7

Specifications

7.1 Introduction

In this chapter we will discuss how the properties of a control system can be specified. This is important for control design because it gives the goals. It is also important for users of control so that they know how to specify, evaluate and test a system so that they know it will have the desired properties. Specifications on a control systems typically include: stability of the closed loop system, robustness to model uncertainty, attenuation of measurement noise, injection of measurement noise, and ability to follow reference signals. From the results of Chapter 5 it follows that these properties are captured by six transfer functions called the Gang of Six. The specifications can be expressed in terms of these transfer functions. Essential features of the transfer functions can be expressed in terms of their poles and zeros or features of time and frequency responses.

7.2 Stability and Robustness to Process Variations

Stability and robustness to process uncertainties can be expressed in terms of the loop transfer function $L = PC$, the sensitivity function and the complementary sensitivity function

$$S = \frac{1}{1+PC} = \frac{1}{1+L}, \qquad T = \frac{PC}{1+PC} = \frac{L}{1+L}.$$

Since both S and T are functions of the loop transfer function specifications on the sensitivities can also be expressed in terms of specifications on the loop transfer function L. Many of the criteria are based on Nyquist's

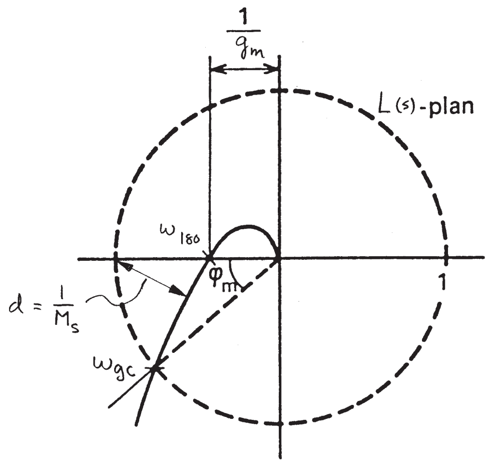

Figure 7.1 Nyquist curve of the loop transfer function L with indication of gain, phase and stability margins.

stability criterion, see Figure 7.1. Common criteria are the maximum values of the sensitivity functions, i.e.

$$M_s = \max_\omega |S(i\omega)|, \quad M_t = \max_\omega |T(i\omega)|$$

Recall that the number $1/M_s$ is the shortest distance of the Nyquist curve of the loop transfer function to the critical point, see Figure 7.1. Also recall that the closed loop system will remain stable for process perturbations ΔP provided that

$$\frac{|\Delta P(i\omega)|}{|P(i\omega)|} \leq \frac{1}{|T(i\omega)|},$$

see Section 5.5. The largest value M_t of the complementary sensitivity function T is therefore a simple measure of robustness to process variations.

Typical values of the maximum sensitivities are in the range of 1 to 2. Values close to one are more conservative and values close to 2 correspond to more aggressive controllers.

Gain and Phase Margins

The gain margin g_m and the phase margin φ_m are classical stability criteria. Although they can be replaced by the maximum sensitivities it is useful to know about them because they are still often used practically.

The gain margin tells how much the gain has to be increased before the closed loop system becomes unstable and the phase margin tells how much the phase lag has to be increased to make the closed loop system unstable.

The gain margin can be defined as follows. Let ω_{180} be the lowest frequency where the phase lag of the loop transfer function $L(s)$ is 180°. The gain margin is then

$$g_m = \frac{1}{|L(i\omega_{180})|} \tag{7.1}$$

The phase margin can be defined as follows. Let ω_{gc} denote gain crossover frequency, i.e. the lowest frequency where the loop transfer function $L(s)$ has unit magnitude. The phase margin is then given by

$$\varphi_m = \pi + \arg L(i\omega_{gc}) \tag{7.2}$$

The margins have simple geometric interpretations in the Nyquist diagram of the loop transfer function as is shown in Figure 7.1. Notice that an increase of controller gain simply expands the Nyquist curve radially. An increase of the phase of the controller twists the Nyquist curve clockwise, see Figure 7.1.

Reasonable values of the margins are phase margin $\varphi_m = 30° - 60°$, gain margin $g_m = 2 - 5$. Since it is necessary to specify both margins to have a guarantee of a reasonable robustness the margins g_m and φ_m can be replaced by a single stability margin, defined as the shortest distance of the Nyquist curve to the critical point -1, this distance is the inverse of the maximum sensitivity M_s. It follows from Figure 7.1 that both the gain margin and the phase margin must be specified in order to ensure that the Nyquist curve is far from the critical point. It is possible to have a system with a good gain margin and a poor phase margin and vice versa. It is also possible to have a system with good gain and phase margins which has a poor stability margin. The Nyquist curve of the loop transfer function of such a system is shown in Figure 7.2. This system has infinite gain margin, a phase margin of 70° which looks very reassuring, but the maximum sensitivity is $M_s = 3.7$ which is much too high. Since it is necessary to specify both the gain margin and the phase margin to endure robustness of a system it is advantageous to replace them by a single number. A simple analysis of the Nyquist curve shows that the following inequalities hold.

$$\begin{aligned} g_m &\geq \frac{M_s}{M_s - 1} \\ \phi_m &\geq 2 \arcsin \frac{1}{2M_s} \end{aligned} \tag{7.3}$$

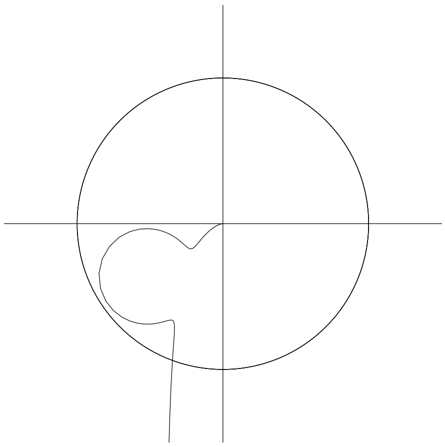

Figure 7.2 Nyquist curve of the loop transfer function for a system with good gain and phase margins but with high sensitivity and poor robustness. The loop transfer function is $L(s) = \frac{0.38(s^2+0.1s+0.55)}{s(s+1)(s^2+0.06s+0.5}$.

A controller with $M_s = 2$ thus has a gain margin of at least 2 and a phase margin of at least 30°. With $M_s = 1.4$ the margins are $g_m \geq 3.5$ and $\phi_m \geq 45°$.

Delay Margin

The gain and phase margins were originally developed for the case when the Nyquist curve only intersects the unit circle and the negative real axis once. For more complicated systems there may be many intersections and it is more complicated to find suitable concepts that capture the idea of a stability margin. One illustration if given in Figure 7.3. In this case the Nyquist curve has a large loop and the Nyquist curve intersects the circle $|L| = 1$ in three points corresponding to the frequencies 0.21, 0.88 and 1.1. If there are variations in the time delay the Nyquist curve can easily enclose the critical point. In the figure it is shown what happens when the time delay is increased from 3 to 4.5 s. This increase corresponds to a phase lag of 0.3 rad at the crossover frequency 0.21 rad/s, the phase lag is however 1.6 rad at the frequency 1.1 rad/s which is marked A in the figures. Notice that the point A becomes very close to the critical point. A good measure of the stability margin in this case is the delay margin which is the smallest time delay required to make the system unstable. For loop transfer functions that decay quickly the delay margin is closely related to the phase margin but for systems where the amplitude ratio of

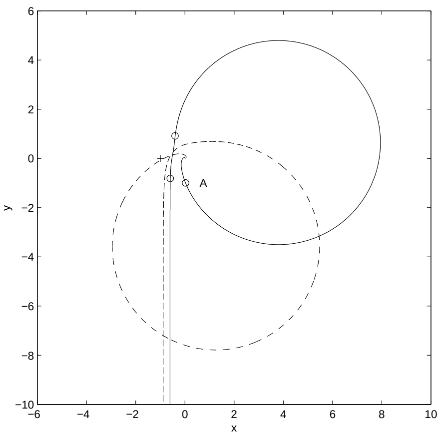

Figure 7.3 Nyquist curve of the loop transfer function $L(s) = \frac{0.2}{s(s^2+0.025s+1)}e^{-3s}$.

the loop transfer function has several peaks at high frequencies the delay margin is a much more relevant measure.

7.3 Disturbances

In the standard system, Figure 5.1, we have used in this book there are two types of disturbances, the load disturbances that drive the system away from its desired behavior and the measurement noise that corrupts the information about the process obtained by the sensors.

Response to Load Disturbances

The response of the process variable to a load disturbance is given by the transfer function

$$G_{xd} = \frac{P}{1+PC} = PS = \frac{T}{C} \tag{7.4}$$

Since load disturbances typically have low frequencies it is natural that the specifications should emphasize the behavior of the transfer function at low frequencies. The loop transfer function $L = PC$ is typically large for small s and we have the approximation

$$G_{xd} = \frac{T}{C} \approx \frac{1}{C}. \tag{7.5}$$

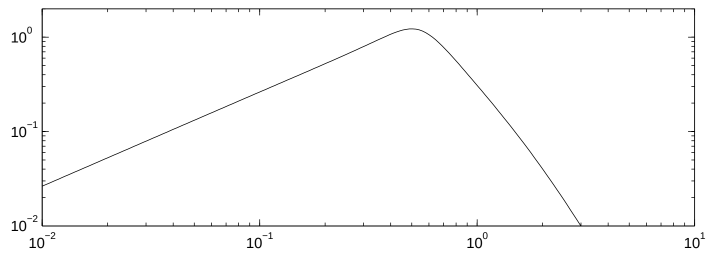

Figure 7.4 Typical gain curve for the transfer function G_{xd} from load disturbance to process output. The gain curve is shown in full lines and the transfer function k_i/s in dotted lines and the process transfer function in full lines.

If $P(0) \neq 0$ and the controller with integral action control we have the following approximation for small s

$$G_{xd} \approx \frac{s}{k_i}.$$

Since load disturbances typically have low frequencies this equation implies that integral gain k_i is a good measure of load disturbance attenuation. Figure 7.4 shows the magnitude curve of the transfer function (7.4) for a PI control of the process $P = (s+1)^{-4}$. The transfer function G_{xd} has typically the form shown in Figure 7.4. The curve can typically be characterized by the low frequency asymptote (k_i), the peak (M_{xd}), the frequency (ω_{xd})where the peak occurs and the high frequency roll-off. It follows from (7.4) that the high frequency behavior is essentially determined by the process and the maximum sensitivity.

Attenuation of load disturbances can also be characterized in the time domain by showing the time response due to a representative disturbance. This is illustrated in 7.5 which shows the response of the process output to a unit step disturbance at the process input. The figure shows maximum error e_{max}, the steady state error e_{ss}, the error of the open loop system e_{ol}, the time to maximum t_{max} and the settling time t_s.

Measurement Noise

An inevitable consequence of using feedback is that measurement noise is fed into the system. Measurement noise thus causes control actions which in turn generate variations in the process variable. It is important to keep these variations of the control signal at reasonable levels. A typical requirement is that the variations are only a fraction of the span of the control signal. The variations in the control variable are also detrimental

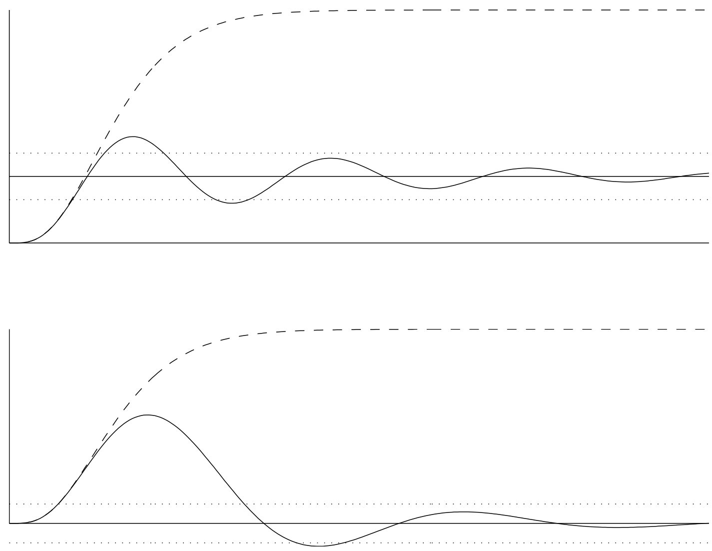

Figure 7.5 Errors due to a unit step load disturbance at the process input and some features used to characterize attenuation of load disturbances. The curves show the open-loop error (dashed lines) and the error (full lines) obtained using a controller without integral action (upper) and with integral action (lower).

by themselves because they cause wear of actuators. Since measurement noise typically has high frequencies the high high frequency gain of the controller is a relevant measure. Notice however that the low frequency gain of the controller is not essential since measurement noise is high frequency. Frequencies above the gain crossover frequency will be regarded as high.

To get a feel for the orders of magnitude consider an analog system where the signal levels are 10V. A measurement noise of 1 mV then saturates the input if the gain is 10^4. If it is only permitted that measurement noise gives control signals of 1V the high frequency gain of the controller must be less than 10^3.

As an other illustration we consider a digital control system with 12 bit AD- and DA-converters. A change of the input of one bit saturates the DA-converter if the gain is 4096. Assume that we permit one bit to give a variation of 0.4% of the output range. The high frequency gain of the controller must then be less than 500. With converters having lower resolution the high frequency gain would be even lower.

High precision analog systems with signal ranges of 1 to 10^4 have been designed. For digital systems the signal ranges are limited by the sensors and the actuators. Special system architectures with sensors and actuators having multiple signal ranges are used in order to obtain systems with a very high signal resolution. In these cases it is possible to have signal ranges up to 1 to 10^6.

The effects of measurement noise can be evaluated by the transfer function from measurement noise to the control signal, i.e.,

$$G_{un} = \frac{C}{1+PC} = CS = \frac{T}{P}. \tag{7.6}$$

Recall that P and C and are the transfer functions of the process and the controller, and that S is the sensitivity function. Notice that when $L = PC$ is large we have approximately $G_{un} \approx 1/C$. Since measurement noise typically has high frequencies and since the sensitivity function is one for high frequencies we find that the response to measurement noise is essentially determined by the high frequency behavior of the transfer function C. A simple measure is given by

$$M_c = \max_{\omega \geq \omega_{gc}} |G_{un}(i\omega)| \leq M_s \max_{\omega \geq \omega_{gc}} |C(i\omega)|$$

where M_c is called the maximum high frequency gain of the controller. When there is severe measurement noise it is advantageous to make sure that the transfer function C goes to zero for large frequencies. This is called high frequency roll-off.

7.4 Reference Signals

The response to set point changes is described by the transfer functions

$$G_{yr} = \frac{FPC}{1+PC} = FT, \qquad G_{ur} = \frac{FC}{1+PC} = FCS$$

Compare with (5.1). A significant advantage with controller structure with two degrees of freedom is that the problem of set point response can be decoupled from the response to load disturbances and measurement noise. The design procedure can then be divided into two independent steps.

- First design the feedback controller C that reduces the effects of load disturbances and the sensitivity to process variations without introducing too much measurement noise into the system

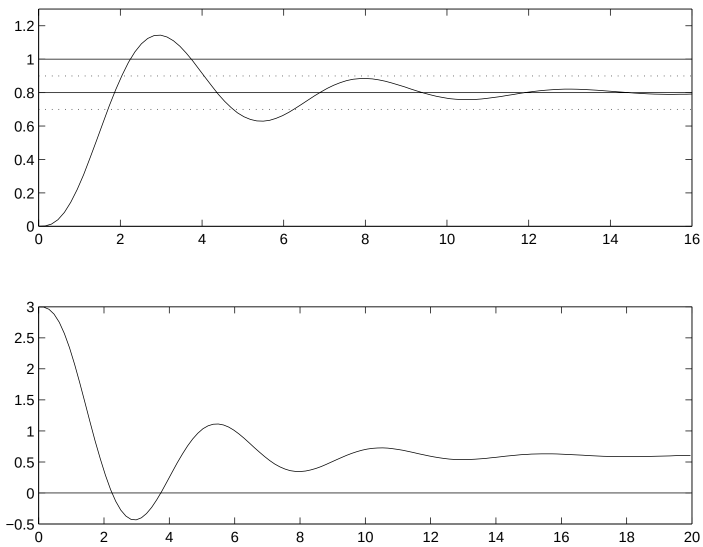

Figure 7.6 Specifications on reference following based on the time response to a unit step in the reference.

- Then design the feedforward F to give the desired response to set points.

Specifications on reference following are typically expressed in the time domain. They may include requirements on rise time, settling time, decay ratio, overshoot, and steady-state offset for step changes in reference. These quantities are defined as follows, see 7.6. These quantities are defined in different ways and there are also different standards.

- The *rise time* t_r is either defined as the inverse of the largest slope of the step response or the time it takes the step to pass from 10% to 90% of its steady state value.

- The *settling time* t_s is the time it takes before the step response remains within p percent of its steady state value. The value $p = 2$ % is commonly used.

- The *delay time* is the time required for the step response to reach 50 % of its steady state value for the first time.

- The *decay ratio* d is the ratio between two consecutive maxima of the error for a step change in reference or load. The value $d = 1/4$, which is called quarter amplitude damping, has been used traditionally. This value is, however, too high as will be shown later.

- The *overshoot* o is the ratio between the difference between the first peak and the steady state value and the steady state value of the step response. In industrial control applications it is common to specify an overshoot of 8%–10%. In many situations it is desirable, however, to have an over-damped response with no overshoot.
- The *steady-state error* e_{ss} is the value of control error e in steady state. With integral action in the controller, the steady-state error is always zero.

Classical specifications have been strongly focused on the behavior of the process output. It is however important to also consider the control signal. Analogous quantities can be defined for the control signal. The overshoot of the control signal is of particular importance, see Figure 7.4.

Step signals are often used as reference inputs In motion control systems it is often more relevant to consider responses to ramp signals or jerk signals. Specifications are often given in terms of the the value of the first non-vanishing error coefficient.

Tracking Slowly Varying Signals - Error Coefficients

Step signals is one prototype of reference signals. There are however situations when other signals are more appropriate. One example is when the reference signal has constant rate of change, i.e.

$$r(t) = v_0 t$$

The corresponding Laplace transform is $R(s) = v_0/s^2$.

For a system with error feedback the error $e = r - y$ has the Laplace transform

$$E(s) = S(s)V(s) = S(s)\frac{v_0}{s^2} \tag{7.7}$$

The steady state error obtained depends on the properties of the sensitivity function at the origin. If $S(0) = e_0$ the steady state tracking error is asymptotically $e(t) = v_0 e_0 t$. To have a constant tracking error it must be required that $S(0) = 0$. With $S(s) \approx e_1 s$ for small s we find that the steady state error is $e(t) = v_0 e_1$ as t goes to infinity. To have zero steady state error for a ramp signal the function $S(s)$ must go to zero faster than s for small s. If $S(s) \approx e_2 s^2$ for small s we find that the error is asymptotically zero. Since

$$L(s) = \frac{1}{S(s)} - 1$$

it follows that the condition $S(s) \approx e_2 s^2$ implies that $L(s) \approx s^{-2}$ for small s. This implies that there are two integrations in the loop. Continuing this reasoning we find that in order to have zero steady state error when tracking the signal

$$r(t) = \frac{t^2}{2}$$

it is necessary that $s(s) \approx e_3 s^3$ for small s. This implies that there are three integrals in the loop.

The coefficients of the Taylor series expansion of the sensitivity $s(s)$ function for small s,

$$S(s) = e_0 + e_1 s + e_2 s^2 + \ldots + e_n s^n + \ldots \tag{7.8}$$

are thus useful to express the steady state error in tracking low frequency signals. The coefficients e_k are called error coefficients. The first non vanishing error coefficient is the one that is of most interest, this is often called the error coefficient.

7.5 Specifications Based on Optimization

The properties of the transfer functions can also be based on integral criteria. Let $e(t)$ be the error caused by reference values or disturbances and let $u(t)$ be the corresponding control signal. The following criteria are commonly used to express the performance of a control system.

$$\begin{aligned}
IE &= \int_0^\infty e(t)dt \\
IAE &= \int_0^\infty |e(t)|dt \\
ITAE &= \int_0^\infty t|e(t)|dt \\
IQ &= \int_0^\infty e^2(t)dt \\
WQ &= \int_0^\infty (e^2(t) + \rho u^2(t))dt
\end{aligned}$$

They are called, IE integrated error, IAE integrated absolute error, ITAE integrated time multiplies absolute error, integrated quadratic error and WQ weighted quadratic error. The criterion WQ makes it possible to trade the error against the control effort required to reduce the error.

7.6 Properties of Simple Systems

It is useful to have a good knowledge of properties of simple dynamical systems. In this section we have summarize such data for easy reference.

First Order Systems

Consider a system where the transfer function from reference to output is

$$G(s) = \frac{a}{s+a} \tag{7.9}$$

The step and impulse responses of the system are

$$h(t) = 1 - e^{-at} = 1 - e^{-t/T}$$
$$g(g) = ae^{-at} = \frac{1}{T}e^{-t/T}$$

where the parameter T is the time constant of the system. Simple calculations give the properties of the step response shown in Table 7.1. The 2% settling time of the system is 4 time constants. The step and impulse responses are monotone. The velocity constant e_1 is also equal to the time constant T. This means that there will be a constant tracking error of $e_1 v = v_0 T$ when the input signal is a ramp $r = v_0 t$.

This system (7.9) can be interpreted as a feedback system with the loop transfer function

$$L(s) = \frac{a}{s} = \frac{1}{sT}$$

This system has a gain crossover frequency $\omega_{gc} = a$. The Nyquist curve is the negative imaginary axis, which implies that the phase margin is 90°. Simple calculation gives the results shown in Table 7.1. The load disturbance response of a first order system typically has the form

$$G_{xd} = \frac{s}{s+a}$$

The step response of this transfer function is

$$h_{xd} = e^{-at}$$

The maximum thus occurs when the disturbance is applies and the settling time is $4T$. The frequency response decays monotonically for increasing frequency. The largest value of the gain is a zero frequency.

Some characteristics of the disturbance response are given in Table 7.2.

Table 7.1 Properties of the response to reference values for the first order system $G_{xr} = a/(s+a)$.

Propety	*Value*
Rise time	$T_r = 1/a = T$
Delay time	$T_d = 0.69/a = 0.69T$
Settling time (2%)	$T_s = 4/a = 4T$
Overshoot	$o = 0$
Error coefficients	$e_0 = 0$, $e_1 = 1/a = T$
Bandwidth	$\omega_b = a$
Resonance peak	$\omega_r = 0$
Sensitivities	$M_s = M_t = 1$
Gain margin	$g_m = \infty$
Phase margin	$\varphi_m = 90^\circ$
Crossover frequency	$\omega_{gc} = a$
Sensitivity frequency	$\omega_{sc} = \infty$

Table 7.2 Properties of the response to disturbances for the first order system $G_{xd} = s/(s+a)$.

Property	*Value*
Peak time	$T_p = 0$
Max error	$e_{max} = 1$
Settling time	$T_s = 4T$
Error coefficient $e_1 = T$	
Largest norm	$\|\|G_{xd}\|\| = 1$
Integrated error	$IE = 1/a = T$
Integrated absolute error	$IAE = 1/a = T$

Second Oder System without Zeros

Consider a second order system with the transfer function

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \tag{7.10}$$

The system has two poles, they are complex if $\zeta < 1$ and real if $\zeta > 1$. The step response of the system is

$$h(t) = \begin{cases} 1 - \dfrac{e^{-\zeta\omega_0 t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi) & \text{for } |\zeta| < 1 \\ 1 - (1+\omega_0 t)e^{-\omega_0 t} & \text{for} \zeta = 1 \\ 1 - \Big(\cosh\omega_d t + \dfrac{\zeta}{\sqrt{\zeta^2-1}}\sinh\omega_d t\Big)e^{-\zeta\omega_d t} & \text{for } |\zeta| > 1 \end{cases}$$

where $\omega_d = \omega_0\sqrt{|1-\zeta^2|}$ and $\phi = \arccos\zeta$. When $\zeta < 1$ the step response is a damped oscillation, with frequency $\omega_d = \omega_0\sqrt{1-\zeta^2}$. Notice that the step response is enclosed by the envelopes

$$e^{-\zeta\omega_0 t} \le h(t) \le 1 - e^{-\zeta\omega_0 t}$$

This means that the system settles like a first order system with time constant $T = \frac{1}{\zeta\omega_0}$. The 2% settling time is thus $T_s \approx \frac{4}{\zeta\omega_0}$. Step responses for different values of ζ are shown in Figure 4.9.

The maximum of the step response occurs approximately at $T_p \approx \pi/\omega_d$, i.e. half a period of the oscillation. The overshoot depends on the damping. The largest overshoot is 100% for $\zeta = 0$. Some properties of the step response are summarized in Table 7.3.

The system (7.10) can be interpreted as a feedback system with the loop transfer function

$$L(s) = \frac{\omega_0^2}{s(s+2\zeta\omega_0)}$$

This means that we can compute quantities such as sensitivity functions and stability margins. These quantities are summarized in Table 7.3.

Second Oder System with Zeros

The response to load disturbances for a second order system with integral action can have the form

$$G(s) = \frac{\omega_0 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

The frequency response has a maximum $1/(2\zeta)$ at $\omega = \omega_0$. The step response of the transfer function is

$$h(t) = \frac{e^{-\zeta\omega_0 t}}{sqrt1-\zeta^2}sin\omega_d t$$

Table 7.3 Properties of the response to reference values of a second order system.

Property	*Value*
Rise time	$T_r = \omega_0 e^{\phi/\tan\phi} \approx 2.2 T_d$
Delay time	T_d
Peak time	$T_p \approx \pi/\omega_D = T_d/2$
Settling time (2%)	$T_s \approx 4/(\zeta\omega_0)$
Overshoot	$o = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
Error coefficients	$e_0 = 0,\ e_1 = 2\zeta/\omega_0$
Bandwidth	$\omega_b = \omega_0\sqrt{1 - 2\zeta^2 + \sqrt{(1-2\zeta^2)^2 + 1}}$
Maximum sensitivity	$M_s = \sqrt{\frac{8\zeta^2+1+(4\zeta Í2+1)\sqrt{8\zeta^2+1}}{8\zeta^2+1+(4\zeta Í2-1)\sqrt{8\zeta^2+1}}}$
Frequency	$w_{ms} = \frac{1+\sqrt{8\zeta^2+1}}{2}\omega_0$
Max. comp. sensitivity	$M_t = \begin{cases} 1/(2\zeta\sqrt{1-\zeta^2}) & \text{if } \zeta \le \sqrt{2}/2 \\ 1 & \text{if } \zeta \le \sqrt{2}/2 \end{cases}$
Frequency	$\omega_{mt} = \begin{cases} \omega_0\sqrt{1-2\zeta^2} & \text{if } \zeta \le \sqrt{2}/2 \\ 1 & \text{if } \zeta \le \sqrt{2}/2 \end{cases}$
Gain margin	$g_m = \infty$
Phase margin	$\varphi_m = 90^\circ - \arctan\omega_c/(2\zeta\omega_0)$
Crossover frequency	$\omega_{gc} = \omega_0\sqrt{\sqrt{4\zeta^4+1} - 2\zeta^2}$
Sensitivity frequency	$\omega_{sc} = \omega_0/\sqrt{2}$

This could typically represent the response to a step in the load disturbance. Figure 7.7 shows the step response for different values of ζ. The step response has its maximum

$$\max_t h(t) = \omega_0 e^{-\zeta//sqrt1-\zeta^2} \tag{7.11}$$

for

$$t = t_m = \frac{\arccos\zeta}{\omega_0}$$

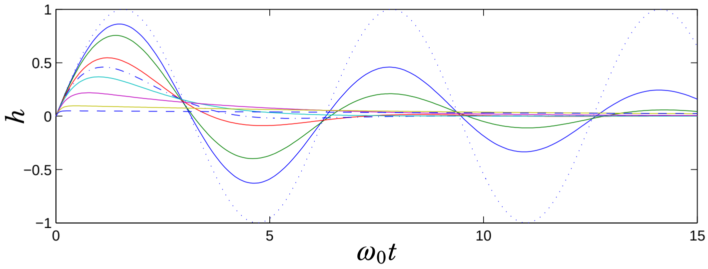

Figure 7.7 Step responses of the transfer function (7.11) for $\zeta = 0$ (dotted), 0.1, 0.2, 0.5, 0.7 (dash-dotted), 1, 2, 5, 10 (dashed).

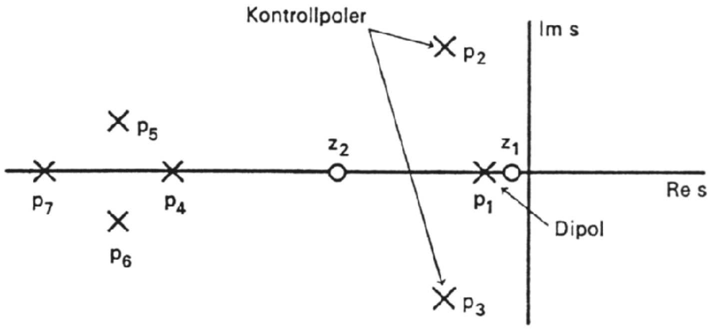

Figure 7.8 Typical configuration of poles and zeros for a transfer function describing the response to reference signals.

Systems of Higher Order

7.7 Poles and Zeros

Specifications can also be expressed in terms of the poles and zeros of the transfer functions. The transfer function from reference value to the output of a system typically has the pole zero configuration shown in Figure 7.8. The behavior of a system is characterized by the poles and zeros with the largest real parts. In the figure the behavior is dominated by a complex pole pair p_1 and p_2 and real poles and zeros. The dominant poles are often characterized by the relative damping ζ and the distance from the origin ω_0. Robustness is determined by the relative damping and the response speed is inversely proportional to ω_0.

- Dominant poles
- Zeros
- Dipoles

7.8 Relations Between Specifications

A good intuition about the different specifications can be obtained by investigating the relations between specifications for simple systems as is given in Tables 7.1, 7.2 and 7.3.

The Rise Time Bandwidth Product

Consider a transfer function $G(s)$ for a stable system with $G(0) \neq 0$. We will derive a relation between the rise time and the bandwidth of a system. We define the rise time by the largest slope of the step response.

$$T_r = \frac{G(0)}{\max_t g(t)} \tag{7.12}$$

where g is the impulse response of G, and let the bandwidth be defined as

$$\omega_b = \frac{\int_0^\infty |G(i\omega)|}{\pi G(0)} \tag{7.13}$$

This implies that the bandwidth for the system $G(s) = 1/(s+1)$ is equal to 1, i.e. the frequency where the gain has dropped by a factor of $1/\sqrt{2}$. The impulse response g is related to the transfer function G by

$$g(t) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} e^{st}G(s)ds = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}G(i\omega)d\omega$$

Hence

$$\max_t g(t) \le \frac{1}{2\pi}\int_{-\infty}^{\infty} \left| e^{i\omega t}G(i\omega)\right| d\omega = \frac{1}{\pi}\int_0^\infty |G(i\omega)|d\omega$$

Equations (7.12) and (7.13) now give

$$T_r\omega_b \ge 1$$

This simple calculation indicates that the product of rise time and bandwidth is approximately constant. For most systems the product is around 2.

7.9 Summary

It is important for both users and designers of control systems to understand the role of specifications. The important message is that it is necessary to have specifications that cover properties of the Gang of Six, otherwise there is really no guarantee that the system will work well. This important fact is largely neglected in much of the literature and in control practice. Some practical ways of giving reasonable specifications are summarized.