Attention as a Performance Measure for Control System Design

Congratulations

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Getting a Handle on Implementation Costs

1. Set up an optimization theory framework that will include implementation costs.

2. Include the costs for communication and computation together with more traditional trajectory performance terms.

3. We will focus on the intuitive idea of attention, used in somewhat the same sense as the term as used in in psychology.

4. Both open loop control and closed loop control require attention. Low attention solutions will turn out to be inexpensive to implement.

5. The best known work in attention is associated with cognitive and sensory attention, priming, etc. but our goals are slightly different.

Consider this talk in the Spirit of Outreach to Computer Science and Psychology (and ourselves).

Books such as those of M. Ito on the cerebellum contain many references to feedback control and have many block diagrams. Even so, the reader from control will conclude that we have not provided psychology with the right tools because the performance of the feedback loops depends so strongly on what is currently important for the task being done.

Robotics has been a successful application of control theory at the hardware level, but the more important problem is software and we have not yet provided a complete theory of language driven systems.

This talk is NOT directed to the "cost is no issue" designers but rather to finding "just enough control" as in washing machines and humans.

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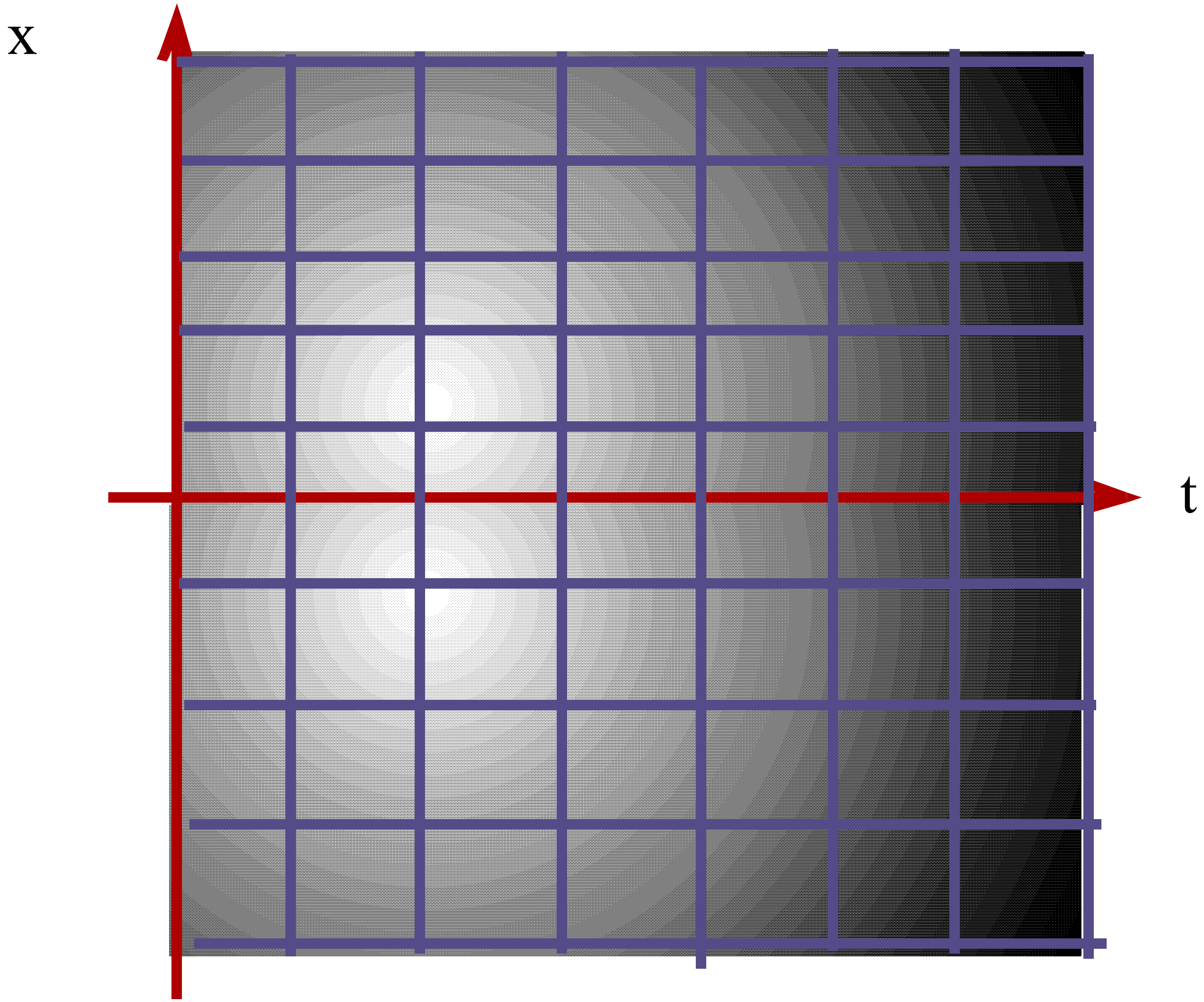
DV - NTSC decompressor
are needed to see this picture.



Representation of the values of u(t,x) and the tiling of space-time that is implicit in any implementation of computer control.

Some Equations to Help Fix Ideas

1. The idea of partial derivatives costing money to implement

$$\text{cost} = a(u_t)^2 + b(u_x)^2$$

2. The number time slices a particular output receives is one measure of the attention the system is receiving.

3. Elementary example

$$dx/dt = u(t,x)$$

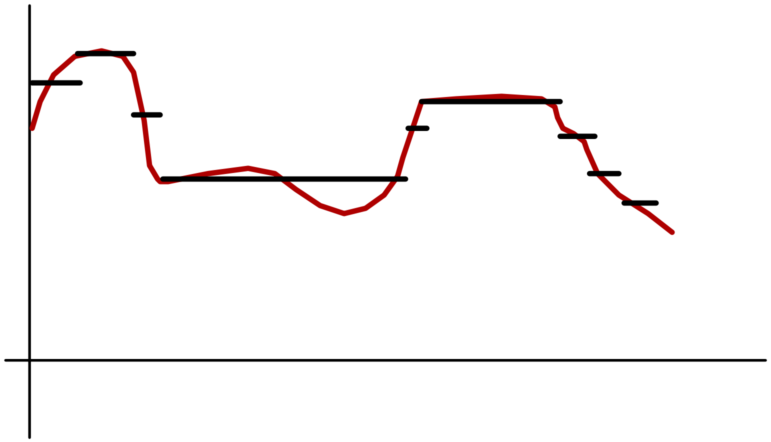
Minimize the integral of $a(u_t)^2 + b(u_x)^2$ plus the integral of $x^2 + u^2$, weighted over a distribution of initial states.

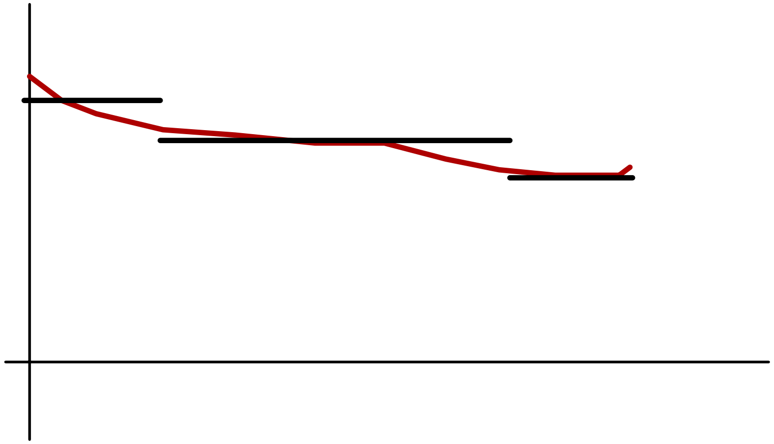
4. For example we might have $u(t,x)=-(t/(1+t^2)\tanh(x)$ which is zero when $t=0$ or when $x=0$.

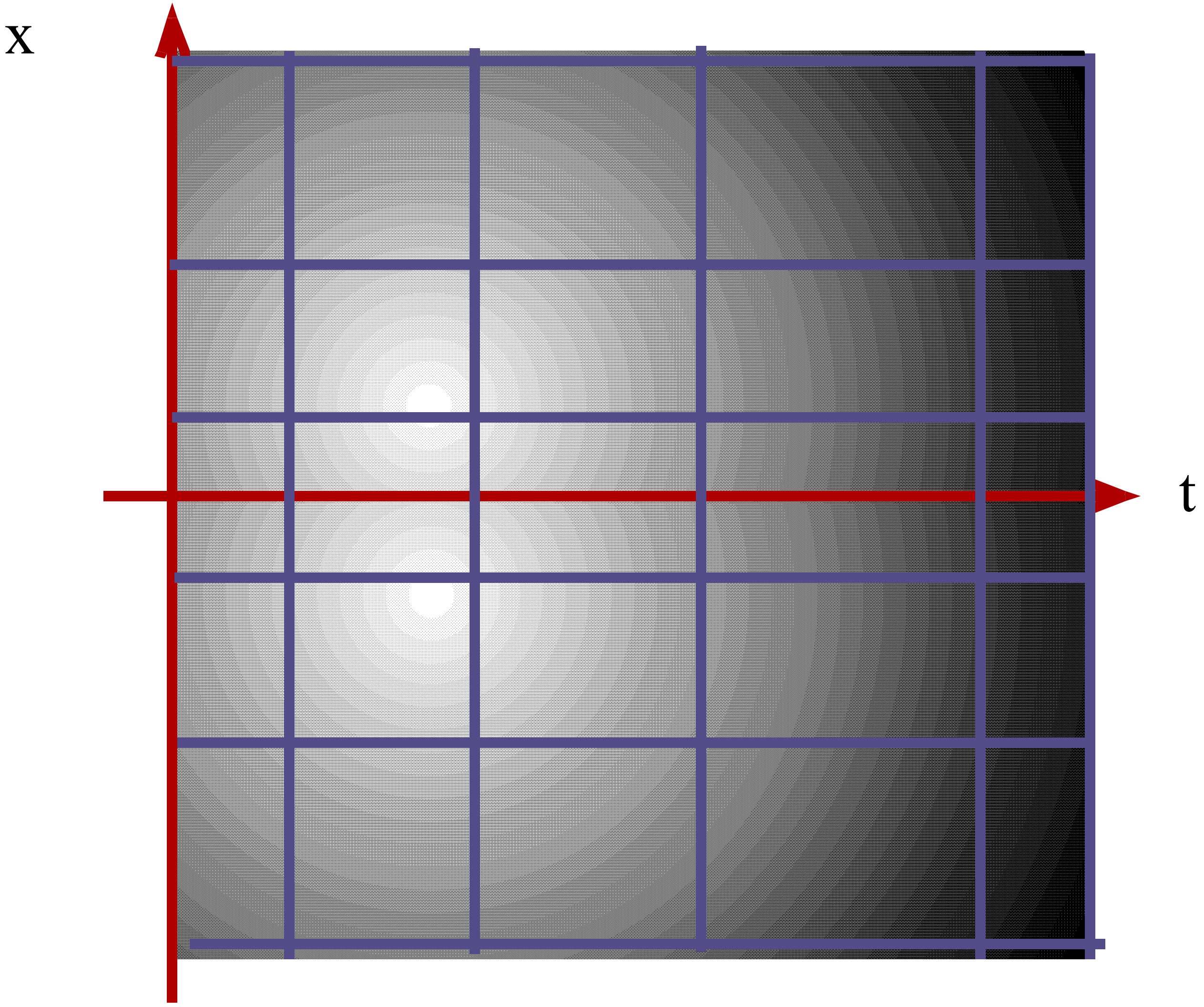
Implementation Cost Factors

1. Number of quantization levels (12 bit, vs. 16 bit, single precision, double precision, etc.
2. Sampling rate, 30 Hz, 100 Hz, …
3. Tolerance to delay, 20 millisecond latency, 60 millisecond latency…
4. Computational complexity of the control law
5. Speed of sensors (time to make a measurement).

If u(y,t) is the desired control then the size of the partial derivatives $u_y(y,t)$ and $u_t(y,t)$ gives a good indication of how hard it will be to approximate u with something that is piecewise constant







We can save resources with little loss in performance by non uniform Quantization when u(t,x) changes slowly outside the normal range of the variables.

The Linearity Trap

Because of saturation we can assert that there are no linear systems. Even so, linear models are very useful and their properties deserve to be known. When do they mislead?

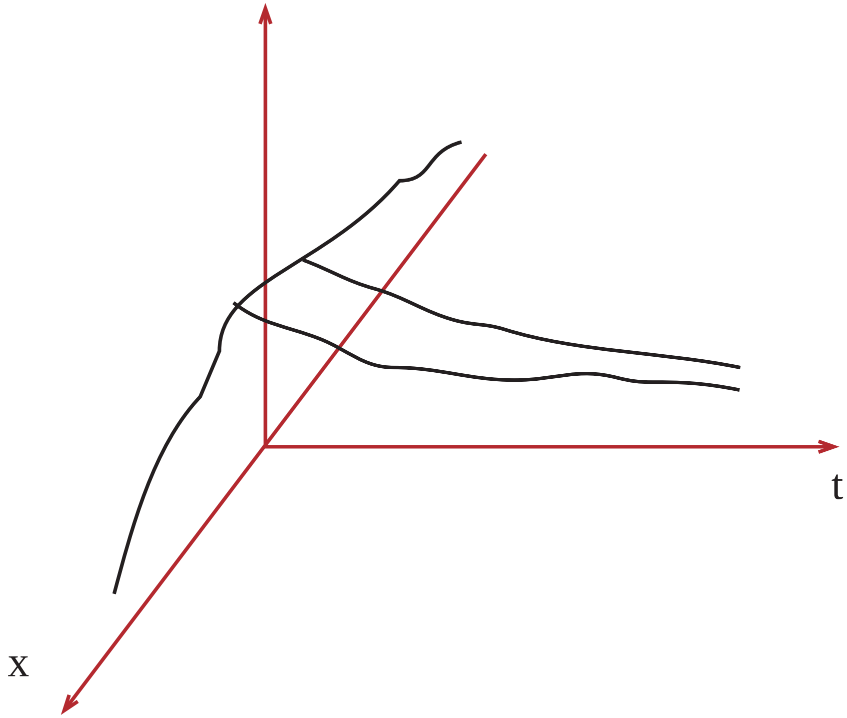
It makes little sense to model many on-off systems such as those that find wide use in low tech control such as the electric valves in dish washers and gasoline pumps as linear systems.

The outcome of a measurement is often a go/no-go decision. In such cases it makes little sense to regard the measurement as being linear.

The default assumption of linearity can be misleading either because of saturation or because of discontinuity, or both.

The Trajectory Optimization Trap

Trajectory optimization is often recommended as an approach to control system design. However, in may happen that the detailed shape of the trajectory is much less important than the reliability and/or cost of the control system and these may over-ride almost all considerations related to the trajectory.



Trajectory and Implementation Costs Formulated Jointly

We are interested in combining two types of terms;

1. A performance term that will insure stability, hitting the target, conserving resources, minimizing time, etc. as dictated by the problem.

2. A implementation term that insures that the control is not excessively sensitive to small changes in the measurements, small errors in the clock, does not require a high sampling rate or ultra fine quantization.

The inclusion of the second term will complicate the mathematics but can give control laws that saturate for large values and are more easily approximated, thus giving more flexibility in their implementation.

An Example

Consider the problem of stabilizing an integrator

$dx/dt = u(t,x)$

The initial condition is modeled as a probability distribution.

$$\rho(x)=(1/Z)\exp-((x^{2})/2\sigma$$

Minimize the expected value of the integral of L(x,u) + the integral over time and space of $u_t^2+u_x^2$. This comes down to controlling the Fokker-Planck equation subject to a quadratic cost term.

A New Type of Optimization Problem

The abstract form of the evolution equations these considerations lead to is

$$dx/dt = (A+BUC)x$$

The performance measure involves linear functionals of x and quadratic functionals of U. Typically A+BUC is a Fokker-Planck operator and u(x,t) enters inside a partial differential operator. One example is

$$\frac{@\check{z}(t;\, x)(t;\, x)}{@t} = \circ\, \frac{@u(t;\, x)\check{z}(t;\, x)(t;\, x)}{@x} + \frac{1}{2}\,\frac{@^2 \check{z}(t;\, x)}{@x^2}$$

The dynamical equation (the controlled Fokker-Planck Eq)

$$\frac{\text{@ž}(t;x)(t;x)}{\text{@}t} = \text{°}\,\frac{\text{@}u(t;x)\text{ž}(t;x)(t;x)}{\text{@}x} + \frac{\text{Æ}}{2}\frac{\text{@}^2\text{ž}(t;x)}{\text{@}x^2}$$

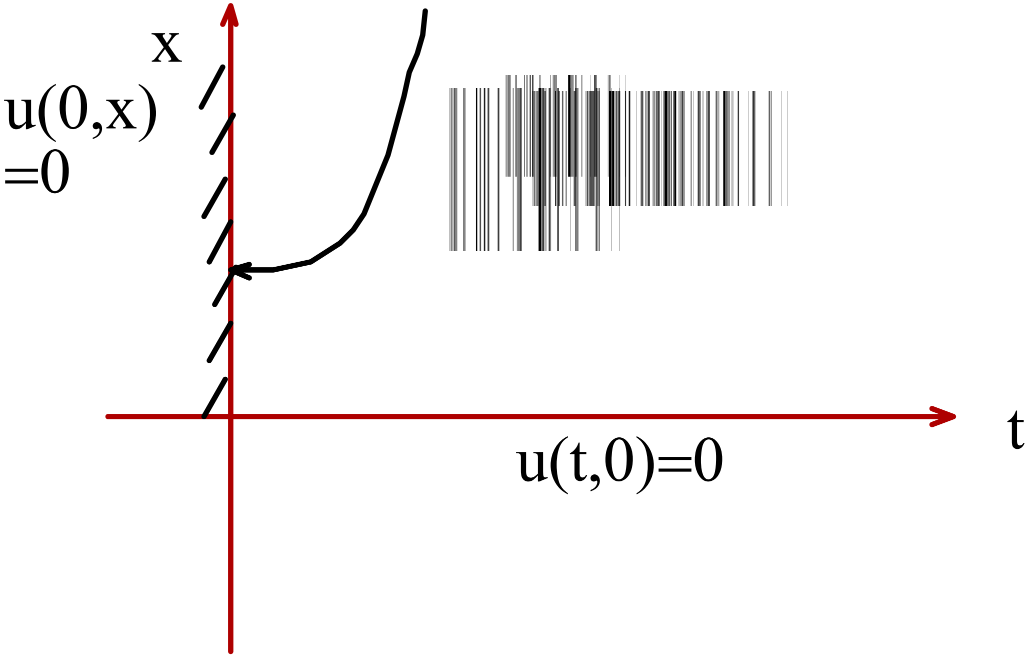
The performance measure

$$\text{¥} = \text{Z}_0^1\,\text{Z}_{\text{°}1}^1\,\text{Ø}(u^2(t;x) + x^2)\text{ž}(t;x) + \text{µ}\frac{\text{@}u}{\text{@}x}\text{Z}^2 + \text{µ}\frac{\text{@}u}{\text{@}t}\text{Z}^2\,dxdt$$

The boundary conditions

$$\text{ž}(0;x) = \text{ž}_0(x)\ \ ;\ \ u(t;0) = 0\ \ ;\ \ u(0;x) = 0$$

Information boundary



x

$u(0,x)$
$=0$

t

$u(t,0)=0$

We can establish an upper bound with the control

$$u(t; x) = \frac{t}{1 + t^2} \tanh x$$

$$u_t = \mu \frac{1}{1 + t^2} \circ \frac{2t^2}{(1 + t^2)^2} \check{Z} \tanh x$$

$$u_x = \frac{1}{1 + t^2} \frac{1}{\cosh^2 x}$$

A Way to Think about Learning and Practice

The optimization problem posed here involves a trade-off between The quality of the trajectory and the implementation costs. The latter involves a trade-off between open loop cost measured by u_t and closed loop costs measured by u_x. A model for what happens when one practices a task is to imagine that the weighting shifts from the open loop term to the closed loop term and from the implementation cost to the trajectory cost

Some Final Points

1. We have framed the problem of optimizing the implementation cost in terms of an optimization problem involving variational problems on (t,x)-space.

2. The solution of such problems will generally lead to saturating control laws, not linear control laws.

3. The smallness of the partial derivatives implies that the control law will change slowly, be relatively insensitive to error, and lend itself to roughly quantized, slowly sampled implementations.

Thanks Marc