DISTRIBUTED PARAMETER SYSTEMS: EARLY THEORY TO RECENT APPLICATIONS

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CENTER FOR RESEARCH IN SCIENTIFIC COMPUTATION

RALEIGH, N.C.

AFOSR WORKSHOP ON
FUTURE DIRECTIONS IN CONTROL
ARLINGTON, VA
APRIL 26-27,2002
TO HONOR MARC Q. JACOBS
ON HIS RETIREMENT

NC STATE University

Center for Research in Scientific Computation CRSC

OUTLINE

- **Some History-1960's to 1990's (as it *might* have been!!)**

- **High Pressure OMCVD Reactors (with real time sensing and feedback control)**

- **Eddy Current Technology Based NDE (reduced order computational methods)**

- **More "History" (Future Directions)-'00's to '20's (at least, as it *oughta* be!!)**

SOME HISTORY-1960's-1990's
Control and Estimation of DPS

- 1960's- Sputnik-Revival of Calculus of Variations via optimal (open loop) control theory-Maximum Principle (Pontryagin,et.al.) DPS ⟶ (late 60's-early 70's) Delay Systems (Bellman-Cooke, Hale, Halanay, Russian school)

- 1968 (1971-English translation) J.L.LIONS, *Optimal Control of Systems Governed by PDE*– abstract operator formulation-semigroups, sesquilinear forms, evolution eqn for elliptic, parabolic, hyperbolic, PDE with delays-open loop optimality leading to Riccati integro-diff eq for feedback gains in LQR with persistent excitation (tracking eqn + Riccati)

- 1969-Lukes and Russell—operator Riccati integral eqn-semigroups-undamped oscillator example (hyperbolic)

1970's

Lions, Ciarlet, Glowinski, students—IRIA(later INRIA)-theoretical and computational foundations—beginning of serious attention to applications-fluids,biology(e.g., Kernevez and Thomas-design, construction, and open loop control of ***active*** bio-membranes)

1976-Duvaut and Lions, *Inequalities in Mechanics and Physics* (semi-permeable media, thermal control, elasticity,viscoelasticity, materials with memory, plasticity, fluids, electromagnetics)

Meanwhile,elsewhere: Delay systems—control, approximation, inverse (parameter estimation) problems,computational methods-- HTB, Jacobs, Burns, Delfour, Mitter, Manitius, Kappel, Herdman, Cliff

Realization that approximation not totally straight forward--standard techniques for simulation may not be adequate for estimation and control design—adjoint convergence?? Preservation of stabilization??

1980's

Large Flexible Structures-CNES(Lions), NASA, AF Labs and AFOSR

General theoretical frameworks for PDE systems along with *increased emphasis on approximation and computation*

a) *Inherent damping important*-Inverse problems and estimation necessary as part of control (Chavent, HTB, Kunisch, Russell, Ito, Inman, ……

b) *Development of abstract ARE theory (including compensator theory) which* ***continued into the 90's***–**Gibson**, Curtain, Pritchard, Salamon, HTB, Kunisch, Lasiecka, Triggiani, Kappel, Ito, Da Prato, Tran, Burns, King, van Keulen, …..

AFOSR Workshops on Flexible Structures(Tampa,'85; Val David,'86); numerous international conferences; 1st AFOSR URI (Brown Univ- '86-'89) was on large flexible structures, etc.

Little, if any, of this was actually applied to solve any large flexible structures control problems! **BUT** ***did provide a sound foundation for real (including experimental) applications in the '90's***

The field of control and estimation of DPS was very fortunate to enjoy strong AFOSR support (*both intellectual and financial*) via some key Directors and Program Managers in the Control Program during the ’80’s and ’90’s:

John Burns--’83 and ’84
Marc Q. Jacobs--’84-’85
Jim Crowley-- ’86-’88
Charley Holland--’88-’91
Marc Q. Jacobs--’90-’01

1990's

i) *Real attention to applications:*

fluids, fluid-structure interactions(see experimental application to structural acoustics in [HTB,R.C.Smith and Y.Wang, *Smart Material Structures: Modeling, Estimation and Control*,1996]), thermal estimation and control, acoustics, precision engineering, electromagnetics, ….

ii) *Advances in nonlinear DPS, systems with hysteresis…*.....

… … … … … …

Design of HPOMCVD reactor with real time sensing and feedback control

Reduced order computational methods for eddy current based nondestructive subsurface damage detection

DESIGN OF A HIGH PRESSURE OGANOMETALLIC CHEMICAL VAPOR DEPOSITION (HPOMCVD) THIN FILM REACTOR

1) High pressure (1 to 100 atms)
2) Real time sensing
3) Real time feedback control

(An DOD/AFOSR MURI Project-1995-2000)

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Objectives

Integration of advanced methods of CVD with state-of-the-art mathematics **to address fundamental scientific issues in the heteroepitaxy of mixed group III nitrides, phosphides and phosphonitrides on silicon and silicon carbide substrates, respectively.**

Utilize **advanced mathematics modeling and control theory** to optimize existing state-of-the-art processing methods (e.g., chemical beam epitaxy (CBE) and remote plasma enhanced CVD and novel CVD processes (e.g., HPOMCVD at superatmospheric pressure) to (i) **provide for real-time process monitoring and control** and (ii) **access processing conditions outside the reach of conventional CVD**.

Project milestones

Computer aided modeling and reactor design history

Real time monitoring and feedback control of PCBE system

- Real time estimate of growth rate and composition from PRS signals
- Output feedback synthesis and nonlinear filter for growth of compositionally, parabolic graded $Ga_{1-x}In_xP$ heterostructures under open- and closed-loop control
- Experimental validation of targeted layer properties (composition and thickness) by SIMS (Secondary Ion Mass Spectroscopy) profile analysis

Control Methodologies for High-Pressure CVD reactors

- Developed proper orthogonal decomposition (POD) techniques as a reduced basis method for the design of feedback controls and compensators in high pressure CVD reactors

- Computationally demonstrated that a reduced order based feedback control is capable of substantial control authority when applied to the full system (an approximation of the physical system)

- Developed and computationally tested nonlinear compensator and nonlinear feedback tracking control of chemical deposition film growth using reduced order models

Achievements

Construction of two new systems for high pressure epitaxy: First prototypes worldwide

Development of two new robust methods of real-time optical process monitoring for use in feedback control of epitaxy of high pressure

Computer aided design of HPCVD reactors

Implementation of nonlinear control and filtering in a PCBE system

Control and estimation methodologies for nonlinear HPCVD systems

Computer Aided Modeling, and Reactor Design History

Objectives

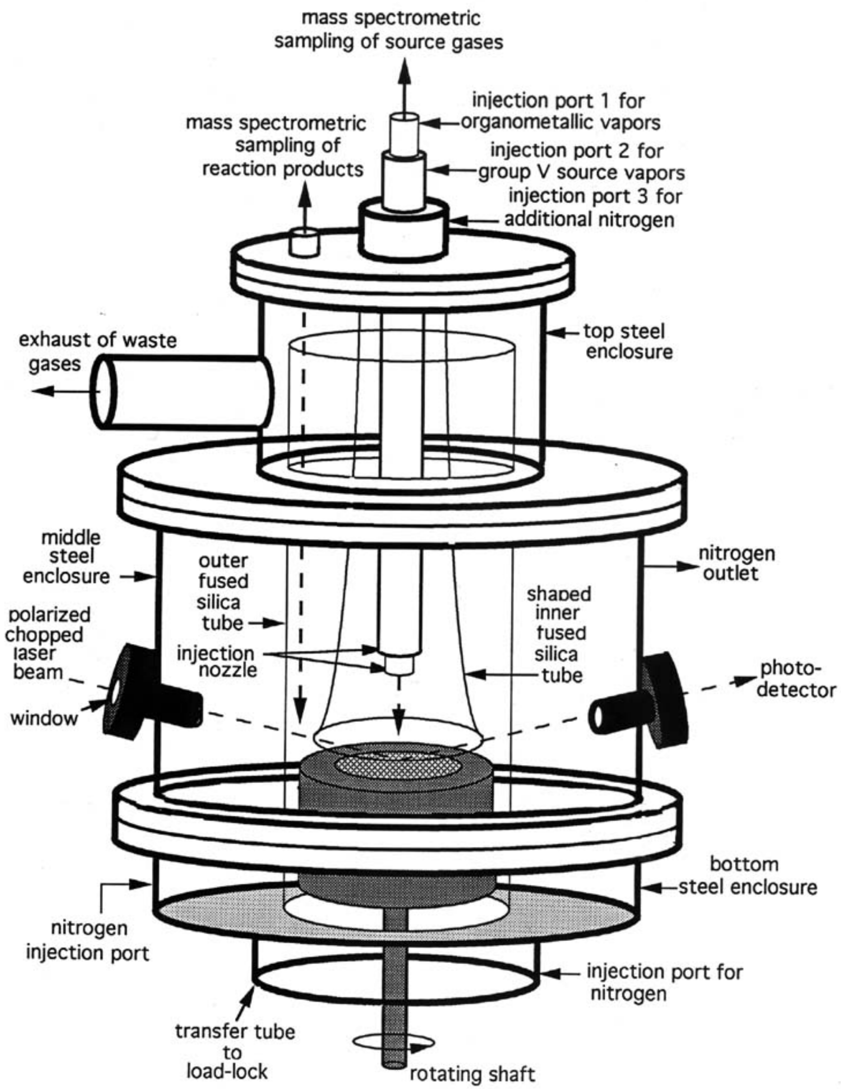
Design, construct and test reactors with

- **Implementable real-time sensing**
- **Real-time controllability**

Significant computational challenges:

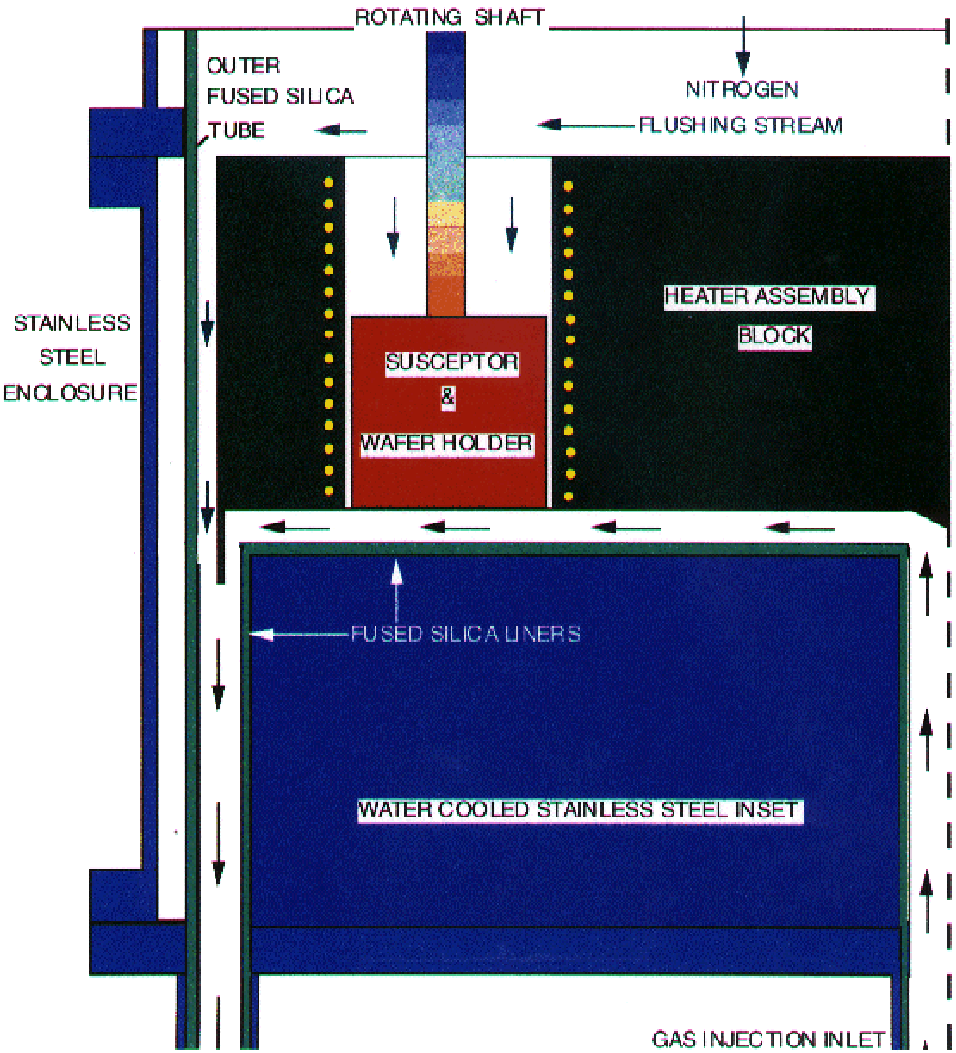
1) Thermal gradients
2) Nonlinear gas (vapor) flow
3) Nonlinear chemical vapor deposition

First Generation Reactor Design

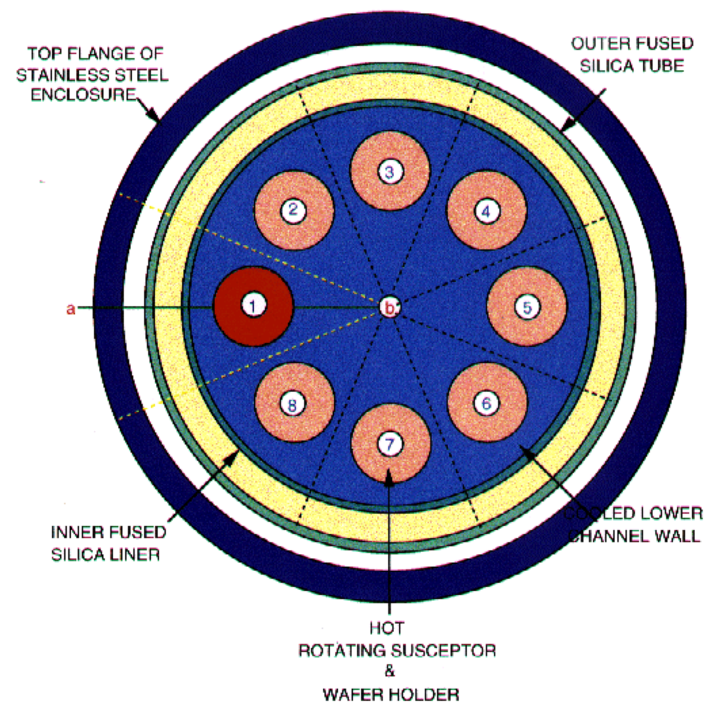


- **Recirculation cells develop at the outer edges of the substrate**

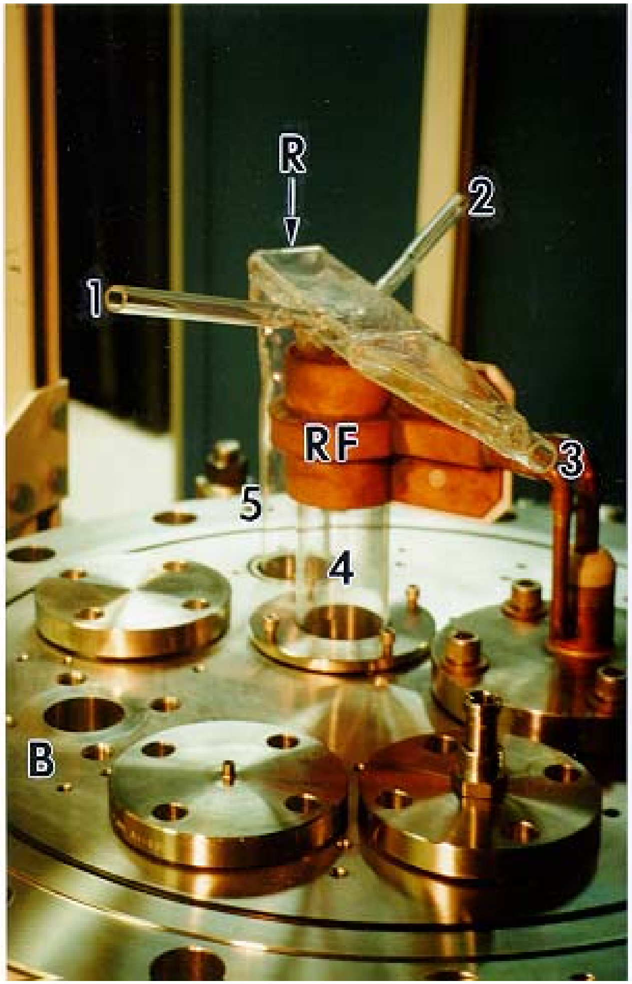
Second Generation Reactor Design

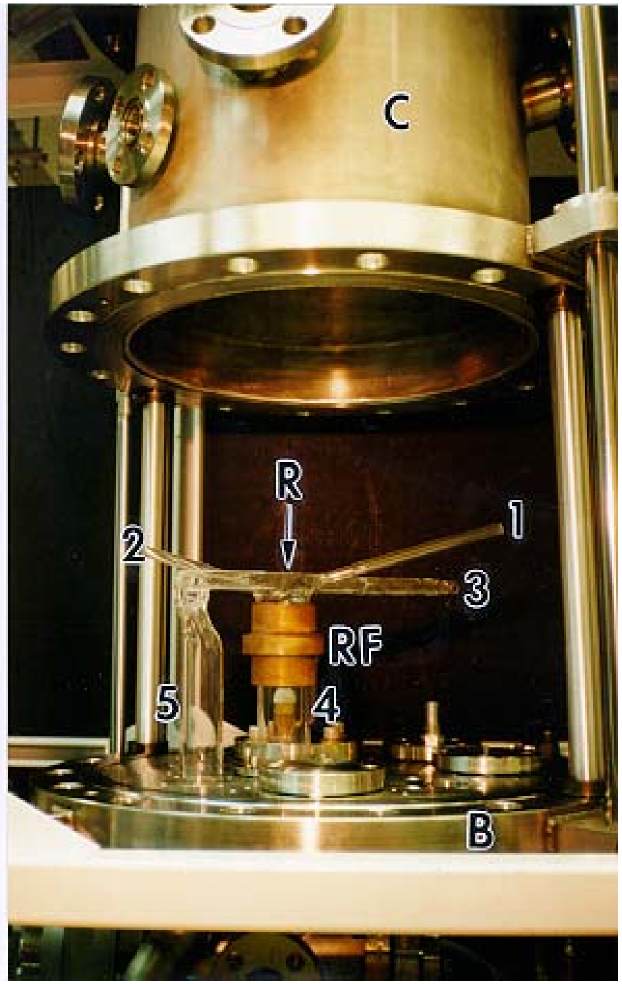


- **Substrate is moved away from the impinging jet**
- **Horizontal flow across the substrate**
- **Multiple wafers capability**



Third Generation Reactor Design

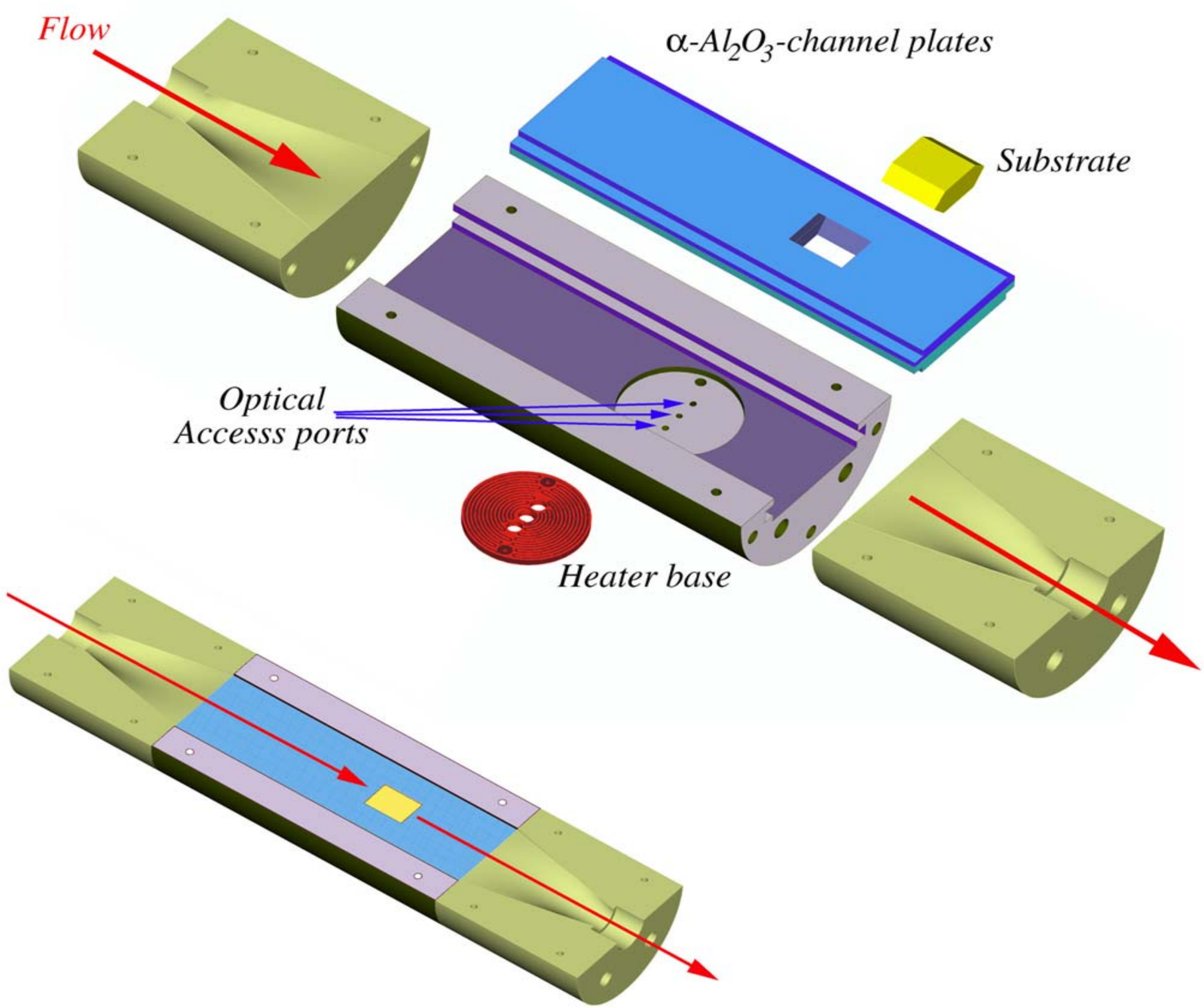




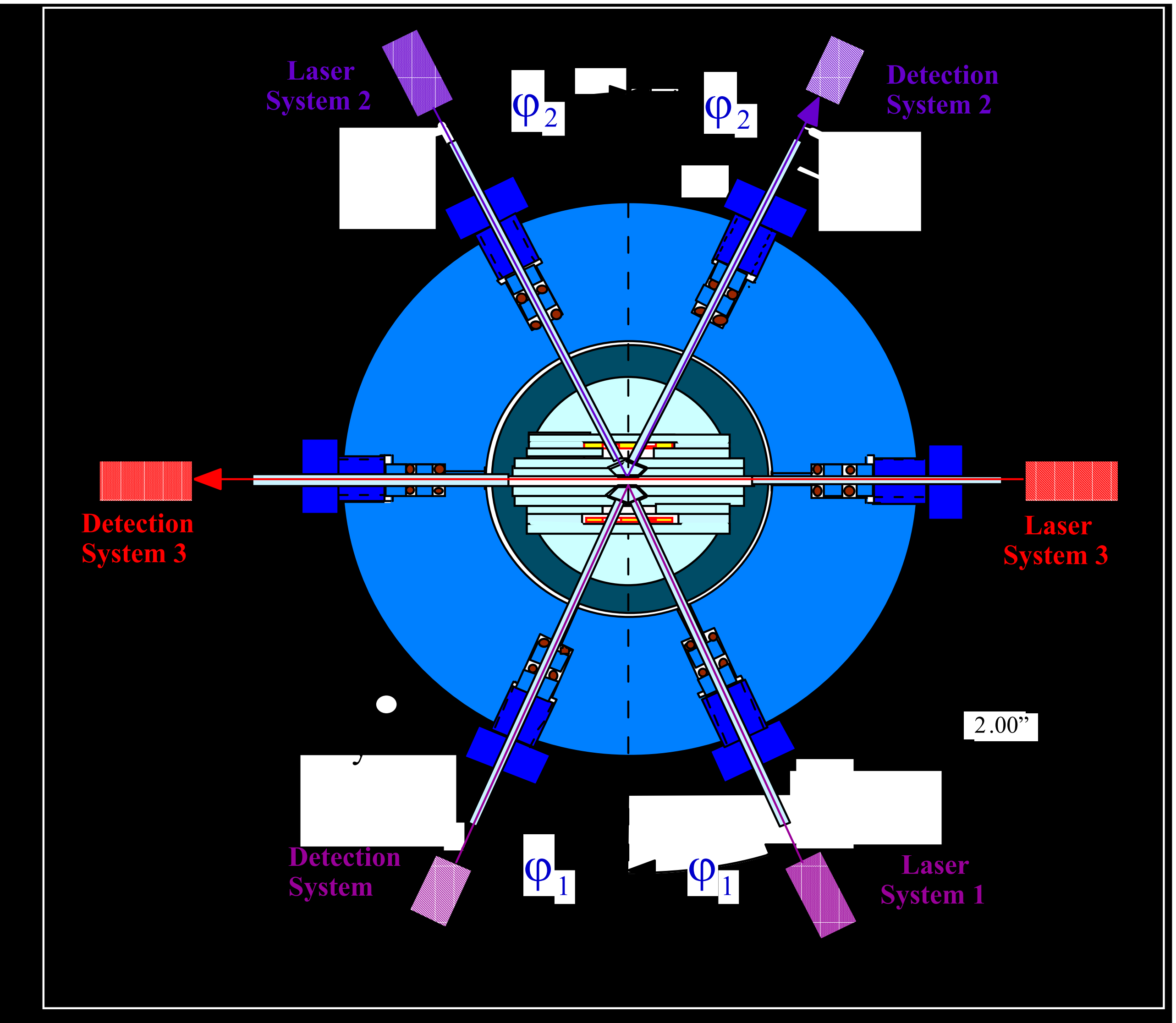
- Differentially Pressure Controlled (DPC) reactor system (5 atm)
- Fused silica reactor with tubular connections to load lock, windows for PRS, gas injection and exhaust
- Stainless steel second confinement shell
- R = fused silica reactor
  1&2 = win. connectors
  C = confinement shell
  3&5 = gas inlet & outlet
  4 = tube on R for
  substrate wafer
  exchange

Fourth Generation Reactor Design

- **Pressure range (up to 100 atm)**
- **Constant cross section**
- **Small channel height (1mm)**
- **Symmetric substrate arrangement**
- **Elimination of competitive polycrystalline deposition**
- **Improved reactor efficiency**

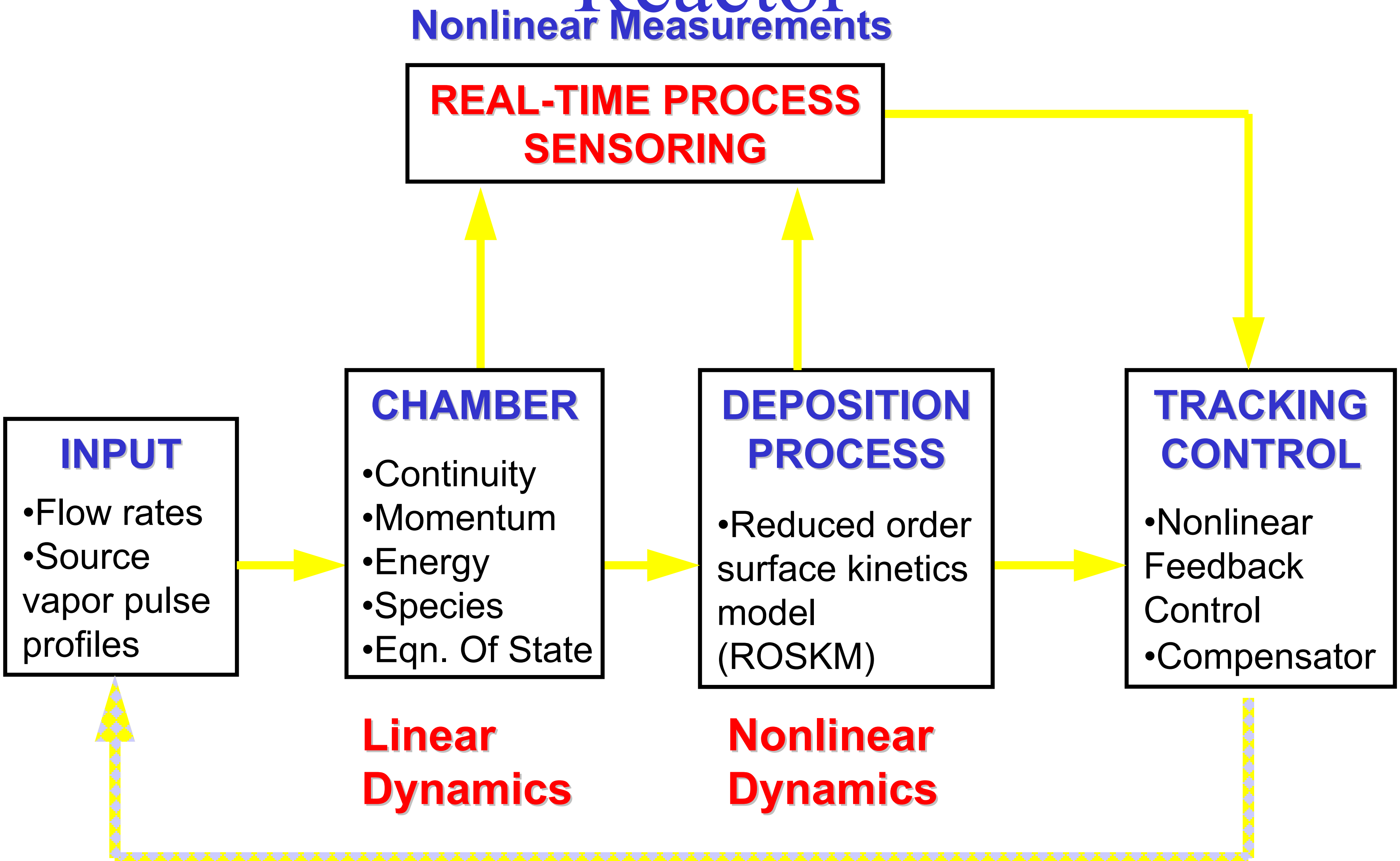


Real-Time Film Growth Sensoring



- Gas phase monitoring
  - Absorption Spectroscopy
- Nucleation kinetics and heteroepitaxial overgrowth monitoring
  - Principle Angle Spectroscopy (PAR)

Real-Time Feedback Control of CVD Reactor



Quasi-Transient Flow

Continuity:

$$\nabla \cdot \left(\rho \vec{u}\right) = 0$$

Eqn. Of State:

$$\rho = \rho_0 \left[1 - \beta_T \left(T - T_0\right)\right]$$

Momentum:

$$\rho \vec{u} \cdot \nabla \vec{u} = -\nabla P + \nabla \cdot \sigma + \left(\rho - \rho_0\right) \vec{g}$$

$$\sigma = -\frac{2}{3} \mu (\nabla \bullet \vec{u}) \vec{I} + \mu (\nabla \vec{u} + \nabla \vec{u}^T)$$

Energy:

$$c_p \rho \vec{u} \cdot \nabla T = \nabla \cdot \left(k_T \nabla T\right)$$

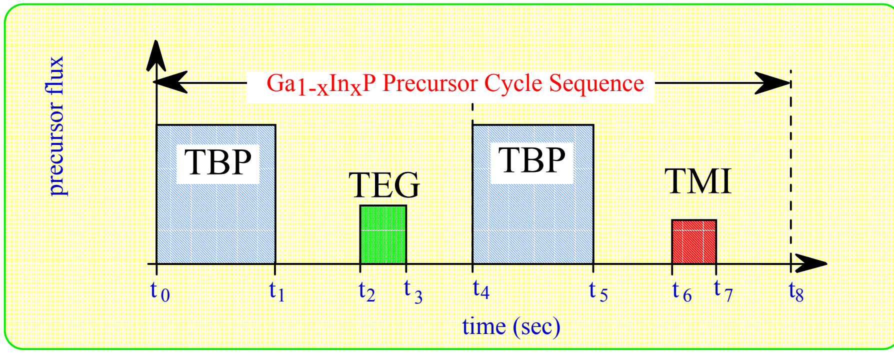
Species:

$$\frac{\partial c_i}{\partial t} + \vec{u} \cdot \nabla c_i = \frac{1}{\rho} \nabla \cdot \left(\rho D_i \nabla c_i\right) + \sum_{i=1}^{N_R} r_{ni}$$

Reduced Order Surface Kinetics (ROSK) Model for $Ga_{1-x}In_xP$ Growth

Thermal decomposition of TBP: for Si(001)

- $C_4H_9PH_2 \rightarrow PH_2 + C_4H_9$ (1)
- $C_4H_9 + C_4H_9PH_2 \rightarrow C_4H_9H + C_4H_{10}$ (2)
- $C_4H_9H \rightarrow H + C_4H_9$ (3)



TEG pyrolysis:

- $Ga(C_2H_5)_3 \rightarrow Ga(C_2H_2)_2 + C_2H_5,$ (4)
- $Ga(C_2H_5)_2 \rightarrow GaC_2H_5 + C_2H_5,$ (5)
- $GaC_2H_5 \rightarrow Ga + C_2H_5,$ (6)

TMI pyrolysis:

- $In(CH_3)_3 \rightarrow In(CH_3)_2 + CH_3,$ (7)
- $In(CH_3)_2 \rightarrow InCH_3 + CH_3,$ (8)
- $InCH_3 \rightarrow In + CH_3,$ (9)

We assume one dominant reaction for the first precursor (TBP) and two dominant reactions for the second and third precursors TEG and TMI). The SRL is treated as an homogenous ideal solution and the surface area is simplified to be constant.

ROSK Model

Simplified first precursor (TBP) approximation:

$$\frac{d}{dt} n_1(t) = n_{TBP} - \tilde{a}_1 n_1(t) - \tilde{a}_4 n_3(t) n_1(t) - \tilde{a}_7 n_6(t) n_1(t)$$

Approximate second precursor (TEG) reactions:

$$\frac{d}{dt} n_2(t) = n_{TEG} - \tilde{a}_2 \; n_2(t)$$

$$\frac{d}{dt} n_3(t) = \tilde{a}_2 n_2(t) - \tilde{a}_3 n_3(t) - \tilde{a}_4 n_3(t) n_1(t)$$

Simplified third precursor (TMI) reactions approximation:

$$\frac{d}{dt} n_5(t) = n_{TMI} - \tilde{a}_5 \; n_5(t)$$

$$\frac{d}{dt} n_6(t) = \tilde{a}_5 n_5(t) - \tilde{a}_6 n_6(t) - \tilde{a}_7 n_6(t) n_1(t)$$

Two incorporation reactions: (for GaP and InP, respectively)

$$\frac{d}{dt} n_{GaP}(t) = \tilde{a}_4 \; n_3(t) \; n_1(t) \quad \text{and} \quad \frac{d}{dt} n_{InP}(t) = \tilde{a}_7 \; n_6(t) \; n_1(t)$$

Note: Surface structure, number of reaction sides and inhomogeneous reactions are approximated at this point in the reaction parameters $\tilde{a}_4$ and $\tilde{a}_7$.

ROSK Model *(continued)*

Composition, x, for $Ga_{1-x}In_xP$:

$$x = \frac{\int \frac{d}{dt} n_{InP}}{\int \left(\frac{d}{dt} n_{GaP} + \frac{d}{dt} n_{InP} \right)}$$

Film growth rate:

$$gr = \frac{1}{A} \left[\tilde{V}_{GaP} \frac{d}{dt} n_{GaP} + \tilde{V}_{InP} \frac{d}{dt} n_{InP} \right]$$

Thickness of the SRL
(as an ideal solution)

$$d_1(t) = \frac{1}{A} \left[n_1 \overline{V}_1 + n_2 \overline{V}_2 + n_3 \overline{V}_3 + n_5 \overline{V}_5 + n_6 \overline{V}_6 \right]$$

$\overline{V}_i$

Effective dielectric function ε_i of the SRL:

$$\varepsilon_i(\omega) = \varepsilon_\infty + \sum_{x \neq 4,7} x_i(t) F_i(\omega) \quad \text{and } x_i(t) = \frac{n_i(t)}{\sum_k n_k(t)}$$

Nonlinear Partial State Observation

Absorption Spectroscopy

$$I = I_0 e^{-\int_0^W \alpha(\vec{x})\, d\vec{x}}$$

$I =$ exiting intensity

$I_0 =$ incident intensity

$W =$ width of the reactor

$$\alpha = \text{absorption coefficient} = \frac{4\pi W_0}{\lambda} \sum_{i=1}^{3} \frac{b_i \; c_i}{W_i}$$

(for a dilute solution)

b_i – imaginary component of the optical response

λ – wavelength, is chosen so that the absorption is sensitive to the concentration of particular gas phase species

Nonlinear Tracking Control Problem

Consider a nonlinear system with linear tracking variable:

$$\frac{d}{dt}x(t) = f(x(t)) + Bv_1(t),\; x(0) = x_0, \qquad x(\bullet) \in R^n$$

$$d(t) = Hx(t)$$

and a cost functional:

$$J(v_1, x_0) = \int_o^\infty [(d - d_T)Q(d - d_T) + v_1 R v_1]dt$$

***$d_T(t)$* – desired tracking profile**

Problem reformulation

Rewrite the nonlinear system with linear tracking variable:
(*J.R. Cloutier, C.N. D'Souza, and C.P. Mracek* (1996))

$$\frac{d}{dt}x(t) = \boxed{A(x(t))x(t)} + Bv_1(t),\ x(0) = x_0, \qquad x(\bullet) \in R^n$$

$$d(t) = Hx(t)$$

and a cost functional:

$$J(v_1, x_0) = \int_o^\infty [(d - d_T)Q(d - d_T) + v_1 R v_1]dt$$

***$d_T(t)$* – desired tracking profile**

Nonlinear Tracking Control

Necessary Optimality Conditions

$$u(t) = -R^{-1}B^{T}\left(\Pi(x(t))x(t) + s(t)\right)$$

SDRE

$$\Pi(x)A(x) + A^{T}(x)\Pi(x) - \Pi(x)BR^{-1}B^{T}\Pi(x) + H^{T}QH = 0$$

Tracking Equation

$$\dot{s} + A^{T}(x)s - \Pi(x)BR^{-1}B^{T}s - H^{T}Q\,d_{T}(t) + \sum_{i=1}^{n} x_{i}\left(\frac{\partial A_{1\ldots n,i}(x)}{\partial x}\right)^{T}\left(\Pi(x)x + s\right) + \dot{\Pi}(x)x = 0$$

$$s(t_{f}) = 0$$

SDRE

$$\Pi(x)A(x)+A^T(x)\Pi(x)-\Pi(x)BR^{-1}B^T\Pi(x)+H^TQH=0$$

Rewrite: (*A. Wernli and G. Cook* (1975))

$$A(x)=A_0+\varepsilon\Delta A(x);$$

$$\Pi(x,\varepsilon)=\Pi(x)|_{\varepsilon=0}+\Pi_\varepsilon(x)|_{\varepsilon=0}\,\varepsilon+\ldots=\sum_{i=0}^{\infty}\varepsilon^i L_n(x)$$

$$\left(\sum_{i=0}^{\infty}\varepsilon^i L_i(x)\right)\left(A_0+\varepsilon\Delta A(x)\right)+\left(A_0^T+\varepsilon\Delta^T A(x)\right)\left(\sum_{i=0}^{\infty}\varepsilon^i L_i(x)\right)$$

$$-\left(\sum_{i=0}^{\infty}\varepsilon^i L_i(x)\right)BR^{-1}B^T\left(\sum_{i=0}^{\infty}\varepsilon^i L_i(x)\right)+H^TQH=0$$

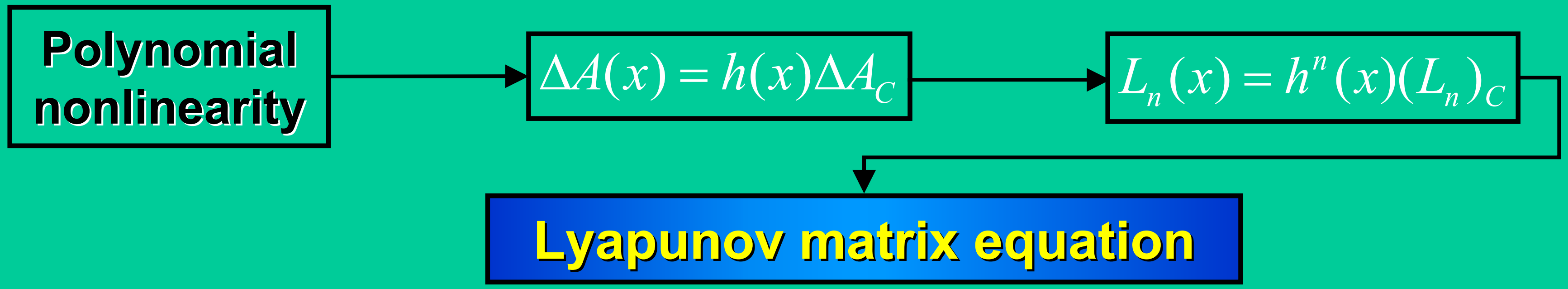
SRDE (continued)

$$L_0 A_0 + A_0^T L_0 - L_0 B R^{-1} B^T L_0 + H^T Q H = 0$$

$$L_1\left(A_0 - BR^{-1}B^T L_0\right) + \left(A_0^T - L_0 B R^{-1} B^T\right) L_1 + L_0 \Delta A + \Delta^T A L_0 = 0$$

$$L_i\left(A_0 - BR^{-1}B^T L_0\right) + \left(A_0^T - L_0 B R^{-1} B^T\right) L_i + L_{i-1} \Delta A + \Delta^T A L_{i-1}$$

$$- \sum_{k=1}^{n-1} \left(L_k B R^{-1} B^T L_{i-k}\right) = 0$$



Nonlinear Compensator

Consider the nonlinear system:

$$\frac{d}{dt}x(t) = f(x(t)) + Bv_1(t)$$

$$z(t) = Cx(t)$$

Rewrite:

$$f(x(t)) = Ax + g(x)$$

Nonlinear State Estimator:

$$\frac{d}{dt}x_e(t) = (A - FC)x_e(t) + g(x_e(t)) + Fz(t) + Bv_1(t)$$

Nonlinear Compensator *(continued)*

$$e(t) = x_e(t) - x(t)$$

$$\frac{d}{dt}e(t) = (A - FC)e(t) + g(x_e(t)) - g(x(t))$$

Assume:

- **(A,C) is observable**
- **g is locally Lipschitz**

$$e(t) \xrightarrow[t \to \infty]{} 0$$

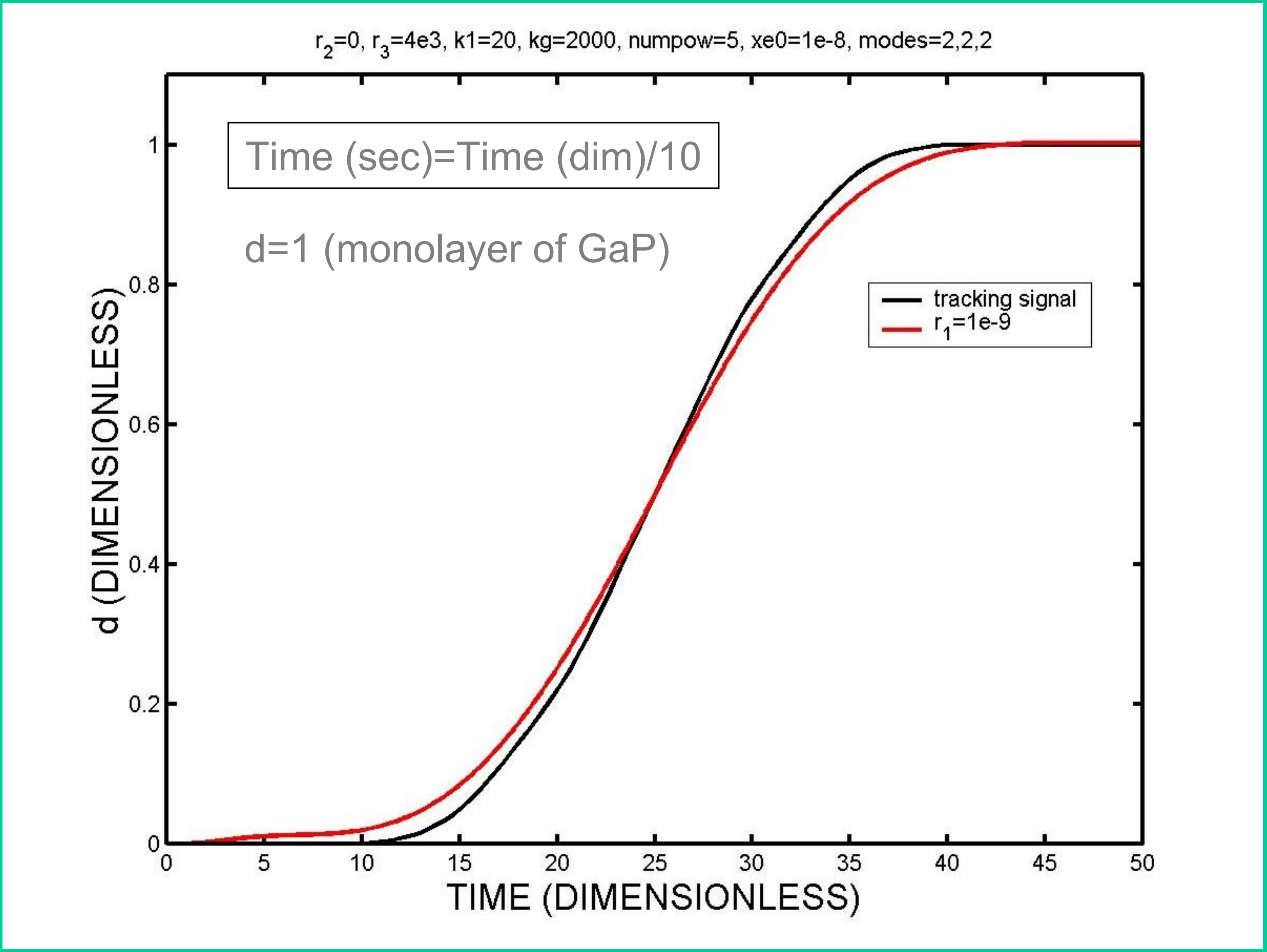
Computational Results

- **The tracking control of thin film thickness involves:**
  - **Linear gas phase dynamics and nonlinear surface kinetics**
  - **Linear tracking variable and nonlinear state observation**

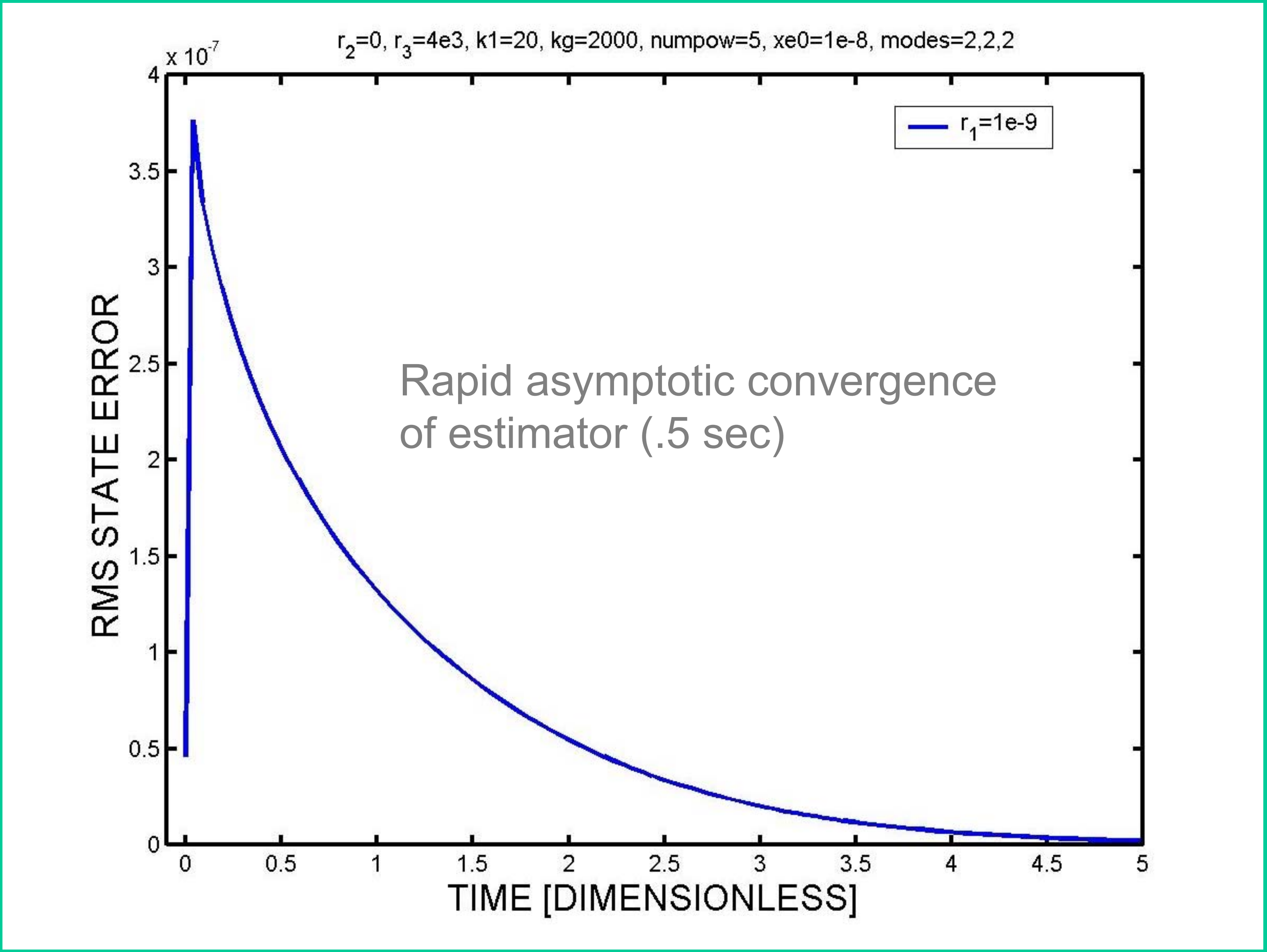
- **Methodology includes:**
  - **Nonlinear compensator based on linearized state observation**
  - **Nonlinear state feedback gain and nonlinear observer gain are solved via SDREs**

- **Design parameters (at <u>10 atm</u>):**
  - $R=identity,\ Q=r_1$ (nonlinear state feedback gain)
  - $U=identity,\ V=r_3$ (nonlinear compensator gain)

Computational Results



Computational Results



Computational Methodology for Eddy Current Based Nondestructive Evaluation Techniques

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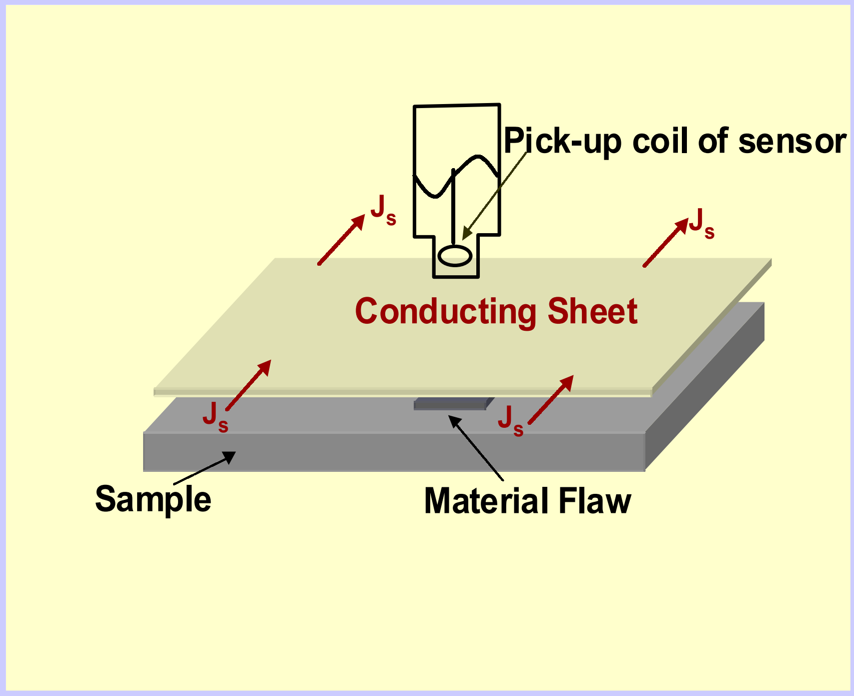
Eddy Current Methods

- A conducting sheet carrying a uniform source current is placed near the sample to be examined.
- The current in the conducting sheet induces a current in the sample, called an eddy current.
- If a defect is present, it disrupts the flow of the eddy current.
- The disturbance in the eddy current manifests itself in the magnetic field data taken by the measuring device.



Maxwell's Equations in Phasor Form

Maxwell's Equations

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + i\omega \mathbf{D}$$

Constitutive Laws

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{H} = \frac{1}{\mu} \mathbf{B}$$

Ohm's Law

$$\mathbf{J} = \sigma \mathbf{E}$$

where

- **B** is the magnetic flux density in T
- **D** is the electric displacement in C/m^2
- **E** is the electric field intensity in V/m
- **H** is the magnetic field intensity in A/m
- **J** is the current density in A/m^2
- ρ is the electric charge density in C/m^3
- ω is the angular frequency in rad/s
- ε is the permittivity in F/m
- μ is the magnetic permeability in H/m
- σ is the electric conductivity in S/m

Boundary Value Problem

The entire boundary value problem is given by

$$\nabla \times \left(\frac{1}{\mu(x,y)} \nabla \times \mathbf{A}(x,y) \right) = \left(\sigma(x,y) + i\omega\varepsilon(x,y)\right)\left(-i\omega\mathbf{A}(x,y) - \nabla\phi\right) \quad \forall (x,y) \in \Omega,$$

$$I_{cs} = \int_{cs} \left(\sigma(x,y) + i\omega\varepsilon(x,y)\right)\left(-i\omega\mathbf{A}(x,y) - \nabla\phi\right) \cdot \mathbf{n}\, da \quad \forall (x,y) \in cs$$

and

$$\nabla\phi = 0 \quad \forall (x,y) \in \Omega \setminus cs$$

with

$$\mathbf{A}(x,-35mm) = \mathbf{0} = \mathbf{A}(x,35mm)$$

$$\nabla\mathbf{A} \cdot \mathbf{n}\big|_{(0mm,y)} = \mathbf{0} = \nabla\mathbf{A} \cdot \mathbf{n}\big|_{(50mm,y)}$$

Potential Difficulties in Inverse Problem

- Our ultimate goal is to determine the feasibility of using a ***portable sensing device*** in conjunction with inverse problem techniques to characterize a damage.
- The inverse problem is a computationally intensive iterative procedure in which the boundary value problem (BVP) must be solved possibly numerous times.
- Using standard finite element methods, the inverse problem would be extremely time consuming and therefore not practical in experimental settings.
- To decrease the computational time, we propose to use the reduced order POD (Proper Orthogonal Decomposition) methodology.

POD Method

- Let $\mathbf{q}_j$ represent a damage and $A(\mathbf{q}_j)$ denote the solution to the boundary value problem given damage $\mathbf{q}_j$. Then the set of N_s snapshots is given by

$$\left\{ A(\mathbf{q}_j) \right\}_{j=1}^{N_s}$$

- We seek POD basis elements Φ_i of the form

$$\Phi_i = \sum_{j=1}^{N_s} V_i(j) A(\mathbf{q}_j)$$

such that each basis element Φ_i, $i=1,\ldots,N_s$ resembles the snapshots in the sense that it maximizes

$$\frac{1}{N_s} \sum_{j=1}^{N_s} \left| \left\langle A(\mathbf{q}_j), \Phi_i \right\rangle_{L^2(\Omega,\mathrm{C})} \right|^2$$

subject to $(\Phi_i\ ,\ \Phi_i\ ) = \|\ \Phi_i\ \|^2 = 1$.

Forming the POD Basis Elements

- The coefficients $V_i(j)$ are found by solving the eigenvalue problem $CV = \lambda V$ where the covariant matrix C is given by
$$[C]_{ij} = \frac{1}{N_s}\left\langle A(\mathbf{q}_i), A(\mathbf{q}_j)\right\rangle_{L^2(\Omega,C)}.$$
- C is a Hermitian positive semi-definite matrix and hence possesses a complete set of orthogonal eigenvectors with corresponding non-negative real eigenvalues which can be ordered according to
$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{N_s} \geq 0.$$
- The set of POD basis elements is given by
$$\left\{\Phi_i\right\}_{i=1}^{N_s}$$
where
$$span\left\{\Phi_i\right\}_{i=1}^{N_s} = span\left\{A(\mathbf{q}_j)\right\}_{j=1}^{N_s}.$$
- The reduced basis is given by
$$\left\{\Phi_i\right\}_{i=1}^{N}$$
where N is chosen so that
$$span\left\{\Phi_i\right\}_{i=1}^{N} \approx span\left\{A(\mathbf{q}_j)\right\}_{j=1}^{N_s}.$$
which can be found by examining
$$\sum_{j=1}^{N}\lambda_j \Big/ \sum_{j=1}^{N_s}\lambda_j$$

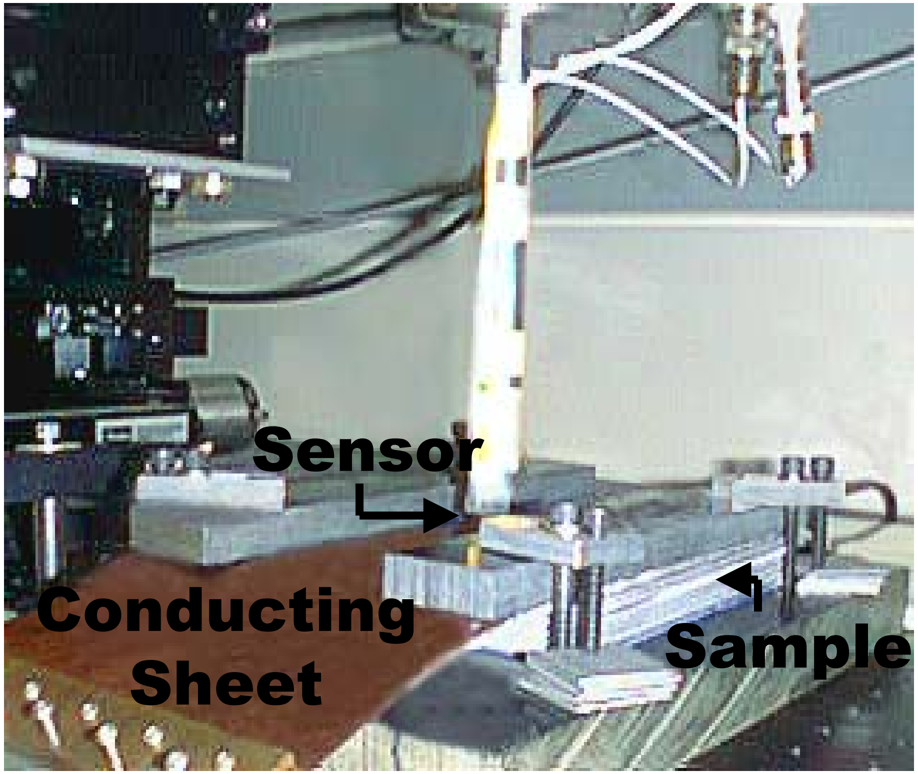
Computational Examples

- Computational simulations were performed in which we estimated length, thickness, and depth separately and length and depth simultaneously.
- We generated "snapshots" of the magnetic vector potential, **A**, by using Ansoft Maxwell 2D Field Simulator to generate "data" across various damages.
- Simulated data was formed in the same way with random noise added to model random measurement error.
- In the parameter estimation problem, a scaled least squares criterion was used.

Conclusions Based on Simulated Results

- The methods were shown to be accurate and robust. Even with data containing considerable noise (10% in our example) the estimated parameters were a good representation of the actual parameters.
- Using a finite element package similar to Ansoft Maxwell 2D Field Simulator, over 7000 finite elements would have to be used in each forward run in the optimization problem where less than 10 POD basis elements are used while still obtaining extremely accurate results. This leads to a significant decrease in computational time.
- The methods were fast. The entire inverse problem took approximately 8 seconds. If one were using Ansoft Maxwell 2D Field Simulator, a single forward simulation would take on average 5-7 minutes. Thus we arrive at a speed up factor ranging from 750-1050, approximately a factor of 10^3.

Experimental Setup



- Sample:

  17 layers of aluminum plates stacked on top of one another

- Damage:

  The damage is formed by cutting out a piece of one of the aluminum plates

- Conducting Sheet:

  Thin sheet of copper

- Sensor:

  GMR sensor

Overall Conclusions

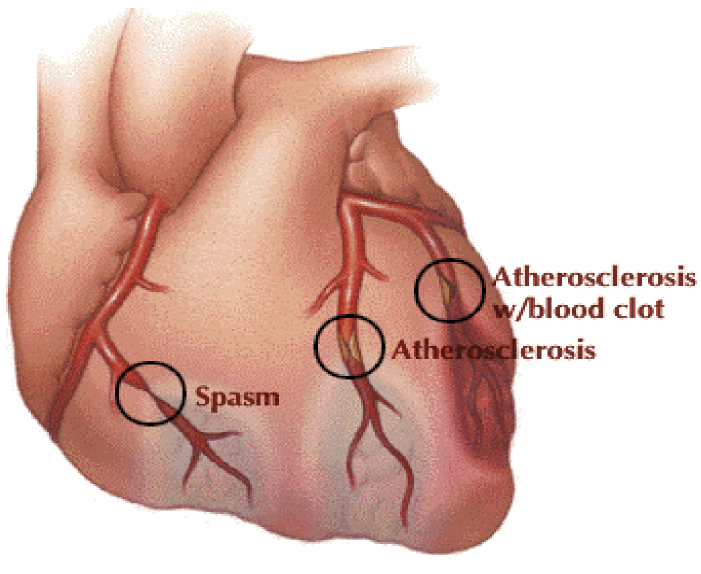
- Taken as a whole, our work indicates that using the POD method in NDE research can be an attractive alternative to standard finite element methods, offering the potential for substantial savings in total computational time.
- Since the method is both fast and accurate, it suggests this method would be beneficial in real-time applications.
- It is possible to either snapshot on FEM simulations or experimental data when forming the POD basis elements and can obtain good results in both cases.

Future Directions
DPS Estimation and Control

- BIOLOGY-
  - a) Biomedical diagnostics
    (e.g.,sensing of distributed systems as in stenosis)
  - b) Biosensing and estimation
    (e.g.,CNS activity via remote electromagnetic sensing of dielectric/ conductivity changes)
  - c) Bioterrorism
    (PBPK type modeling for viral, toxic threats and responses distributed over populations and geography)

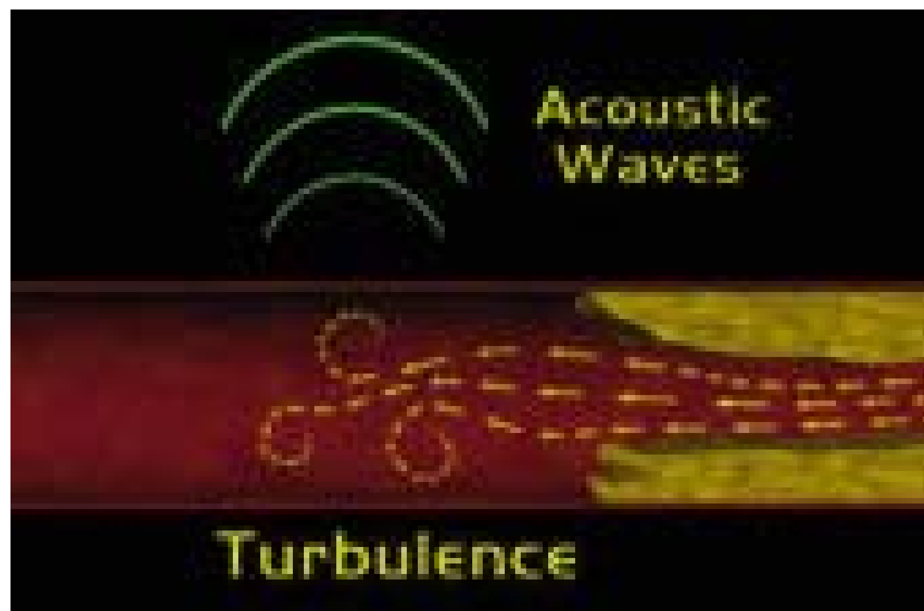
Combining DPS estimation and control with statistical techniques to include uncertainty will be essential--robust methodology required

Coronary Artery Diseases

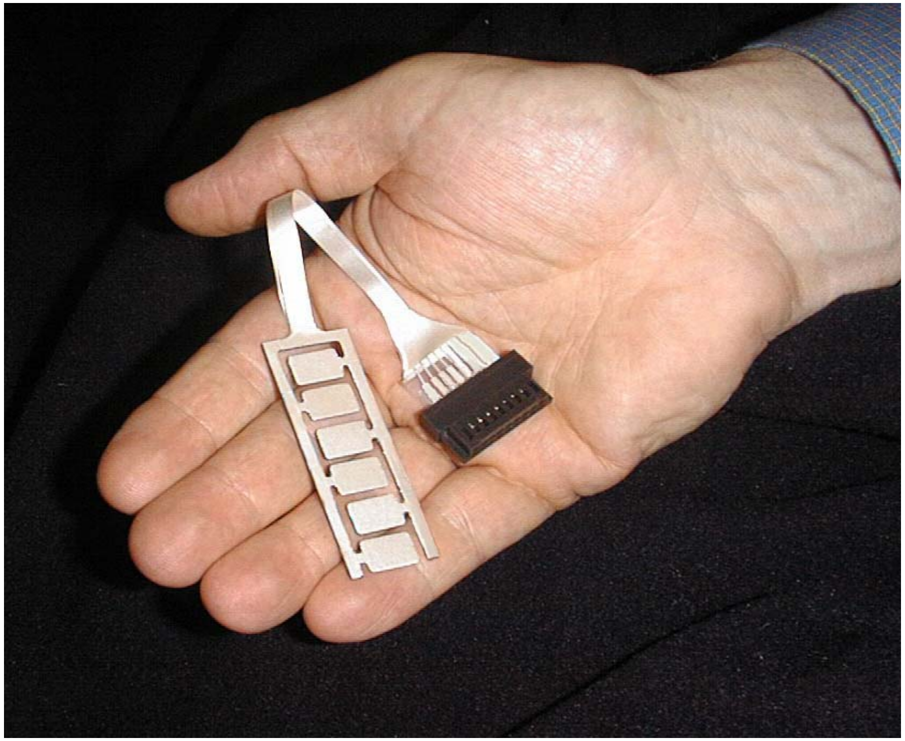


Arterial stenosis is the buildup of plaque (fat and calcium deposits) on the interior walls of arteries

Turbulence induced acoustic waves



Non-Invasive and Localization of CAD



- **Array of sensors placed on the patient's chest that will detect acoustic shear waves (propagated thru heterogeneous medium-tissue and bone)**
- **Inexpensive and (as effective as angiogram)**
- **No risk to patient and easy to administer**

MedAcoustics

Center for Research in Scientific Computation
CRSC

Future Directions
DPS Estimation and Control

- TELECOMMUNICATIONS/INTERNET

  DPS and statistics-PDE with discontinuities and uncertainty-robust control methodology

- NANOTECHNOLOGY/ADAPTIVE STRUCTURES

  design and control of materials-chemical,bio,composites,fluids,etc. will require multiscale modeling of systems:

  molecular to structural—micro to macro response

- ELECTROMAGNETICS AND OPTICS

  sensing and detection, remote surveillance and interrogation, quantum computing

Combining DPS estimation and control with statistical techniques essential

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