CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110
Homework Set #8

R. M. Murray
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All students should complete the following problems:

1. Consider the dynamics of the magnetic levitation system from lecture. The transfer function from the electromagnet input voltage to the IR sensor output voltage is given by

$$P(s) = \frac{k}{s^2 - r^2}$$

with $k = 4000$ and $r = 25$ (these parameters are slightly different than those used in the MATLAB files distributed with the lecture).

   (a) Design a stabilizing compensator for the process, assuming unity feedback. Compute the poles and zeros for the loop transfer function and for the closed loop transfer function between the reference input and measured output.

   (b) Plot the Nyquist plot corresponding to your compensator and verify that the Nyquist criterion is satisfied.

   (c) Plot the log of the magnitude of the sensitivity function, $\log |S(j\omega)|$, versus ω on a *linear* scale and numerically verify that the Bode integral formula is (approximately) satisfied. (Hint: you can do the integration numerically in MATLAB, using the trapz function. Make sure to choose your frequency range sufficiently large.)

2. Consider a second order system with transfer fuction

$$P(s) = \frac{s-1}{(s+10)^2}.$$

   (a) Plot the Bode plot for the system. Find another transfer function with the same magnitude but whose phase lag is less than the phase lag of $P(s)$.

   (b) Consider a proportional controller $C(s) = K_p$. Compute the range of gains for which the controller stabilizes the system and show that as $K_p \to \infty$, one of the poles of the closed loop transfer function approaches the zero at $s = 1$.

Only CDS 110a students need to complete the following additional problems.

3. For the control systems below, design a PID control law that stabilizes the sytem. You may you any method (loop shaping, Ziegler-Nichols, sisotool, etc). For the closed loop system, determine that steady state error, the maximum frequency for which the closed loop system can track with less than 5% error, and the approximate bandwidth of the system.

   (a) Disk drive read head positioning system:

$$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}$$

   (b) Second order system:

$$P(s) = \frac{100}{(100s+1)(s+1)}$$

4. In this problem we will design a PID compensator for the pitch axis of the Caltech ducted fan. Use the following transfer function to represent the vehicle dynamics:

$$P(s) = \frac{r}{Js^2 + bs + mgl}$$

$g = 9.8 \text{ m/sec}^2$ $\quad m = 1.5 \text{ kg}$ $\quad b = 0.05 \text{ kg/sec}$

$l = 0.05 \text{ m}$ $\quad J = 0.0475 \text{ kg m}^2$ $\quad r = 0.25 \text{ m}$

   (a) Design a PID compensator that stabilizes the system. You can use any method that you choose. Plot the pole zero diagram, frequency response, and step response for the *closed loop* system.

   (b) Plot the root locus plot for the system. Mark the open and closed loop pole locations corresponding to the PID compensator at the default gain (from part (a)).

   (c) Use the root locus plot to choose a new gain such that the dominant poles have a settling time of half of the value of the settling time for the original compensator designed in part (a). Show the location of the the poles with the new gains on your root locus plot.