Robust Control System: Asymmetric Distributed Games

Adversary can destroy C3I structure.

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Adversary Changes Your Command and Control Network

Islands of communication at $t$

Islands of communication at $t'$
Asymmetric Adversary

Given centralized algorithm with intact command structure

Agent j:
Sensor value: $X[j]$
Actuator value: $X^*[j]$

What should agents within communication islands do when part of the command structure is destroyed?
MURI Challenge

• Software verification, control systems have been studied for decades.
• MURI Challenge: extend V&V theories of classical control to multi-agent systems, dynamic sets of agents, incorporating sensor fusion with an asymmetric adversary.
• Extending tool set to new DoD challenges
Asymmetric Adversary: Gaming Strategy

Options:
2. Pre-specify plan for every possible group of agents. With N agents: make $2^N$ plans.
3. Each group follows same strategy independent of command network.
4. Pre-specify plans based on types of agent sets: 1 red agent, 2 blue agents…

What should agents within communication islands do when part of the command structure is destroyed?
Asymmetric Adversary: Oblivious Control Strategy

Agents in each communication island execute EXACTLY same steps as agents in centralized solution oblivious of all other agents.
State Changes in Oblivious Control

Islands in interval $t_0$ to $t_1$

Communication structure changes at time $t_1$

Islands in interval $t_1$ to $t_2$

Each agent in same state at $t_1^-$ and $t_1^+$

Each island goes through sequence of state changes in its oblivious algorithm: each agent changes state from $t_0$ to $t_1$

Each island goes through sequence of state changes in its oblivious algorithm: each agent changes state from $t_1$ to $t_2$
Oblivious Control

Agent $j$:
- Sensor value: $X[j]$
- Actuator value: $X^*[j]$
- $X^* = f(X)$

Lyapunov, Variant Penalty, function

$h$ $X^*$ $X$
Steps in Oblivious Control

- Islands in interval $t_0$ to $t_1$
  - Island 1
  - Island 2
  - Island 3
  - Island 4
- State of island at $t_0$
- State of island at $t_1^-$
- State of new island at $t_1^+$

Islands in interval $t_1$ to $t_2$
- Island 1
- Island 2

$h_1$ $h_2$ $h_3$ $h_4$

$x_1$ $x_2$ $x_3$ $x_4$
Discrete and Continuous Systems

**Example of discrete system**: Sort array in ascending order
Penalty function \( h \): number of out-of-order pairs:
\[ h(X) = \text{cardinality of } \{ (j, k) \mid j < k \text{ AND } X[j] > X[k] \} \]

E.g., \( h([10 \ 6 \ 8 \ 2]) = 5 \)

**Example of continuous system**: Compute mean
\( h(X) = \text{sum-of-squares of deviation of points from mean} \)
Research Questions

• For what class of problems does oblivious control work? Not work?
• Is there a generic method for showing correctness? What is it? How easy is it?
Problems: Obliviousness Works

Gaussian mean

Convex hull
Problems: Obliviousness Works

Agent movement

Sorting

**Initial**

Agent movement

| 9 4 7 | 6 3 1 0 | 8 5 |

Sorting

| 0 1 3 | 4 5 6 7 | 8 9 |
Problems for which Obliviousness Doesn’t Work Directly

Regression: compute regression line for variables

Each point \((x, y, \text{temperature}, \text{pressure})\)

Minimum circumscribing circle
Solve with Obliviousness First Step

Regression: First compute means using obliviousness

Minimum circumscribing circle: First compute convex hull using obliviousness

Each point \((x, y, \text{temperature, pressure})\)
V&V: Distributed Prediction and Planning in Adversarial Situation

Generator predicts (likelihood of) event.

Measured values

Predictive event

Predicted trajectory
Methodology

- Proof obligation: Any step of a set (communication island) $B$ of agents is also a step of set $B \cup \{a\}$ for any agent $a$. 

\begin{align*}
\text{Set } B \text{ of agents takes an optimization step } z & \quad & \text{Set } B \text{ of agents takes an optimization step } z \text{ and new agent } a \text{ takes no step is equivalent to set } B \cup \{a\} \text{ taking an optimization step } z' \\
\end{align*}
E.G. Weighted Mean with SOS

Set $B$ of agents takes an optimization step

Set $B$ of agents takes an optimization step is also a step taken by set $B \cup \{a\}$

$[0, 4]$ changes to $[1, 3]$ reduces penalty while maintaining feasibility
Progress Requirements

• If there is a state transition from a state $S$ to a different state $S'$ then the system does not remain forever in state $S$.

• Continuous case: each state transition converges to optimum solution or decreases $h$ by at least Delta (for some arbitrary positive constant Delta).
What to do when your command structure is attacked?

Communication cells are chaotic and fluid. What strategies will bring total system to its goal?
Further Work:
Robust Distributed Control

• Extend description of oblivious algorithms for continuous system to notations based on UNITY/graph grammars --- see Klavins.
• Adversary blocks communication in game – theoretic situation --- see Parillo.
• Consensus control algorithms --- see Murray
• Identify problem class in which h is SOS --- see Doyle
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