CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110
Homework Set #7

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All students should complete the following problems:

1. For the control systems below, determine that steady state error, the maximum frequency for which the closed loop system can track with less than 5% error, and the approximate bandwidth of the system.

   (a) Disk drive read head positioning system, using lead compensator:

   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10} \qquad C(s) = 1000\frac{s+1}{s+10}$$

   (b) Second order system with PD compensator:

   $$P(s) = \frac{100}{(100s+1)(s+1)} \qquad C(s) = s + 10$$

2. In this problem we will design a lead compensator for the pitch axis of the Caltech ducted fan. Use the following transfer function to represent the vehicle dynamics:

   $$P(s) = \frac{r}{Js^2 + bs + mgl} \qquad \begin{array}{lll} g = 9.8 \text{ m/sec}^2 & m = 1.5 \text{ kg} & b = 0.05 \text{ kg/sec} \\ l = 0.05 \text{ m} & J = 0.0475 \text{ kg m}^2 & r = 0.25 \text{ m} \end{array}$$

   Design a unity feedback controller with the following specifications

   - Steady state error of less than 1%
   - Tracking error of less than 5% from 0 to 1 Hz (remember to convert this to rad/sec).
   - Closed loop step response with maximum overshoot of 20%.
   - Closed loop frequency response with no more than 3 dB gain at all frequencies.
   - High frequency disturbance rejection from reference to output (H_{yr}) of at least 10X above 100 Hz.

   If you cannot meet all of the specifications, you should prioritize them in the order listed.

   (a) Plot the open loop Bode plot for the system and mark on the plot the various frequency domain constraints in the above specification.

   (b) Design a compensator for the system that satisfies the specification. You should include plots to show that all specifications are met.

   (c) *Optional*: Write down the state space representation of your controller. (I.e., write down a state space system whose transfer function is the control law you designed.)

   Hint: start with the controller given in class and see if you can modify it to satisfy the specification.

Only CDS 110 students need to complete the following additional problems:

3. Continuing the previous problem, we will now investigate the stability of your control law under perturbations.

(a) Determine whether your control law is robustly stable with respect to added sensor dynamics

$$G(s) = \frac{1}{s+1}.$$

These dynamics should be inserted in the feedback loop, as we did in Lecture 7.1. If your controller is not stable with for the perturbed system, redesign your control law to provide as much performance as you can for the nominal plant and still maintain stability for the perturbed plant.

(b) For the control law that provided robust stability in (a), check to see if the performance specifications are satisfied for the perturbed plant. If your controller is not robust, redesign your control law to provide robust stability and robust performance to the specified sensor dynamics. If necessary, relax the specifications according to the priorities.

(c) Consider now the case where we have a time delay of 1 second instead of $G(s)$. Determine whether your control law from part (b) provides robust stability and/or robust performance in the presence of the specified time delay. (You just need to check to see if it works; you don't need to redesign it.)