All students should complete the following problems:

1. Plot the Nyquist and Bode plots for the following systems and compute the gain and phase margin of each. You should annotate your plots to show the gain and phase margin computations. For the Nyquist plot, mark the branches corresponding to the following sections of the Nyquist "D" contour: negative imaginary axis, positive imaginary axis, semi-circle at infinity (the curved part of the "D").

   (a) Disk drive read head positioning system, using lead compensator (we'll learn about how to design these in a week or two):

   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10} \qquad C(s) = 1000\frac{s+1}{s+10}$$

   (b) Second order system with PD compensator:

   $$P(s) = \frac{100}{(100s+1)(s+1)} \qquad C(s) = s + 10$$

   Note: you may find it easier to sketch the Nyquist plot from the Bode plot (taking some liberties with the scale) rather than relying on MATLAB.

2. In this problem we will design a PI controller for a cruise control system, building on the example shown in class. Using the following transfer function to represent the vehicle and engine dynamics:

   $$P(s) = \frac{ra/m}{(s+a)(s+b/m)}$$

   where $r = 5$ is the transmission gain (this was labelled k in previous sets), $a = 0.2$ is the engine lag coefficient, $m = 1000$ kg is the mass of the car, and $b = 50$ N sec/m is the viscous damping coeffient.

   (a) Consider a proportional controller for the car, $u = K_p(r - y)$. Assuming a unity gain feedback controller, this gives
   $$C(s) = K_p.$$

   Set $K_p = 100$ and compute the steady state error, gain and phase margins, and poles/zeros for the closed loop system. Remember that the gain and phase margins are computed based on the loop transfer function $L(s) = P(s)C(s)$.

   (b) Consider a proportional + integral controller for the car,

   $$C(s) = K_p + \frac{K_i}{s}.$$

   Fill in the following table (make sure to show your work):

K_p	K_i	Stable?	Gain Margin	Phase Margin	Steady State Error	Bandwidth
500	100					
50	1000					
50	1					
5	1					

   For each entry in the table, plot the pole zero diagram (pzmap) for the *closed loop* system and the step response.

Only CDS 110a students need to complete the following additional problems:

3. Continuing the previous problem, we will now insert a small amount of time delay into the feedback path of the system. A pure time delay of τ seconds satisfies the equation

$$y(t) = u(t - \tau)$$

This system is a linear input/output system and it can be shown that its transfer function is

$$G(s) = e^{-s\tau}.$$

Unfortunately, MATLAB is not able to perfectly represent a time delay in this form, and so we have to use a Pade approximation, which gives a constant gain transfer function with phase that approximates a time delay. Using a 2nd order Pade approximation, we can approximate our time delay as

$$G(s) = \frac{1 - T_d s/2 + (T_d s)^2/12}{1 + T_d s/2 + (T_d s)^2/12}$$

(this function can be computed using the `pade` function in MATLAB).

Assume that there is a time delay of τ seconds, which we will insert between the output of the plant and the controller (as we did in Monday's lecture).

(a) For the case $K_p = 50$, $K_i = 1$, insert time delays of $\tau = 0.25$ sec and $\tau = 0.75$ seconds. Using a Pade approximation, compute the resulting gain and phase margin for each case and compute the overshoot and settling time (2%) for the step responses.

(b) Repeat part (a) using $K_p = 20$, $K_i = 0.5$ and time delays of 0.75 sec and 1.5 sec.

(c) *Optional:* Plot the Nyquist plot for $K_p = 20$, $K_i = 0.5$, $\tau = 0.75$ (with the exact time delay, not the Pade approximation).

4. Consider a simple DC motor with inertial J and damping b. The transfer function is

$$P(s) = \frac{1}{Js^2 + bs}.$$

For simplicity, choose $J = 2$, $b = 1$. In this problem you will design some simple controllers to achieve a desired level of performance.

(a) Design a proportional control law, $C(s) = K_p$, that gives stable performance and has a bandwidth of at least 1 rad/s and a phase margin of at least 30 degrees. Plot the step response for the closed loop system using your controller.

(b) Design a proportional + derivative controller (PD) of the form

$$C(s) = K_p + K_d s$$

that gives closed loop bandwidth $\omega = 10$ rad/sec and has phase margin of at least 30 degrees. Plot the step response for the closed loop system using your controller.