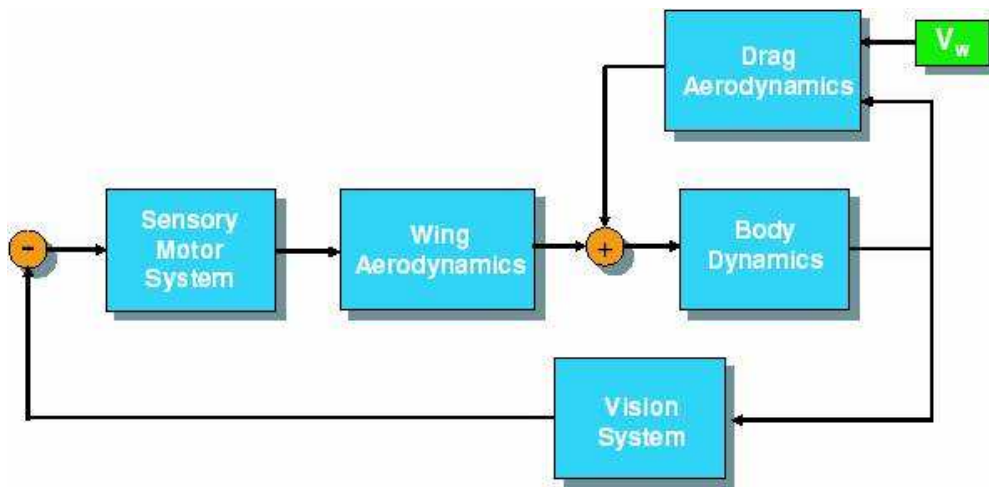
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Due: 11 Oct 04

Note: In the upper left hand corner of the first page of your homework set, please put the class you are taking (CDS 101, CDS 110) and the number of hours that you spent on this homework set (including reading).

All students should complete the following problems:

1. Consider the following block diagram of the flight control system of a fly:



Using the paper “Vision as a Compensatory Mechanism for Disturbance Rejection in Upwind Flight” by Reiser et al. (available via the course web page), identify the state, input, outputs, and dynamics for each block in the diagram. You may give your answer in words, but be precise as possible. (Hint: not all of the blocks are “dynamic”; some are static maps with inputs and outputs, but no state.)

2. (MATLAB/SIMULINK) In this problem you will build a model of a vehicle in SIMULINK and control the vehicle using feedback control. The vehicle will consist of a body (chassis + wheels) and a drive train (engine + transmission). Assume that the vehicle dynamics are of the form

$$m\dot{v} = -bv + F_{\text{engine}} + F_{\text{hill}}$$

where $m = 1000$ kg is the mass of the vehicle, $b = 50$ N sec/m is the viscous damping coefficient, and F_{engine} and F_{hill} represent the forces on the vehicle due to the engine and the terrain, respectively. We can implement this in SIMULINK as a two input, one output system, written in state space form as

$$\begin{aligned} \dot{x}_v &= [-b/m] x_v + [1/m \quad 1/m] u_v \\ y_v &= x_v \end{aligned} \tag{1}$$

where $x_v = v$ is the vehicle state, $u_v = [F_{\text{engine}} \quad F_{\text{hill}}]^T$ is the vehicle input (two dimensional), and $y_v = v$ is the vehicle output (velocity). You should make this into a single SIMULINK block using the “State Space” block (under **Simulink**→**Continuous**→**State Space** in the Simulink Library Browser). You may also want to use the Mux block (under **Signals & Systems**).

We will model the engine dynamics as a “first order lag”. Let τ represent the engine torque and assume the engine has the following dynamics:

$$\begin{aligned}\dot{\tau} &= -a\tau + u_e \\ y_e &= K\tau\end{aligned}\tag{2}$$

where $a = 0.2$ is the lag coefficient, $K = 5$ is the conversion factor between engine torque and force applied to the vehicle (representing the transmission) and u_e is the accelerator input (which we will assume has the proper units). You should also create a SIMULINK block for this subsystem.

Finally, we include the effects of a hill. The hill simply exerts a force on the car that is based on the angle of the hill:

$$F_{\text{hill}} = -mg \sin(\theta)\tag{3}$$

where $g = 9.8 \text{ kg m/sec}^2$ and $\theta = \pi/18$ is the angle of the hill (10 degrees).

- (a) Plot the output of the open loop vehicle model (1) for a step input of $F_{\text{engine}} = 500$ Newtons (assume $F_{\text{hill}} = 0$). What is the rise time (0 to 95% of the final value)?
- (b) Plot the output of the open loop engine model (2) for a step input of $u_e = 100 \text{ Nm}$. What is the rise time?
- (c) In the homework from last week, you built a simple cruise controller. Replace the vehicle/engine model in that system with your vehicle and engine models and plot the response for the default gains ($K_i = 50$, $K_p = 1000$). Make sure to set your simulation time to be sufficiently long.
- (d) Now include the effect of a hill on your system. You should model the system so that the car is initially on a flat surface and then encounters the hill at $T = 100$ seconds. Plot the response of the system and compute the rise time.

Note: if you are having trouble figuring out how to create these blocks in SIMULINK, take a look at “hw1cruise.mdl” from last week’s homework and see if you can modify it appropriately. It has all of the subsystems you will need (except for the Mux block). You may also find the following web-based tutorial helpful:

<http://www.engin.umich.edu/group/ctm/examples/cruise/cc.html>

(ignore the sections on transfer functions; we will get to these later in the class).

Only CDS 110a students need to complete the following additional problems:

3. Consider the following discrete time system

$$\begin{aligned}z[k+1] &= Az[k] + Bu[k] \\ y[k+1] &= Cz[k+1]\end{aligned}$$

where

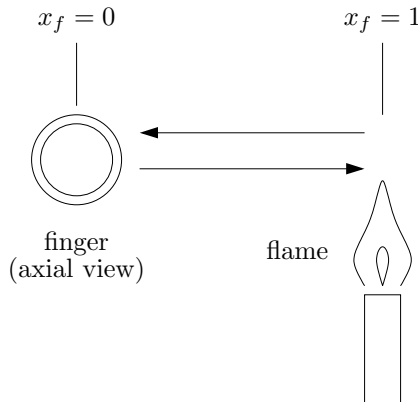
$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [1 \ 0]$$

In this problem, we will explore some of the properties of this discrete time system as a function of the parameters, the initial conditions, and the inputs.

- Assume that the off diagonal element $a_{12} = 0$ and that there is no input, $u = 0$. Write a closed form expression for the output of the system from a nonzero initial condition $z[0] = (z_1[0], z_2[0])$ and give conditions on a_{11} and a_{22} under which the output gets smaller as k gets larger.
- Now assume that $a_{12} \neq 0$ and write a closed form expression for the response of the system from a nonzero initial conditions. Given a condition on the elements of A under which the output gets smaller as k gets larger.
- Write a MATLAB program to plot the output of the system in response to a unit step input, $u[k] = 1, k \geq 0$. Plot the response of your system with $z[0] = 0$ and A given by

$$A = \begin{bmatrix} 0.5 & 1 \\ 0 & 0.25 \end{bmatrix}$$

- In this problem we will look at how to play with fire without getting burned. The system we want to consider is a finger being moved back and forth across a flame, as shown below:



The description of the system is as follows:

- The temperature of a finger is regulated by an internal feedback mechanism. To first order, we will say that heat is convected away by blood flow, at a rate

$$F_b = \alpha_b(T_f - T_b)$$

where T_f is the temperature of the fingertip, T_b is the temperature of the blood, and α_b is the convection coefficient (the F signifies the heat flux).

- A flame gives off heat into the ambient air, and we assume a steady-state temperature field around the flame. The ambient air far from the flame is at 25 degrees Celsius.
- The flame is fixed at $x_F = 1$, and fingertip begins at a position $x_f = 0$, where the ambient air is precisely at the same temperature as the blood.

- Suppose that the temperature of the air varies exponentially with distance from the flame, so

$$T_a(x) = 25 + (T_F - 25) \left(\frac{T_b - 25}{T_F - 25} \right)^{(x-1)^2}$$

where T_F is the flame temperature.

- Heat convects into the finger from the ambient air at a rate

$$F_a = \alpha_a(T_a - T_f).$$

- The dynamics of the fingertip temperature is given by

$$c_f \dot{T}_f = -F_b + F_a$$

where c_f is the fingertip thermal capacity.

- The fingertip is rapidly passed into and out of the flame, according to

$$x_f(t) = \sin(\omega t).$$

Using the MATLAB `ode45` function (or something similar), build a model for the system and solve the following:

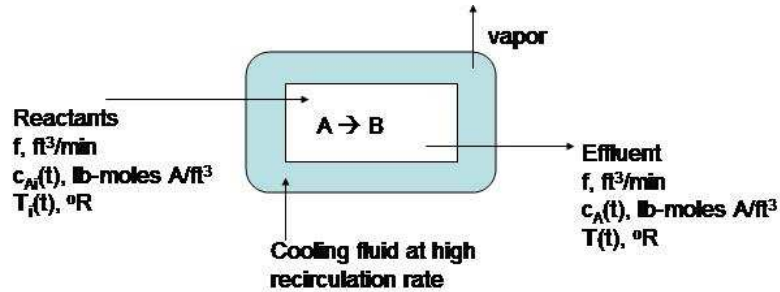
- Assume that the finger moves sinusoidally in and out of the flame at frequency $\omega = 1$ rad/sec. Plot the temperature of the finger as a function of time and identify the transient and steady state response.
- Plot the *steady state* amplitude of the finger temperature as a function of the ω for ω ranging from 1 to 100 rad/sec. You should get something similar to the frequency response plot shown in lecture on Monday. You should compute at least 5 points in your graph.
- Double the “gain” of the temperature control system by increasing α_b by a factor of 2. Replot the frequency response from part 4b and describe in words how it differs from the original gain (i.e., where is the response bigger, smaller or unchanged and what is the reason).

You should use the following parameter values in your simulations:

- $T_b = 37$, $T_F = 1400$ degrees Celsius.
- $\frac{\alpha_a}{c_f} = 1 \text{ s}^{-1}$
- $\frac{\alpha_b}{c_f} = 20, 40 \text{ s}^{-1}$

Supplemental problems: If you like, you may do any one problems of the following problems in place of Problem 4. These problems make use of domain-specific knowledge and so you should only do them if you are comfortable with that problem area. In addition, these problems are experimental in nature and you should ask questions quickly if you get stuck (it might not be your fault!).

5. Consider a chemical reactor in which species A undergoes a first-order, exothermic conversion to species B. To remove the heat of reaction, a jacket surrounds the reactor where a coolant is maintained at 100 °F. Suppose that such a reactor is performing at steady-state conditions provided in the table below:



Process Information

$$\begin{aligned}
 V &= 13.26 \text{ ft}^3 & A &= 36 \text{ ft}^2 \\
 E &= 27,820 \text{ Btu/lbmole} & R &= 1.987 \text{ Btu/lbmole}\cdot^\circ\text{R} \\
 \rho &= 55 \text{ lbm}/\text{ft}^3 & C_p &= 0.88 \text{ Btu}/\text{lbm}\cdot^\circ\text{F} \\
 \Delta H_r &= -12,020 \text{ Btu/lbmole} & U &= 75 \text{ Btu}/(\text{h}\cdot\text{ft}^2\cdot^\circ\text{F}) \\
 k_o &= 1.73515 \times 10^{13}/\text{min}
 \end{aligned}$$

Steady-State Values

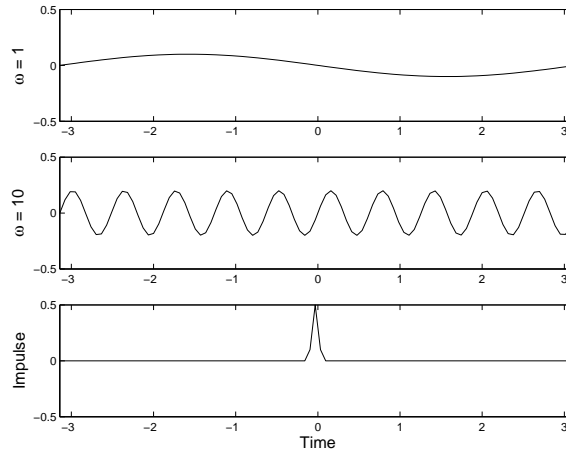
$$\begin{aligned}
 c_{A_i}(t) &= 0.8983 \text{ lbmole}/\text{ft}^3 & T_i(t) &= 578 ^\circ\text{R} \\
 T_c &= 560.0^\circ\text{R} & f &= 1.3364 \text{ ft}^3/\text{min} \\
 c_A(t) &= 0.08023 \text{ lbmole}/\text{ft}^3 & T(t) &= 690.0^\circ\text{R}
 \end{aligned}$$

Inevitably, under normal process conditions, the reactor will experience disturbances in the inlet temperature ($T_i(t)$) and concentration of species A ($c_{A_i}(t)$) in the input stream. Thus, we would like to know what impact these fluctuations in inlet conditions might have on the concentration of species A ($c_A(t)$) and the temperature ($T(t)$) of the effluent stream.

Suggestions: Assume that the reactor contents are well-mixed and that the heat capacity (C_p) and density (ρ) of reactants and products are equal.

- Develop a set of equations that could be used to predict temporal changes in effluent temperature and species A concentration ($T(t)$ and $c_A(t)$, respectively).
- Since we are interested in *deviations* in process variables, it is useful to reformulate the above equations in terms of deviation variables. A deviation variable (Y') for a process variable (Y) is defined as $Y' \equiv Y - \bar{Y}$ where \bar{Y} is the steady-state value. Reformulate equations in terms of such deviation variables, and solve for $c'_A(t)$ and $T'(t)$.
- Plot $c'_A(t)$ and $T'(t)$ versus time for the following cases: (a) $T'_i(t) = -5^\circ\text{R}$ and (b) $T'_i(t) = -10^\circ\text{R}$. Explain the observed behavior of the reactor. Does it always return to the same steady-state value? Is the dynamic response “smooth” or oscillatory?

6. In this problem we will explore a simple example of a suspension system for a vehicle. In general, mechanical systems respond differently to different inputs. One categorization of inputs is in terms of “frequency content”. We will analytically study a simple mechanical system to understand its response to the three kinds of inputs shown in the figure below:



One aspect of a car’s dynamics is the interaction of the car chassis with the suspension. As a crude model, we will treat the chassis as a lumped mass m_b , and the suspension as a linear stiffness with constant k_s . We will also assume that the suspension includes a viscous damper (shock-absorber) with damping coefficient b_s . Newtonian mechanics gives the following differential equation for the body dynamics:

$$m_b \ddot{q} + b_s \dot{q} + k_s q = f(t).$$

Here, $f(t)$ is the input force from the ground. The three inputs shown represent relatively smooth road, very bumpy road, and a single isolated bump.

- Setting $f(t) = \sin(\omega t)$, analytically solve the differential equation for the steady-state solution, in terms of the system parameters m_b, b_s, k_s , and the driving frequency ω . Assume $b_s^2 < 4m_b k_s$ so that the system is underdamped (typical of this kind of mechanical system).
- Sketch the amplitude of the steady-state response as function of the driving frequency ω . Identify the maximum of this plot in terms of the model parameters.
- The parameters that enter a model influence its behavior. Consider the limiting cases $\omega \ll \sqrt{\frac{k_s}{m_b}}$ and $\omega \gg \sqrt{\frac{k_s}{m_b}}$. What does your amplitude plot tell you about the response to these two different frequencies? What about inputs near this frequency?
- The third input shown is called an “impulse”, and is usually modeled as discontinuously altering the velocity of the system. This happens, for example, when driving over a sharp step in elevation. Setting $f(t) = 0$, solve the differential equation with initial conditions $q(0) = 0, \dot{q}(0) = 1$, as a function of the system parameters. Again assume the system is underdamped.
- Do you see any relationship between the oscillation frequency of this solution and the amplitude plot for the steady-state solution?
- What do your answers above tell you about the role of the car’s mass in reducing the effect of impulsive disturbances?