CDS 101: Principles of Feedback and Control
Course Overview

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Abstract

CDS 101 is a first year graduate class in control for non-specialists. The course is structured to introduce the *principles* and *tools* of control to students in a diverse set of disciplines. The level of mathematical abstraction is kept to a minimum, while attempting to maintain a careful and rigorous approach to the subject. The main structure of the course is presented and a detailed description of the course contents is given.

1 Introduction

CDS 101 is an experimental course being developed in the Conrol and Dynamical Systems Department at Caltech. This document provides an overview of the course, including the pedagogical approach taken in the course. This document is intended to serve as documentation for instructors and TAs for CDS 101, as well as others interested in developing similar courses.

1.1 Course Audience and Goals

CDS 101 is intended for advanced students in science and engineering who are interested in the principles and tools of feedback control, but not the analytical techniques for design and synthesis of control systems. Special attention is payed to insuring that the course is accessible to students from biolgical, physical, and information sciences. These students have varying levels of mathematical sophistication, especially with regards to continuous mathematics.

The goal of the course is to enable students to use the principles and tools of feedback and control in their research activities. In particular, after taking this course, students should be able to build control-oriented models of physical, biological, or information systems and simulate those models in the time-domain; analyze stability, performance and robustness of the models; and design rudimentary feedback control systems in the time and frequency domain. Special emphasis is given to state space methods for analysis and synthesis since these techniques are needed for systems that are nonlinear and asynchronous.

1.2 Course Administration

CDS 101 is taught as a one hour/week main lecture, a weekly homework set, and a (currently optional) tutorial/application lecture. The course is taught in 10 weeks, with 8 homework sets, a midterm, and a final.

The main lecture (Mondays, 2-3 pm) is a prepared presentation that covers the main topics for the week. Hardware demonstrations are used to convey concepts when possible, along with simulations and interactive MATLAB sessions. Each lecture introduces the main MATLAB commands needed to implement the concepts. A printed copy of the lecture presentation is handed out at the beginning of each lecture so that students can take notes. For many lectures, a printout of the MATLAB code used for the examples used in the lecture is also included.

The homework set for CDS 101 consists of 2 problems per week, with at least one of these being a computer exercise. The computer exercises use MATLAB and SIMULINK, and consider examples that are moderate complexity, to allow the power of the tools to be demonstrated. The homeworks are designed to require approximately 2-3 hours to solve. Students were asked to report the number of hours spent on the homework on the first page of the homework, so that the amount of time used could be tracked and the difficulty of the homeworks could be adjusted appropriately.

Lectures are given each week (Friday, 2-3 pm) to provide application examples and review selected material in advance of the midterm and final. The application lectures are given by faculty and are intended to provide a bridge between the topics given in classes and the research activities of the faculty. For 2002, the lectures include:

- Insect flight modeling and control, Michael Dickinson (BE)
- Congenstion control of the Internet, Eric Klavins (CS)
- Quantum feedback control, Hideo Mabuchi (Ph)
- Control of the Keck and CELT telescopes, Doug MacMartin (UTRC)

These lectures are advertised to the campus so that anyone can attend. They are currently optional for the students, but will be a required part of the course in future years.

In addition, some Friday lecture slots are used for reviews by the head TA. The first Friday lecture of the term is a tutorial on the use of MATLAB. The lectures on the fifth and tenth weeks of the term are reserved for a review of the material that will be covered on the midterm and final.

CDS 101 is co-taught with the first term of a more traditional controls course, CDS 110ab. CDS 110a shares the Monday lecture with CDS 101, but has an additional two hours of lectures on Wednesday (1-3 pm) that goes into more detail on the selected topics. The lectures are designed such that very little information is repeated in the Wednesday lectures. This structure for the course requires that the Monday lectures be self-contained, yet serve as an effective introduction to the more detailed concepts provided in the Wednesday lectures.

1.3 Pre-course

Since CDS 101 is intended to be taken by students with a diverse mathematics background, a special *pre-course* is offered in the week before the course begins. This pre-course gives a concise introduction to three main topics: linear algebra, ordinary differential equations, and dynamical systems. The pre-course spans two days, with three major topics: modeling of physical systems, ordinary differential equations, and linear algebra. Two one hour lectures are given in each topic, with homework sets and computer exercises assigned (intended to be done that same day).

In 2002, the pre-course had 25 students attend the lectures. The pre-course was advertised through flyers, e-mail to faculty, and e-mail to mailing lists for various groups, centers, and departments. The most effective mechanism for announcing the course was via established mailing lists that went to students.

1.4 Mud cards

"Mud cards" are a simple tool for allowing students to get additional information on topics that they didn't understand in the lecture. 3×5 cards are handed out at the beginning of each lecture and the students are instructed to write on the cards the "muddiest" part of the lecture. These cards are collected at the end of the class and the TAs sort them into categories and write up answers to the questions that arose. The answers are posted on the web the evening of the lecture, rapidly providing additional information for students who had questions.

The database used for storing mud card responses allows the questions and responses to be posted on the web page for each lecture, so that students could see what other questions had been asked for each lecture. In addition, the database allows responses to frequently asked questions on the homeworks, which could be posted on the web for students who were working on the homework sets.

Mud cards have turned out to be very heavily used by the students. In 2002, there were an average of 13 mud card responses for each Monday lecture, with a peak of 22 (these numbers are for both CDS 101 and 110 students). The questions ranged from clarification of details in the lecture to conceptual questions regarding the material to administrative questions/comments about the course.

1.5 Additional resources

In addition to the specific resources described above, the course uses a variety of additional techniques to increase the effectiveness of the course. Most of these are fairly standard in modern courses, but are listed here for completeness.

Background survey A survey is distributed on the first day of class to determine the background of the students in the class. The survey asks for the year and option of the student, as well as the courses that the student has previously taken (chosen from a list). In addition, a list of topics are given and the students are asked about their familiarity with each. The topics range from those that are pre-requisites for the course, to the topics that will be covered in the course, to advanced topics in control. The results of the survey provide information about the preparation and level of the students.

TA office hours There are two sets of office hours for the course. The head TA holds office hours on Fridays at 3 pm, after the optional lecture. In addition, there are office hours on Sundays (the day before the homework is due) from 5–7 pm. The TAs also respond to e-mail queries. Frequently asked questions are posted on the web.

All students are required to attend office hours at least once in the first three weeks of the course. The only obligation is to show up, introduce themselves, and get marked for attending. The purpose of this requirement is to let all of the students know when and where the office hours are, as well as introduce them to (a subset of) the TAs.

Course videos All lectures (including the optional lectures) are videotaped and made available to students in the course.

Course web page The course web page contains links to all of the lectures (in PowerPoint format), the reading assignments, streaming video for the lectures (posted w/in one week after the lecture), copies of the homeworks and solutions. The head TA is responsible for maintaining the web page.

Midterm and final surveys Surveys are distributed as part of the midterm and final exams. These surveys ask about the utility of the various elements of the course and ask for specific suggestions on how to improve the course. on the midterm survey, the number of hours spent per week is also requested, along with a list of the aspects of the course that take the most time.

2 Course Approach and Contents

The approach taken in CDS 101 is to focus on the main principles of feedback and control and the mathematical and computer tools available to implement these principels. This section gives a high level overview of the primary topics of the course.

2.1 Principles

The course emphasizes two main principles:

1. Robustness to uncertainty through feedback
2. Design of dynamics through feedback

These particular principles are emphasized because of their broad relevance to many different domains.

Although control is not formally introduced until about half way through the course, feedback is used in the examples and homeworks starting in the first lecture. This allows the concepts of control to be studied throughout the course, building intuition for the eventual formal analysis of feedback systems.

2.2 Tools

The course makes use of a number of tools for modeling and analysis. MATLAB is used as the primary software for the course, due to its broad use in science and engineering.

In addition to SIMULINK and the Controls Toolbox, some custom software has been developed as part of the course:

- `phaseplot.m`
- `cruise.mdl`

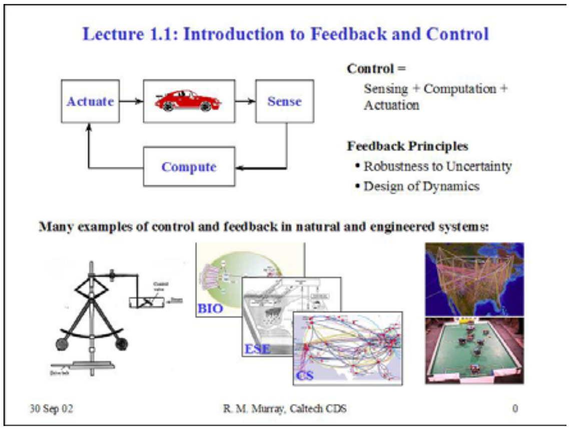
2.3 Lecture Descriptions

This section gives a more detailed description of the topics covered in CDS 101. For each lecture, a list of the goals for the lecture, a copy of the summary chart for the lecture, and a short description of the lecture are given.

Lecture 1.1: Introduction to Feedback and Control

Goals:

- Define what a control system is and learn how to recognize its main features
- Describe what control systems do and the primary principles of control
- Give an overview of CDS 101/110; describe course structure and administration



Lecture 1.1: Introduction to Feedback and Control

Actuate → Sense → Compute

Control =
Sensing + Computation + Actuation

Feedback Principles
- Robustness to Uncertainty
- Design of Dynamics

Many examples of control and feedback in natural and engineered systems:

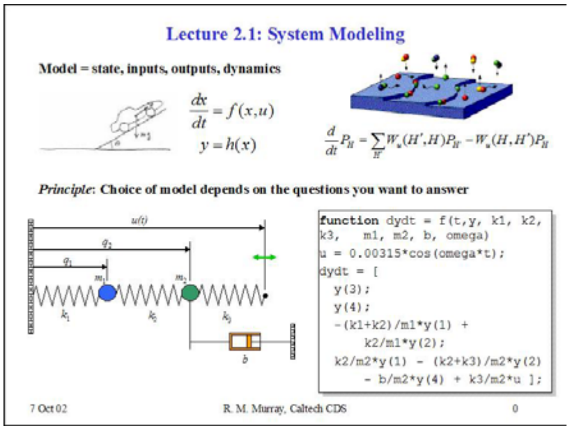
30 Sep 02 R. M. Murray, Caltech CDS 0

This lecture provides an overview of the basic ideas in feedback and control, including the major principles of feedback and many examples of applications. The goal of this lecture is to introduce some of the basic ideas in feedback systems and provide examples that will allow students to identify and recognize control systems in their everyday world. CDS 101 course administration is also covered in the second half of the lecture.

Lecture 2.1: System Modeling

Goals:

- Describe what a model is and what types of questions it can be used to answer
- Introduce the concepts of state, dynamics, inputs and outputs
- Provide examples of common modeling techniques: finite state automata, difference equations, differential equations, Markov chains
- Describe some common modeling tradeoffs



Lecture 2.1: System Modeling

Model = state, inputs, outputs, dynamics

$$\frac{dx}{dt} = f(x,u)$$
$$y = h(x)$$

$$\frac{d}{dt}P_H = \sum_{H'} W_u(H',H)P_{H'} - W_u(H,H')P_H$$

***Principle*: Choice of model depends on the questions you want to answer**

```
function dydt = f(t,y, k1, k2,
k3,   m1, m2, b, omega)
u = 0.00315*cos(omega*t);
dydt = [
  y(3);
  y(4);
  -(k1+k2)/m1*y(1) +
      k2/m1*y(2);
  k2/m2*y(1) - (k2+k3)/m2*y(2)
      - b/m2*y(4) + k3/m2*u ];
```

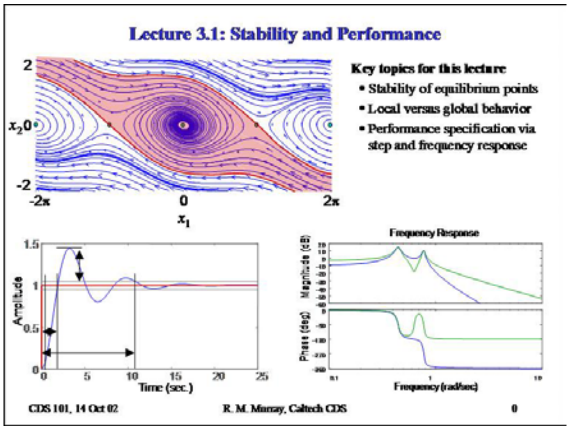
7 Oct 02 R. M. Murray, Caltech CDS 0

This lecture provides an overview of modeling for control systems. We discuss what a model is and what types of questions it can be used to answer. The concepts of state, dynamics, inputs and outputs are described, including running examples to demonstrate the concepts. Several different modeling techniques are summarized: finite state automata, difference equations, differential equations, and Markov chains. Two examples are included to demonstrate the main concepts.

Lecture 3.1: Stability and Performance

Goals:

- Describe different types of local stability of an equilibrium point
- Explain the difference between local stability, global stability, and related concepts
- Describe performance measures for (controlled) systems, including transients and steady state response



Lecture 3.1: Stability and Performance

Key topics for this lecture
- Stability of equilibrium points
- Local versus global behavior
- Performance specification via step and frequency response

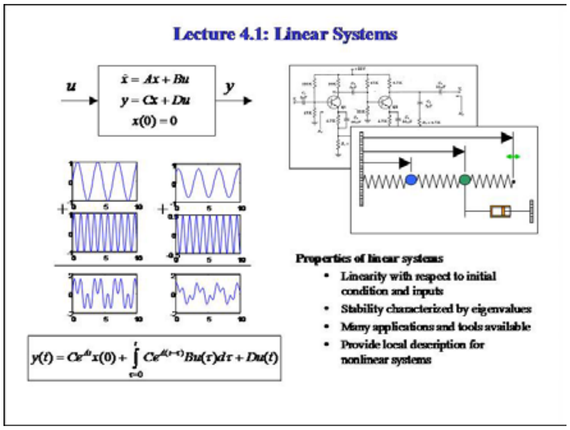
CDS 101, 14 Oct 02 R. M. Murray, Caltech CDS 0

This lecture provides and introduction to stability and performance of (nonlinear) control systems. A formal definition of stable systems is given and phase portraits are introduced to help visualize the concepts. Local and global behavior of nonlinear systems is discussed, using a damped pendulum and the preditor-prey problem as examples. Performance of control systems is presented for both transient (step respones) and steady state (frequency domain) specifications.

Lecture 4.1: Linear Input/Output Systems

Goals:

- Describe linear system models: properties, examples, and tools
- Characterize the stability and performance of a linear system in terms of eigenvalues
- Compute linearization of a nonlinear systems around an equilibrium point



Lecture 4.1: Linear Systems

$$y(t) = Ce^{At}x(0) + \int_{\tau=0}^{t} Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

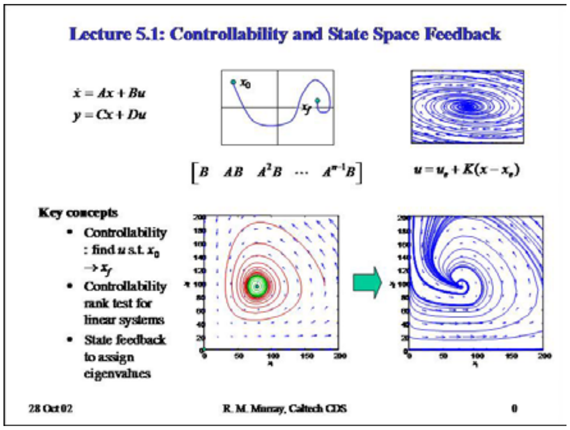
Properties of linear systems
- Linearity with respect to initial condition and inputs
- Stability characterized by eigenvalues
- Many applications and tools available
- Provide local description for nonlinear systems

This lecture gives an introduction to linear control systems. The main properties of linear systems are given and the matrix exponential is used to provide a formula for the output response given an initial condition and input signal. Linearization of nonlinear systems as an approximation of the dynamics is also covered.

Lecture 5.1: Controllability and State Feedback

Goals:

- Define controllability of a control system
- Give tests for controllability of linear systems and apply to examples
- Describe the design of state feedback controllers for linear systems



Lecture 5.1: Controllability and State Space Feedback

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$\begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$

$u = u_e + K(x - x_e)$

Key concepts

- Controllability : find u s.t. $x_0 \to x_f$
- Controllability rank test for linear systems
- State feedback to assign eigenvalues

28 Oct 02

R. M. Murray, Caltech CDS

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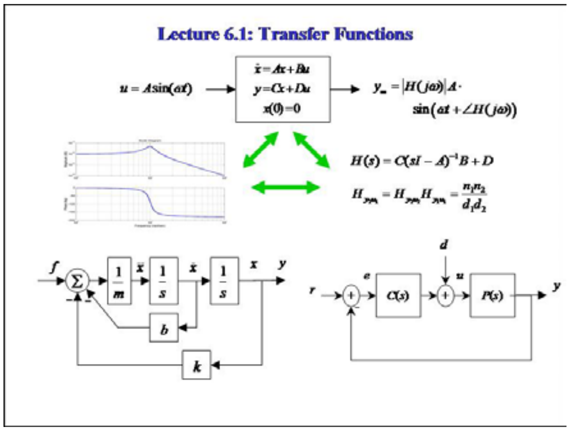
This lecture introduces the concept of controllability and explores the use of state space feedback for control of linear systems. Controllability is defined as the ability to move the system from one condition to another over finite time. The controllability matrix test is given to check if a linear system is controllable, and the test is applied to several examples. The concept of (linear) state space feedback is introduced and the ability to place eigenvalues of the closed loop system arbitrarily is related to controllability. A cart and pendulum system and the preditor prey problem are used as examples.

Lecture 6.1: Transfer Functions

Goals:

- Define the input/output transfer function of a linear system
- Derive the transfer function corresponding to a system in state space form
- Build transfer functions for interconnected systems, via block diagram algebra



Lecture 6.1: Transfer Functions

$u = A\sin(\omega t)$

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$x(0) = 0$

$y_{ss} = |H(j\omega)| A \cdot \sin(\omega t + \angle H(j\omega))$

$H(s) = C(sI - A)^{-1}B + D$

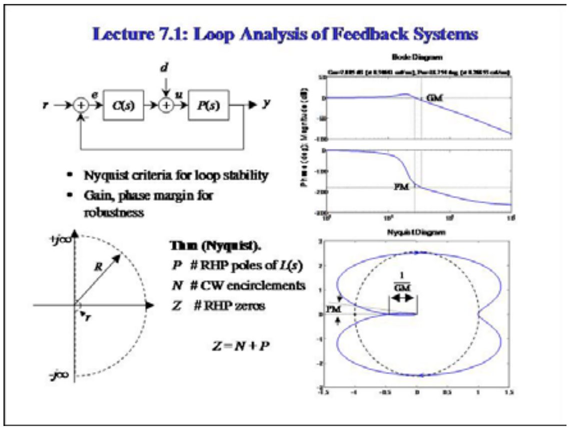
$H_{y_2u_1} = H_{y_2u_2}H_{y_1u_1} = \frac{n_1n_2}{d_1d_2}$

This lecture is the first lecture of the course to introduce transfer functions in a formal way (we have already seen frequency response and body plots). The lecture uses the example of a spring, mass, damper system to show how transfer functions can be used to compute the frequency response of an interconnected system of components. We also define poles and zeros and indicate how they affect the frequency response of a system. Finally, we introduce the general computations of block diagram algebra.

Lecture 7.1: Loop Analysis

Goals:

- Show how to compute closed loop stability from open loop properties
- Describe the Nyquist stability criterion for stability of feedback systems
- Define gain and phase margin and determine it from Nyquist and Bode plots



Lecture 7.1: Loop Analysis of Feedback Systems

- Nyquist criteria for loop stability
- Gain, phase margin for robustness

Thm (Nyquist).

P # RHP poles of $L(s)$

N # CW encirclements

Z # RHP zeros

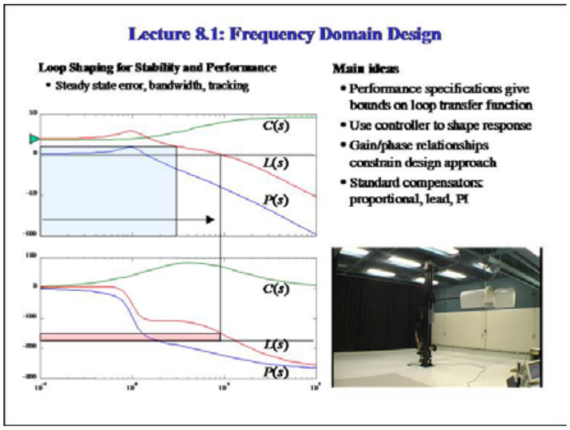
$$Z = N + P$$

This lecture describes how to analyze the stability and performance of a feedback system by looking at the open loop transfer function. We introduce the Nyquist criteria for stability and talk about the gain and phase margin as measures of robustness. The cruise control system is used as an example throughout the lecture.

Lecture 8.1: Loop Shaping

Goals:

- Describe the use of frequency domain performance specification
- Show how to use loop shaping to achieve a performance specification
- Work through a detailed example of a control design problem



Lecture 8.1: Frequency Domain Design

Loop Shaping for Stability and Performance

- Steady state error, bandwidth, tracking

Main ideas

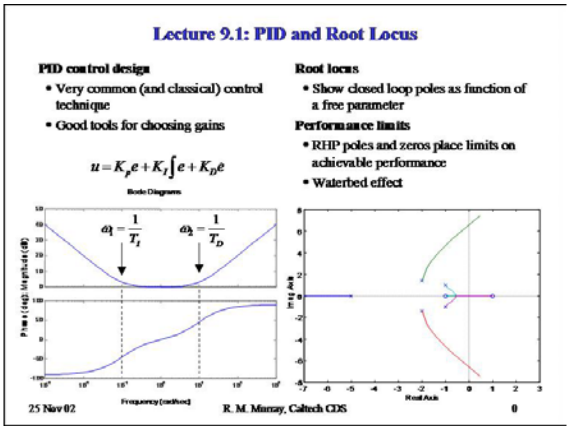
- Performance specifications give bounds on loop transfer function
- Use controller to shape response
- Gain/phase relationships constrain design approach
- Standard compensators: proportional, lead, PI

This lecture covers the basic tools in frequency domain control design through "loop shaping". After reviewing the role of the controller on the loop shape and the relationship between the gain and the phase, we consider several standard controller types (proportional, lead, PI) and apply them to design a high performance controller for the pitch axis of the Caltech ducted fan.

Lecture 9.1: PID and Root Locus

Goals:

- Define PID controllers and describe how to use them
- Introduce the root locus technique and describe how to use it to choose loop gain
- Show some of the limitations of feedback due to RHP poles and zeros



Lecture 9.1: PID and Root Locus

PID control design

- Very common (and classical) control technique
- Good tools for choosing gains

$$u = K_P e + K_I \int e + K_D \dot{e}$$

Root locus

- Show closed loop poles as function of a free parameter

Performance limits

- RHP poles and zeros place limits on achievable performance
- Waterbed effect

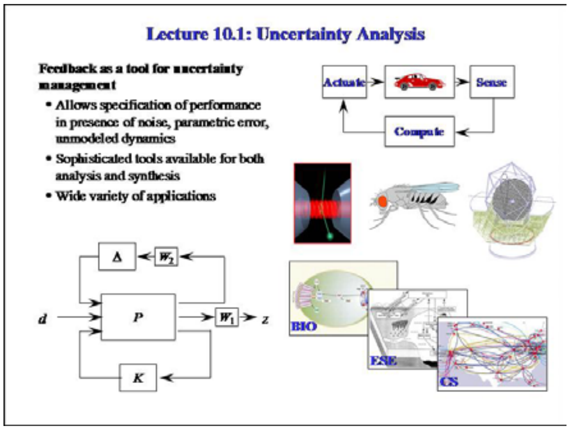
25 Nov 02 R. M. Murray, Caltech CDS 0

This lecture describes PID control, including the use of Ziegler-Nichols rules to design the gains. The root locus technique is introduced for determining the loop gain and the effect of pole location on performance is described in this context. Finally, some of the implications of right half plane zeros are discussed using the root locus plot.

Lecture 10.1: Uncertainty Management

Goals:

- Describe how feedback and control are used as tools for uncertainty management
- Summarize the main principles and tools for the course



Lecture 10.1: Uncertainty Analysis

Feedback as a tool for uncertainty management

- Allows specification of performance in presence of noise, parametric error, unmodeled dynamics
- Sophisticated tools available for both analysis and synthesis
- Wide variety of applications

This final lecture provides a desription of how control can be used for designing feedback systems that provide robustness to uncertainty. This ties together the various concepts discussed in the class and illustrates the main ideas of the ducted fan example. A summary of the main topics of the course is also given.