CDS 101/110

R. M. Murray   **Homework Set #2**   Issued: 6 Oct 03
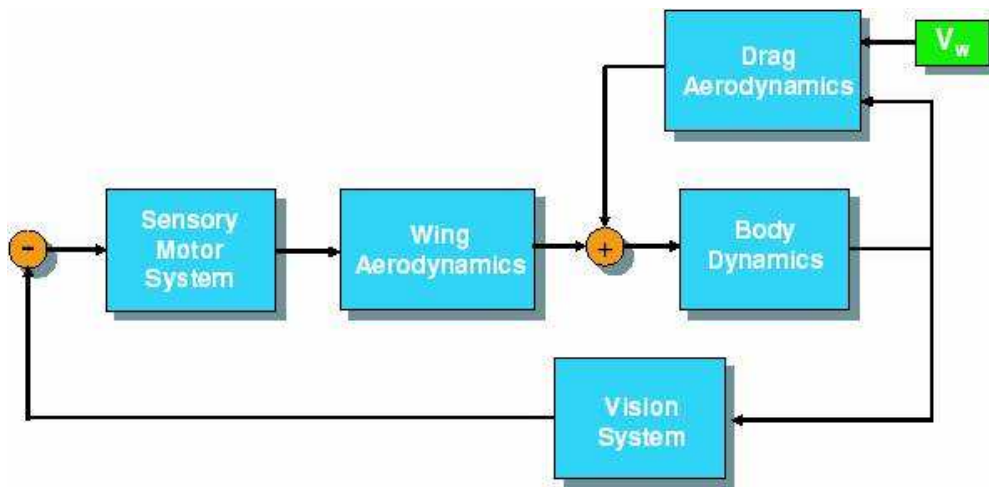
Fall 2003   Due: 13 Oct 03

Note: In the upper left hand corner of the first page of your homework set, please put the class you are taking (CDS 101, CDS 110) and the number of hours that you spent on this homework set (including reading).

All students should complete the following problems:

1. Consider the following block diagram of the flight control system of a fly:

   Using the paper "Vision as a Compensatory Mechanism for Disturbance Rejection in Upwind Flight" by Reiser et al. (available via the course web page), identify the state, input, outputs, and dynamics for each block in the diagram. You may give you answer in words, but be precise as possible. (Hint: not all of the blocks are "dynamic"; some are static maps.)

2. (MATLAB/SIMULINK) In this problem you will build a model of a vehicle in SIMULINK and control the vehicle using feedback control. The vehicle will consist of a body (chassis + wheels) and a drive train (engine + transmission). Assume that the vehicle dynamics are of the form

$$m\dot{v} = -bv + F_{\text{engine}} + F_{\text{hill}}$$

   where $m = 1000$ kg is the mass of the vehicle, $b = 50$ N sec/m is the viscous damping coefficient, and F_{engine} and F_{hill} represent the forces on the vehicle due to the engine and the terrain, respectively. We can implement this in SIMULINK as a two input, one output system, written in state space form as

$$
\begin{aligned}
\dot{x}_v &= \begin{bmatrix} -b/m \end{bmatrix} x_v + \begin{bmatrix} 1/m & 1/m \end{bmatrix} u_v \\
y_v &= x_v
\end{aligned}
\tag{1}
$$

where $x_v = v$ is the vehicle state, $u_v = \begin{bmatrix} F_{\text{engine}} & F_{\text{hill}} \end{bmatrix}^T$ is the vehicle input (two dimensional), and $y_v = v$ is the vehicle output (velocity). You should make this into a single SIMULINK block using the "State Space" block (under `Simulink→Continuous→State Space` in the Simulink Library Browser). You may also want to use the `Mux` block (under `Signals & Systems`).

We will model the engine dynamics as a "first order lag". Let τ represent the engine torque and assume the engine has the following dynamics:

$$\dot{\tau} = -a\tau + u_e$$
$$y_e = K\tau$$
(2)

where $a = 0.2$ is the lag coefficient, $K = 5$ is the conversion factor between engine torque and force applied to the vehicle (representing the transmission) and u_e is the accelerator input (which we will assume has the proper units). You should also create a SIMULINK block for this subsystem.

Finally, we include the effects of a hill. The hill simply exerts a force on the car that is based on the angle of the hill:

$$F_{\text{hill}} = -mg\sin(\theta)$$
(3)

where $g = 9.8$ kg m/sec^2 and $\theta = \pi/18$ is the angle of the hill (10 degrees).

(a) Plot the output of the open loop vehicle model (1) for a step input of $F_{\text{engine}} = 500$ Newtons (assume $F_{\text{hill}} = 0$). What is the rise time (0 to 95% of the final value)?

(b) Plot the output of the open loop engine model (2) for a step input of $u_e = 100$ Nm. What is the rise time?

(c) In the homework from last week, you built a simple cruise controller. Replace the vehicle/engine model in that system with your vehicle and engine models and plot the response for the default gains ($K_i = 50$, $K_p = 1000$). Make sure to set your simulation time to be sufficiently long.

(d) Now include the effect of a hill on your system. You should model the system so that the car is initially on a flat surface and then encounters the hill at $T = 100$ seconds. Plot the response of the system and compute the rise time.

Note: if you are having trouble figuring out how to create these blocks in SIMULINK, take a look at "hw1cruise.mdl" from last week's homework and see if you can modify it appropriately. It has all of the subsystems you will need (except for the `Mux` block). You may also find the following web-based tutorial helpful:

http://www.engin.umich.edu/group/ctm/examples/cruise/cc.html

(ignore the sections on transfer functions; we will get to these later in the class).

Only CDS 110a students need to complete the following additional problems:

3. Consider the following discrete time system

$$z[k + 1] = Az[k] + Bu[k]$$
$$y[k + 1] = Cz[k + 1]$$

where

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

In this problem, we will explore some of the properties of this discrete time system as a function of the parameters, the initial conditions, and the inputs.

(a) Assume that the off diagonal element $a_{12} = 0$ and that there is no input, $u = 0$. Write a closed form expression for the output of the system from a nonzero initial condition $z[0] = (z_1[0], z_2[0])$ and give conditions on a_{11} and a_{22} under which the output gets smaller as k gets larger.

(b) Now assume that $a_{12} \neq 0$ and write a closed form expression for the response of the system from a nonzero initial conditions. Given a condition on the elements of A under which the output gets smaller as k gets larger.

(c) Write a MATLAB program to plot the output of the system in response to a unit step input, $u[k] = 1$, $k \geq 0$. Plot the response of your system with $z[0] = 0$ and A given by

$$A = \begin{bmatrix} 0.5 & 1 \\ 0 & 0.25 \end{bmatrix}$$

4. Consider the coupled mass spring system show in the figure below (the same one considered in class on Monday):
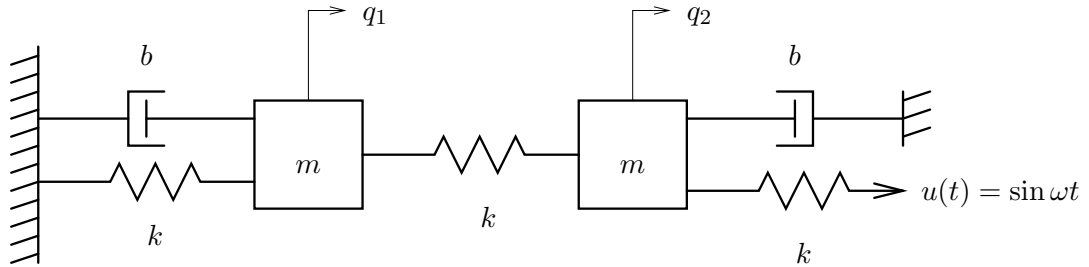
The input to this system is the sinusoidal motion of the end of rightmost spring and the output is the position of each mass, q_1 and q_2.

(a) Write the equations of motion for the system, using the positions and velocities of each mass as states.

(b) Rewrite the dynamics in terms of $z_1 = \frac{1}{2}(q_1 + q_2)$ and $z_2 = \frac{1}{2}(q_1 - q_2)$.

(c) Note that the resulting equations are diagonal. Solve these linear ODEs for $z_1(t)$ and $z_2(t)$ given initial conditions and input.

(d) Setting $m = 250$, $k = 50$, $b = 10$, plot the motion of the first and second masses in response to an input motion $u = A\sin(\omega t)$ with $\omega = 1$ rad/sec and $A = 1$ cm. Determine the amount of time required for the system to reach steady state oscillations.

(e) Plot the *steady state* amplitude of the motion of the first and second masses as a function of the input frequency, ω. You should get something similar to the plot shown in lecture on Monday.