Controlling Structured Spatially Interconnected Systems

Raffaello D'Andrea

Mechanical & Aerospace Engineering
Cornell University

“Recent” Developments...

- Proliferation of actuators and sensors
- “Moore’s Law”
- Embedded systems, CAN, Bluetooth

MORE PERFORMANCE!!!

What is, and will be, needed:

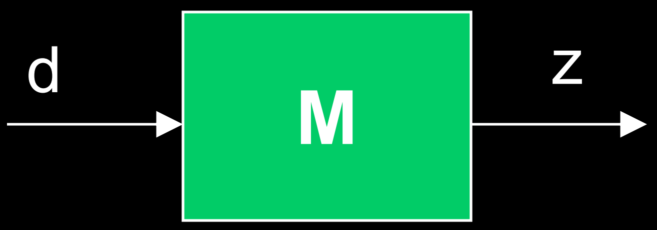
NEW CONTROL TOOLS:

- LARGE numbers of actuators and sensors
- Distributed computation
- Limited connectivity

- Robustness
- Performance
- Flexibility
- Etc.

Modeling Interconnected Systems

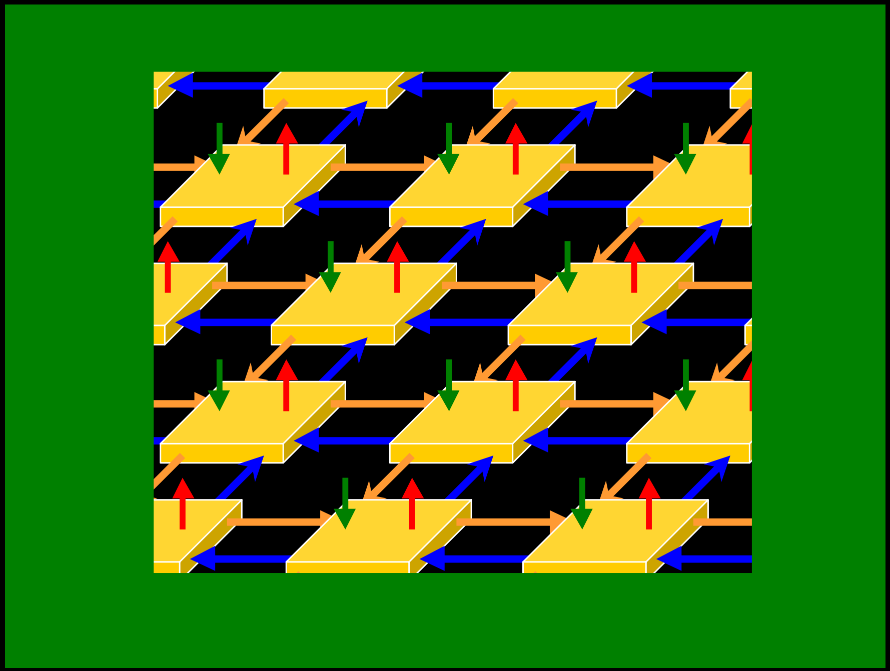
$$\dot{x}(t) = f(x(t), d(t))$$
$$z(t) = h(x(t), d(t))$$



x(t),d(t), and z(t) live in a Hilbert space:

$$d(t) = d(t, s_1, s_2, \cdots, s_L) = d(t, \mathbf{s})$$

$$\|d(t)\|_{l_2}^2 := \sum_{s_1=-\infty}^{\infty} \cdots \sum_{s_L=-\infty}^{\infty} d^*(t, \mathbf{s}) d(t, \mathbf{s})$$



Restrict the model class: local interconnection

WHY?

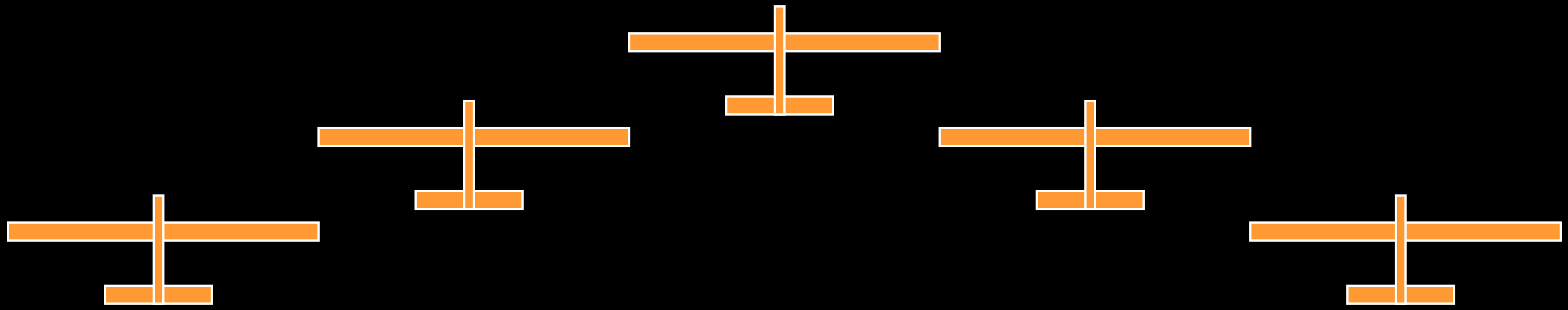
- Large class of systems, non-trivial behavior:
  - vehicle platoons
  - finite difference approximations of PDEs
  - cellular automata, artificial life, etc.
  - behavior of groups, swarm intelligence, etc.

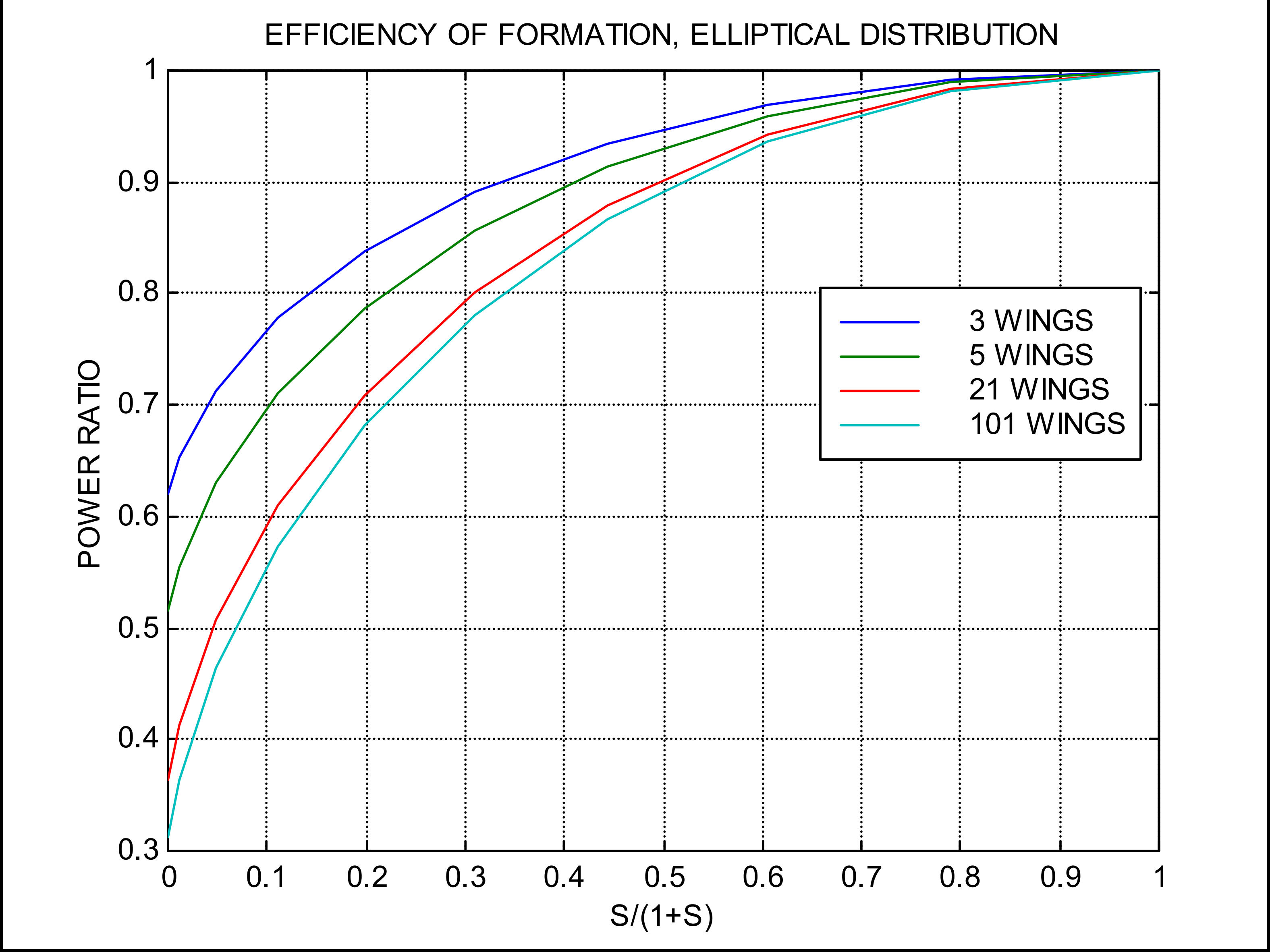
Case Study: Formation Flight

Use upwash created by neighbouring craft to provide extra lift

MOTIVATION

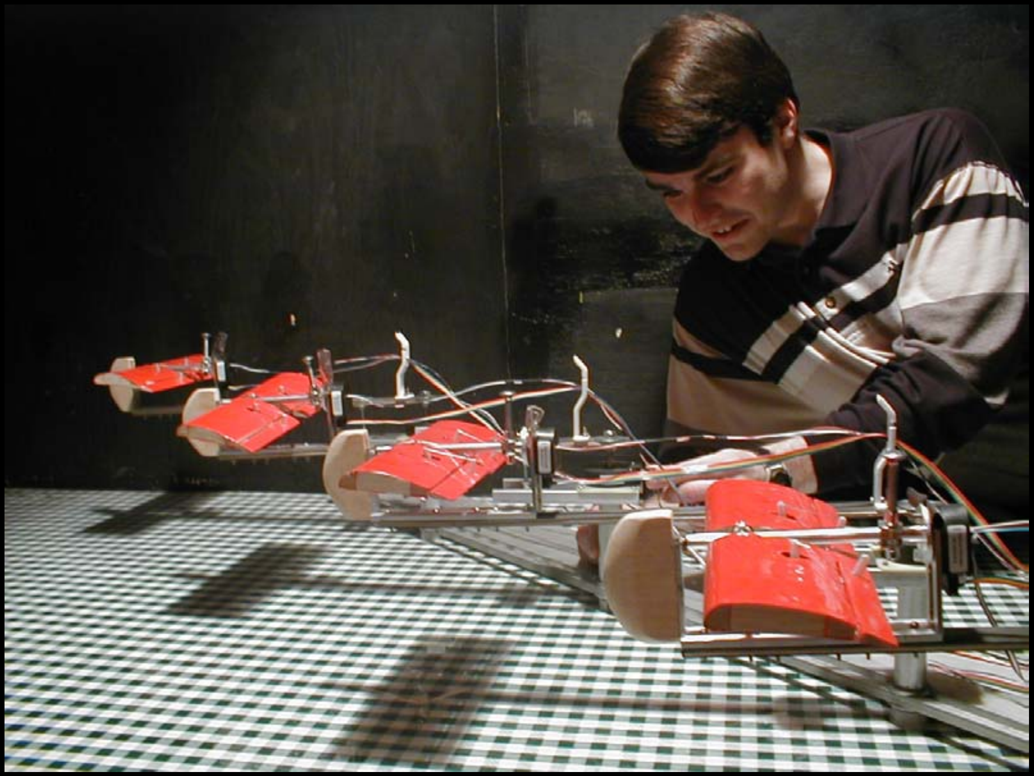
- "satellite" type of applications
  (Wolfe, Chichka and Speyer '96)
- MAVs and UAVs, extend range

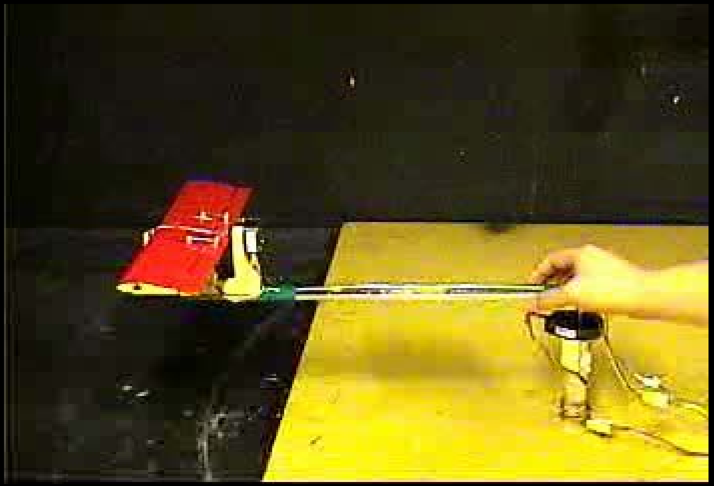




Lissaman and Shollenberger ‘70: Formation Flight of Birds

Formation Flight Test-Bed





Student:
Jeff Fowler (ME)

$$\ddot{y}(t,s) = c_1 y(t,s) - c_2 \theta(t,s)$$

$$\ddot{\theta}(t,s) = -c_3 \dot{\theta}(t,s) + u(t,s) - c_4 u(t,s+1) + c_5 \dot{\theta}(t,s+1) + c_6 \left(y(t,s) - y(t,s+1) \right)^2$$

Define shift operator **S**:

$$(\mathbf{S}u)(t,s) := u(t,s+1)$$

yields

$$\ddot{y} = c_1 y - c_2 \theta$$
$$\ddot{\theta} = -c_3\dot{\theta} + u - c_4 \mathbf{S}u + c_5 \mathbf{S}\dot{\theta} + c_6 (y - \mathbf{S}y)^2$$

In general:

$$\Delta = \begin{bmatrix} \frac{d}{d\mathbf{t}} I & & & & \\ & \mathbf{S}_1 I & & & \\ & & \mathbf{S}_1^{-1} I & & \\ & & & \ddots & \\ & & & & \mathbf{S}_L^{-1} I \end{bmatrix}$$

$$(\Delta x)(t,\mathbf{s}) = F(x(t,\mathbf{s}), d(t,\mathbf{s}), \mathbf{s})$$
$$z(t,\mathbf{s}) = H(x(t,\mathbf{s}), d(t,\mathbf{s}), \mathbf{s})$$

d(t,**s**), x(t, **s**), z(t, **s**) are FD,
F and H NL functions on FD space

Special cases...

$$(\Delta x)(t,\mathbf{s}) = F(x(t,\mathbf{s}), d(t,\mathbf{s}), \mathbf{s})$$
$$z(t,\mathbf{s}) = H(x(t,\mathbf{s}), d(t,\mathbf{s}), \mathbf{s})$$

- $\Delta = \frac{\mathrm{d}}{\mathrm{d}\mathbf{t}} I,$ $F(\cdot) = Ax + Bd, H(\cdot) = Cx + Dd$ $\Rightarrow$ $\dot{x} = Ax + Bd,$ $z = Cx + Dd$

- $F(\cdot) = Ax + Bd, H(\cdot) = Cx + Dd$ Linear, spatial invariant systems

- $\Delta = \frac{\mathrm{d}}{\mathrm{d}\mathbf{t}} I$ Family of completely decentralized systems

"CONVENIENT" FRAMEWORK FOR CAPTURING STRUCTURE

Recent Related Work

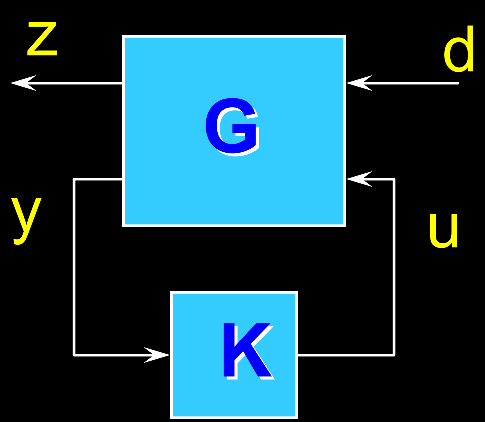
Siljak et al: Decentralized control of complex systems

Bamieh, Paganini, Dahleh: Spatially Invariant Systems

Cheng, Yang, Zhai, Peterson, Savkin, …: Decentralized Control of IC systems.

Stewart, Gorinevski, Dumont: Cross directional control

Control Design and Analysis: Spatially Invariant Systems



d(t,**s**): disturbances
z(t,**s**): errors
y(t,**s**): sensors
u(t,**s**): actuators

Closed Loop:

$$\begin{aligned} \Delta x &= Ax + Bd \\ z &= Cx + Dd \end{aligned}$$

Stable: $(\Delta - A)^{-1}$ exists and is bounded

Contractive: $\left\| D + C(\Delta - A)^{-1} B \right\| < 1$

Analysis

$$\dot{x}_{\mathrm{T}} = A_{TT}x_{\mathrm{T}} + A_{TS}x_{\mathrm{S}} + B_T d$$
$$\Delta_{\mathrm{S}} x_{\mathrm{S}} = A_{ST}x_{\mathrm{T}} + A_{SS}x_{\mathrm{S}} + B_S d$$
$$z = C_T x_{\mathrm{T}} + C_S x_{\mathrm{S}} + Dd$$

$$\Delta_{\mathrm{S}} = \begin{bmatrix} \mathbf{s}_1 I & & & \\ & \mathbf{s}_1^{-1} I & & \\ & & \ddots & \\ & & & \mathbf{s}_{\mathrm{L}}^{-1} I \end{bmatrix}$$

$$\mathrm{X}_{\mathrm{S}} = \begin{bmatrix} \mathrm{X}_1^{++} & \mathrm{X}_1^{+-} & & & \\ \mathrm{X}_1^{-+} & \mathrm{X}_1^{--} & & & \\ & & \ddots & & \\ & & & \mathrm{X}_{\mathrm{L}}^{++} & \mathrm{X}_{\mathrm{L}}^{+-} \\ & & & \mathrm{X}_{\mathrm{L}}^{-+} & \mathrm{X}_{\mathrm{L}}^{--} \end{bmatrix}$$

Stable and Contractive if there exists $\mathrm{X}_{\mathrm{T}}>0$ and structured X_{S} s.t.

$$\begin{bmatrix} I & 0 & 0 \\ A_{ST}^- & A_{SS}^- & B_S^- \\ 0 & 0 & I \end{bmatrix}^* \begin{bmatrix} A_{TT}^*\mathrm{X}_{\mathrm{T}} + \mathrm{X}_{\mathrm{T}}A_{TT} & \mathrm{X}_{\mathrm{T}}A_{TS}^+ & \mathrm{X}_{\mathrm{T}}B_T \\ \left(\mathrm{X}_{\mathrm{T}}A_{TS}^+\right)^* & -\mathrm{X}_{\mathrm{S}} & 0 \\ \left(\mathrm{X}_{\mathrm{T}}B_T\right)^* & 0 & -I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ A_{ST}^- & A_{SS}^- & B_S^- \\ 0 & 0 & I \end{bmatrix} +$$

$$\begin{bmatrix} I & 0 & 0 \\ A_{ST}^+ & A_{SS}^+ & B_S^+ \\ C_T & C_S & D \end{bmatrix}^* \begin{bmatrix} 0 & \mathrm{X}_{\mathrm{T}}A_{TS}^- & 0 \\ \left(\mathrm{X}_{\mathrm{T}}A_{TS}^-\right)^* & \mathrm{X}_{\mathrm{S}} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ A_{ST}^+ & A_{SS}^+ & B_S^+ \\ C_T & C_S & D \end{bmatrix} < 0$$

Theorem: There exists a controller such that the analysis LMI is satisfied if and only if there exists structured **Y** and **X** such that

$$U^*\begin{bmatrix} A\mathbf{Y}+\mathbf{Y}A^* & \mathbf{Y}C_1^* & B_1 \\ C_1\mathbf{Y} & -I & D_{11} \\ B_1^* & D_{11}^* & -I \end{bmatrix}U<0$$

$$V^*\begin{bmatrix} A^*\mathbf{X}+\mathbf{X}A & \mathbf{X}B_1 & C_1^* \\ B_1^*\mathbf{X} & -I & D_{11}^* \\ C_1 & D_{11} & -I \end{bmatrix}V<0$$

$$\begin{bmatrix} \mathbf{X_0} & \mathrm{I} \\ \mathrm{I} & \mathbf{Y_0} \end{bmatrix}>0$$

Controller implementation:

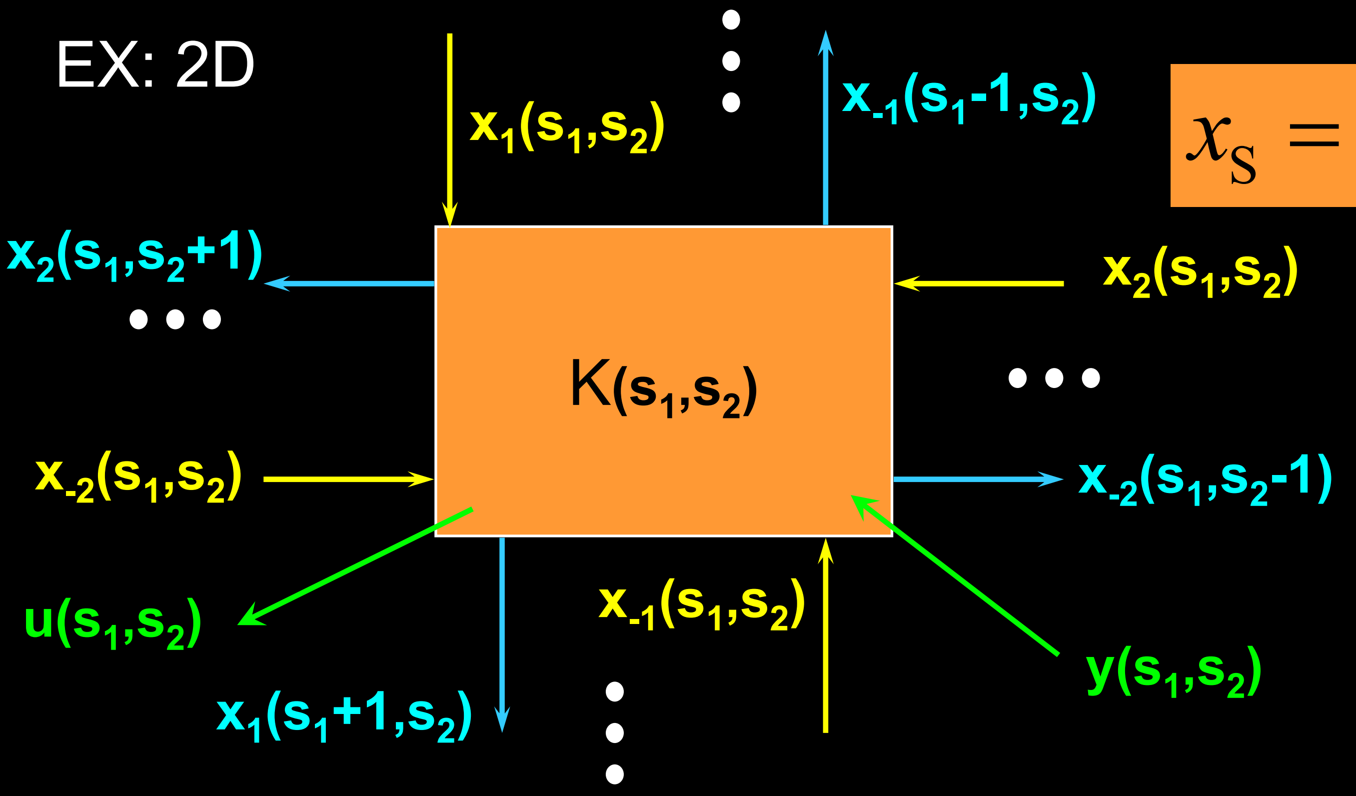
$$\dot{x}_T = A_{TT} x_T + A_{TS} x_S + B_T y$$

$$\Delta_S x_S = A_{ST} x_T + A_{SS} x_S + B_S y$$

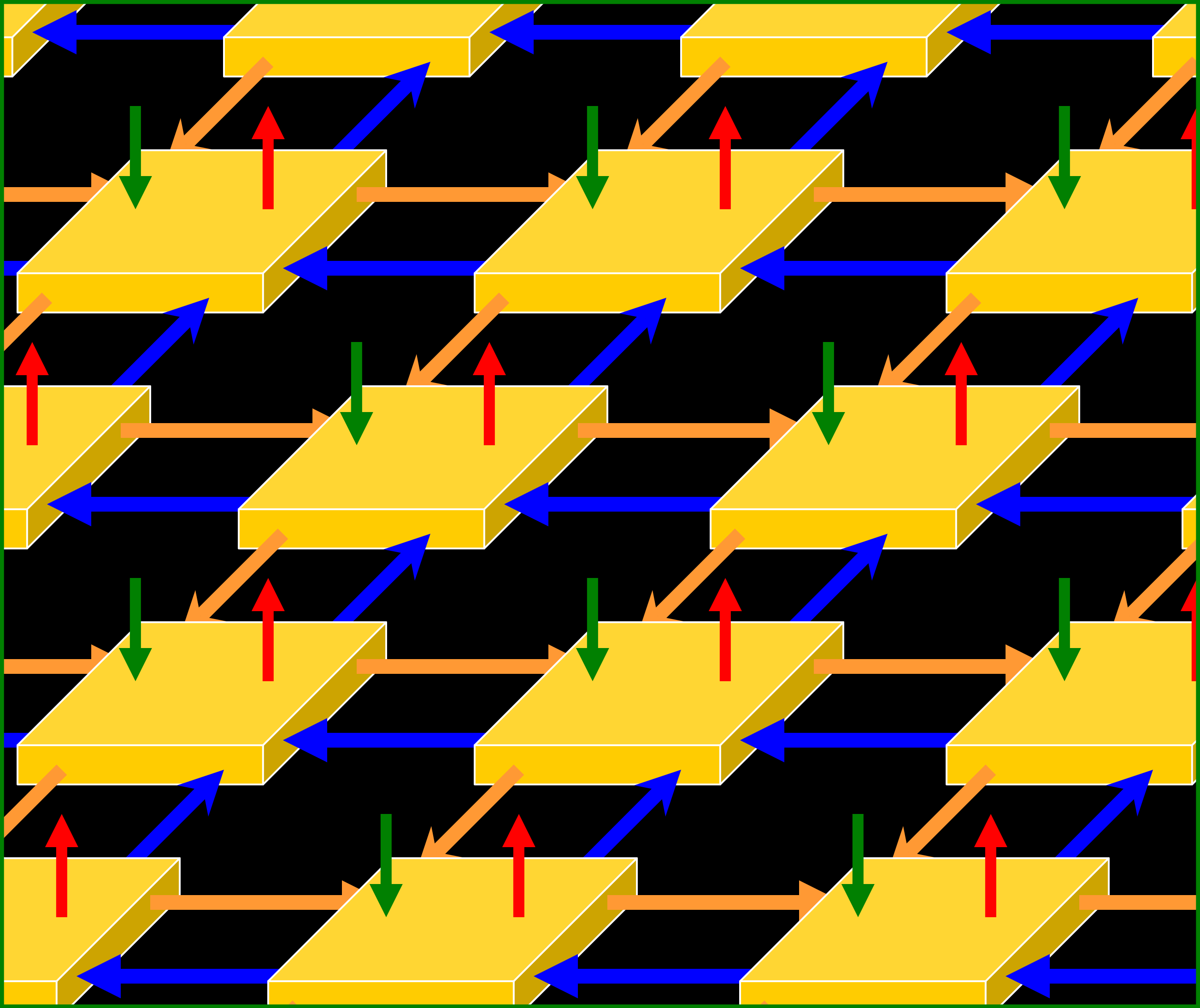
$$u = C_T x_T + C_S x_S + D y$$

EX: 2D

$$x_S = (x_1, x_{-1}, x_2, x_{-2})$$



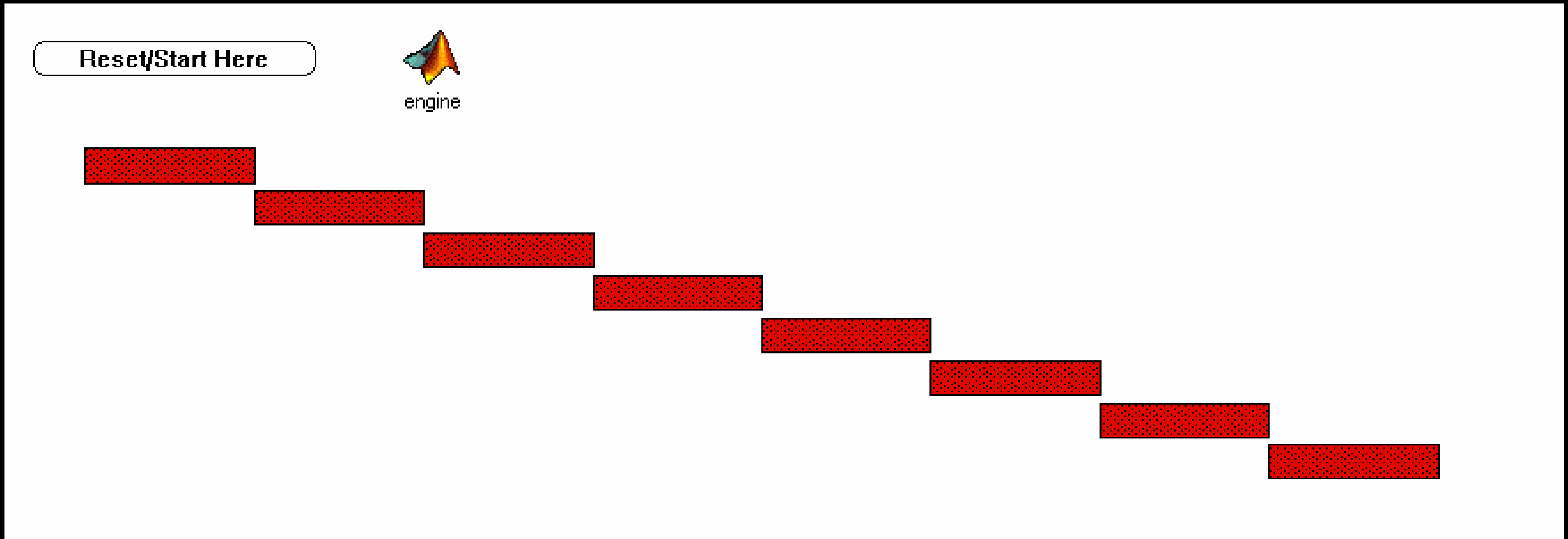
Control Architecture



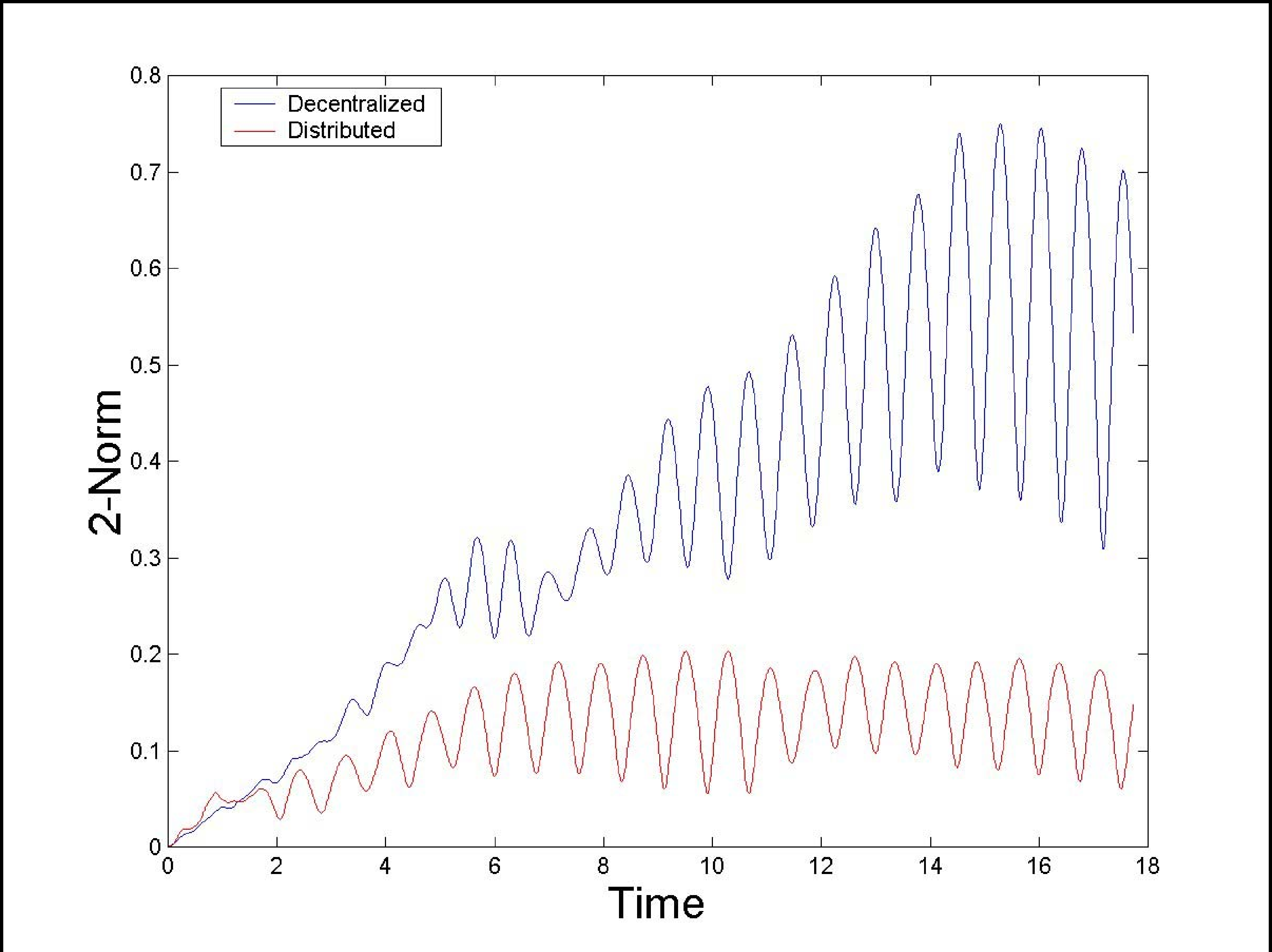
Decentralized Control

Decentralized Control

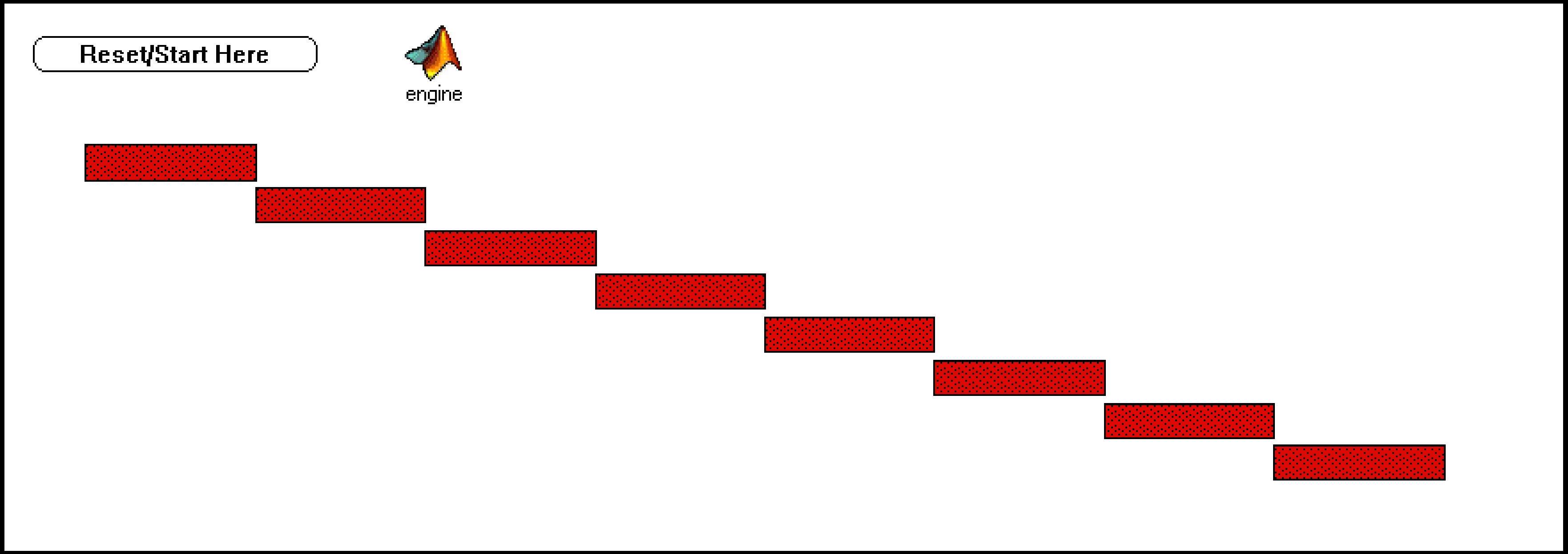
Distributed Control



COMPARISON OF SPATIAL 2-NORM, ROLL ANGLE



STRONG NONLINEAR COUPLING



Nonlinear Spatially Interconnected Systems:

$$\Delta x = f(x) + g(x)d$$
$$z = h(x)$$

- Feedback linearization
- Backstepping
- etc.

Other ongoing work

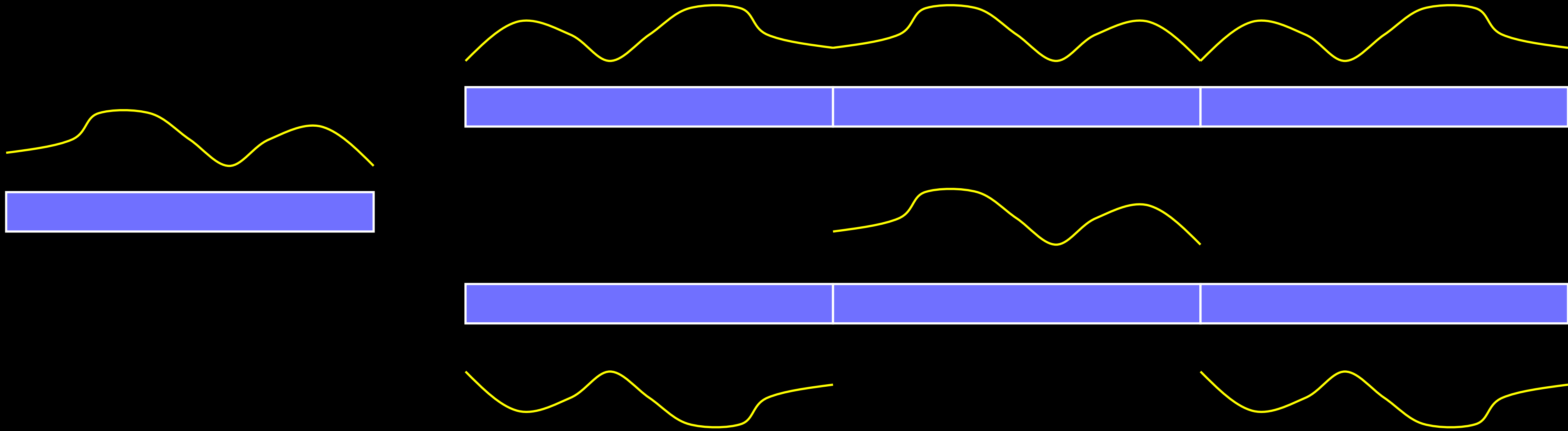
Spatially and Time Varying Systems:

- non-homogeneous properties
- finite boundary conditions

$$\begin{aligned} \Delta x &= A(\mathbf{s})x + B(\mathbf{s})d \\ z &= C(\mathbf{s})x + D(\mathbf{s})d \end{aligned}$$

TOOLS:

- LTI to LTV machinery (GEIR DULLERUD, UIUC)
- method of images, etc.
- LPV tools



Framework for Robust Control of IC systems

$$(\Delta x)(t,\mathbf{s}) = F(x(t,\mathbf{s}), d(t,\mathbf{s}), \mathbf{s})$$

$$z(t,\mathbf{s}) = H(x(t,\mathbf{s}), d(t,\mathbf{s}), \mathbf{s})$$

Delta contains temporal operators, spatial operators, AND uncertainty.

Student: Ramu Chandra (AE)

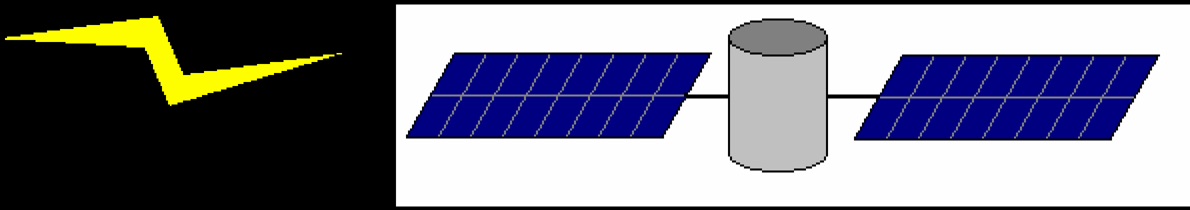
Model Reduction (CAROLYN BECK, UIUC)

Cross-Directional Control (GREG STEWART, HONEYWELL)

Phased Array Antennas for AFV Communication

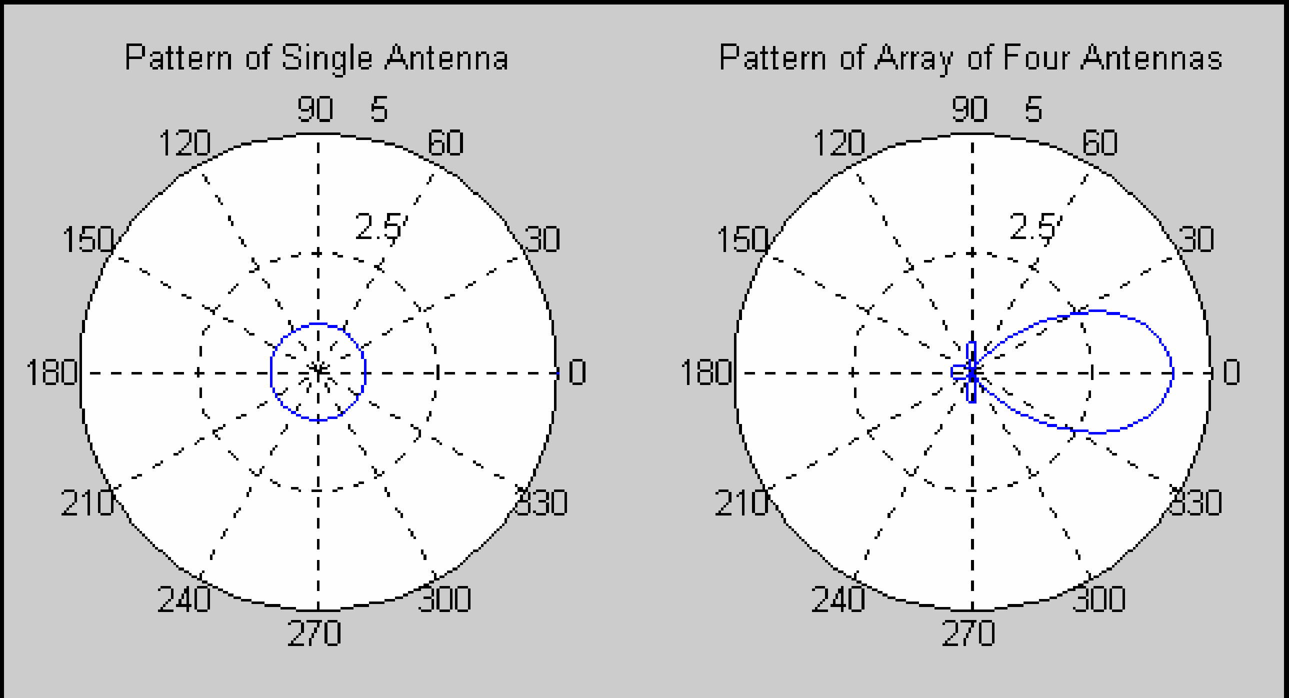
Student: Sean Breheny (ECE)

- High data rate comms between AFVs and base station/satellite (video, etc.)
- Difficult to put a high gain antenna on an AFV (size constraint)
- Since it may be advantageous to use groups of AFVs anyway, why not investigate whether a formation of AFVs, each carrying a low gain antenna, could form a high gain phased array?



What is a Phased Array Antenna?

- Exploit EM wave interference among several antennas.
- For the simplest case (where array elements are not strongly coupled to each other), gain increases roughly linearly in N, the number of elements.
- Channel capacity increases linearly when the maximum bandwidth is used.



Example: Endfire Array