MURI Review
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Caltech

Verification and Performance in Networked Embedded Systems

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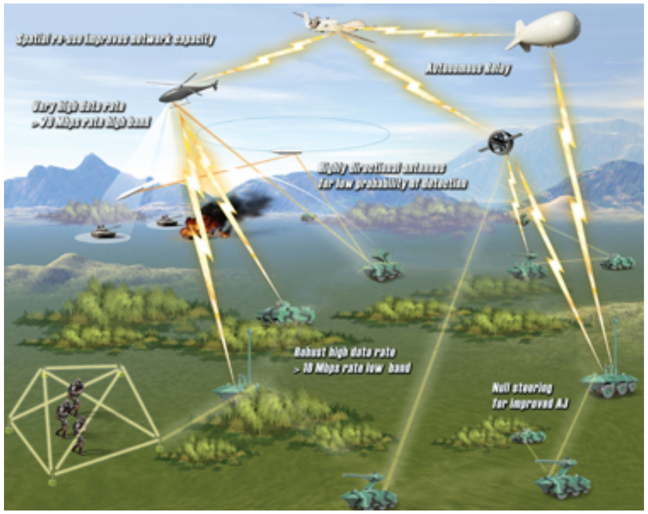
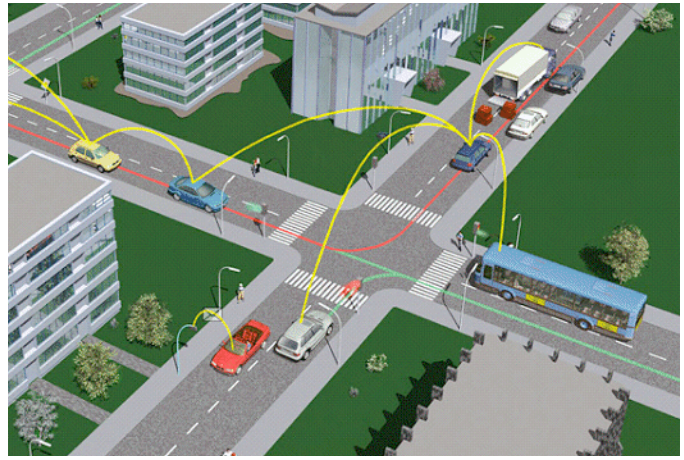
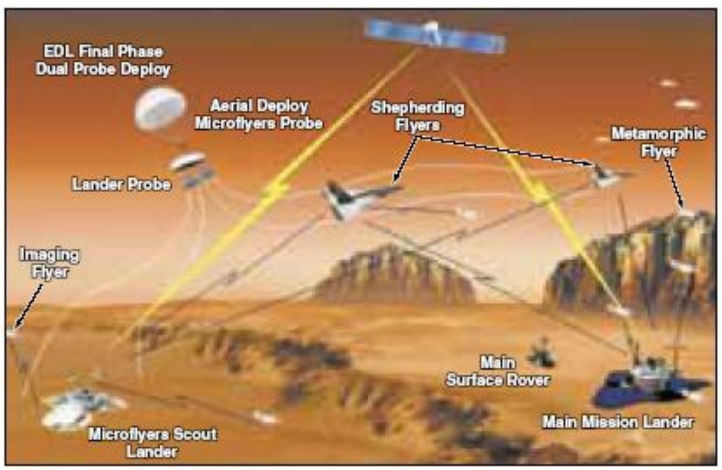
Setting

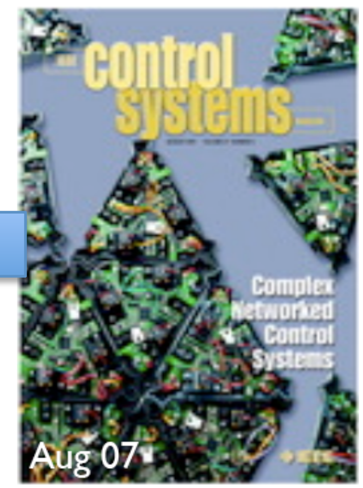




- Heterogeneity
- Local communication
- Motion
- Changing environment
- Stochasticity
- Complex Tasks / Subtasks

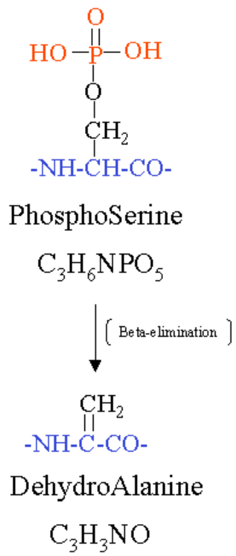


Metabolobtics



Programmable Parts

Challenges



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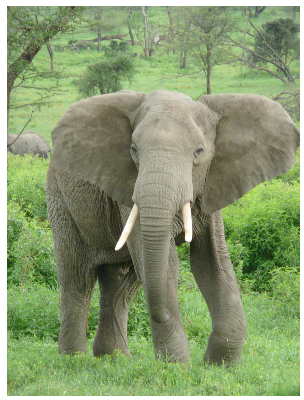


- Abstraction
- Programming and design
- Specification
- Verification
- Performance

```
Send(strTestMessage.GetBuffer(),strTestMessage.GetLength()+1)
```

?

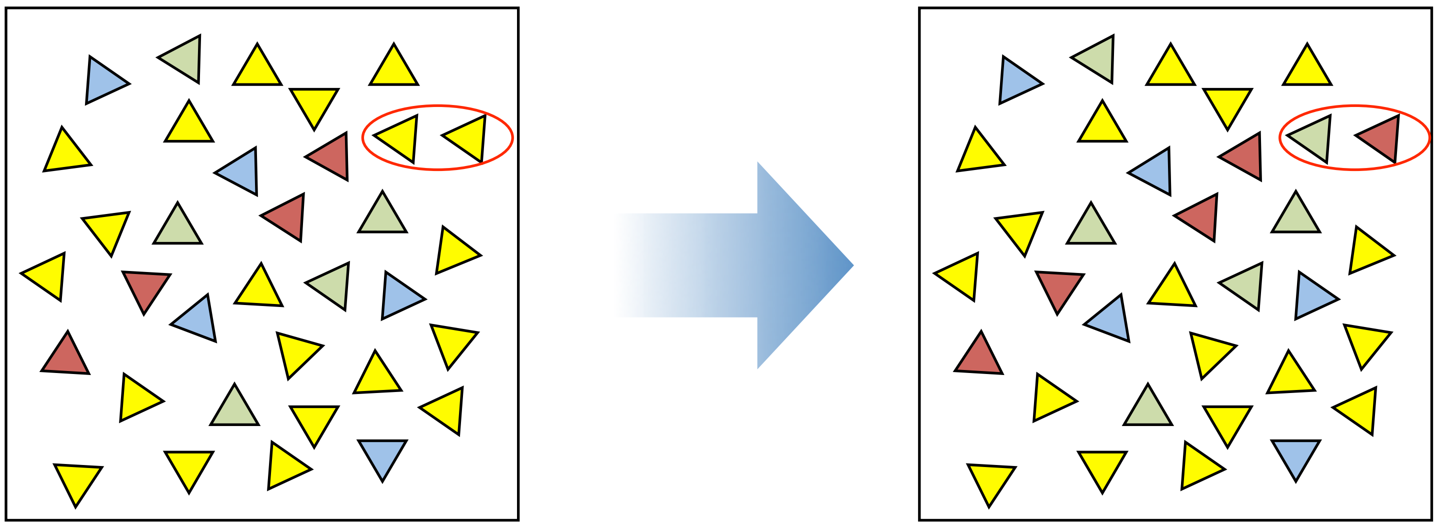
Overview of UW Progress

1. Testbeds: PPT, EFRI
2. Suite of examples specified and verified
3. Lyapunov function co-design
4. Separation of continuous and discrete design
5. Separation of verification and performance tuning
6. Pseudometrics to compare, diagnose and optimize stochastic systems
7. Diagnosis / observation of stochastic process with low-dimensional, stochastic output

This talk (Overview of McNew's Thesis)

David's talk

Abstraction: Concurrency

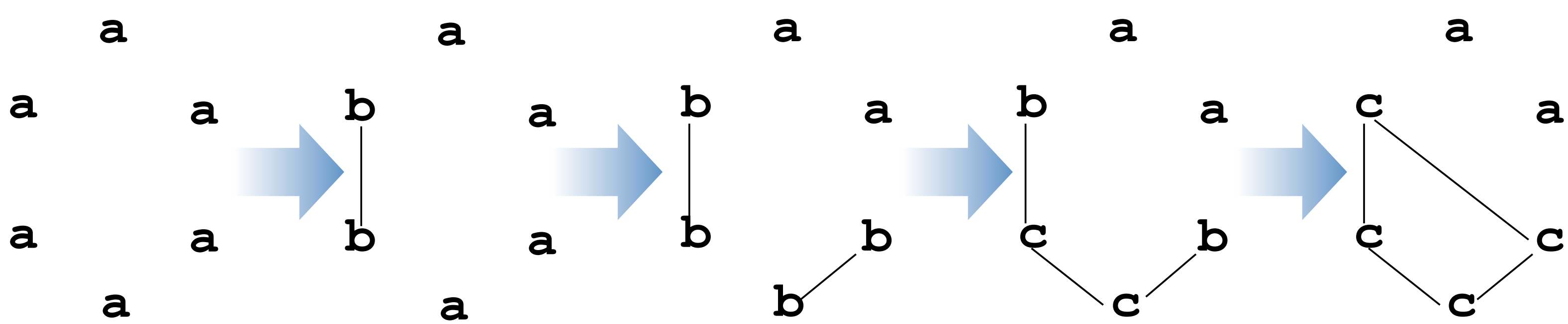


A concurrent program:
Each guards and rule can be evaluated by small groups of agents without global knowledge.

$$\left\{\begin{array}{l} guard_1 : rule_1 \\ guard_2 : rule_2 \\ . \\ . \\ . \\ guard_n : rule_n \end{array}\right.$$

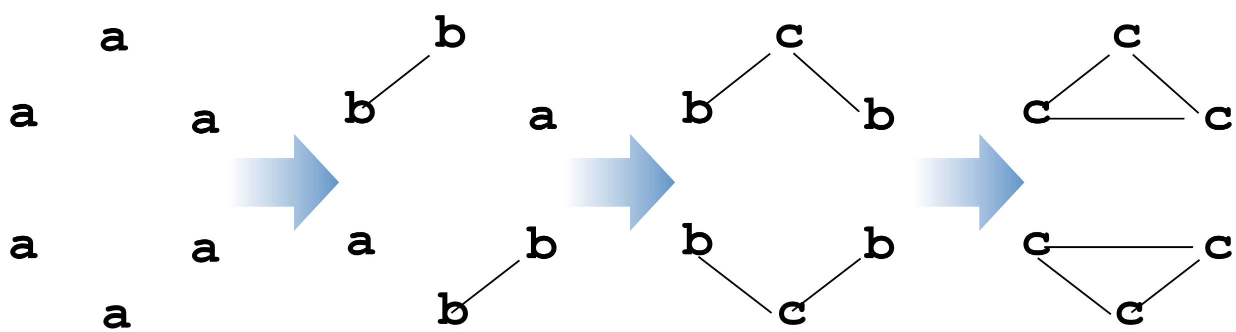
Non-determinism: Which rule? Where to apply it? When to apply it?
Abstraction: How is the rule applied? – That's an implementation issue!

Example: Graph Grammars



$$
\Phi = \begin{cases} a \quad a \quad \rightharpoonup \quad b - b, \\ a \quad b \quad \rightharpoonup \quad b - c, \\ b \quad b \quad \rightharpoonup \quad c - c \end{cases}
$$

In general, each node carries a data structure, and rules operate on pairs of data structures.



E. Klavins, R. Ghrist and D. Lipsky, A Grammatical Approach to Self-Organizing Robotic Systems, IEEE Transactions on Automatic Control. Jun. 2006, Vol. 51, No. 5, pp. 949-962.

E. Klavins, Programmable Self-Assembly. Control Systems Magazine, Aug. 2007. Vol. 24, No. 4, pp. 43-56.

Graph Grammar Rules More Generally

Rule name

Unbound node indices

Existential variable

Precondition on data structures maintained by i, j and k

Precondition local network

Postcondition on network
(there may also be postconditions on data)

Rule r_3 *(Merge Branches)*

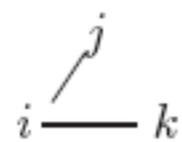
Vertices : i, j, k

Variables : b_i

Precondition:

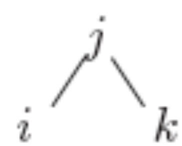
$$i.mode = t \wedge j.mode = a \wedge k.mode = a$$
$$ij.order(j) < b_i \wedge ik.order(k) < b_i$$

$E \; =$



Effect:

$E \; :=$

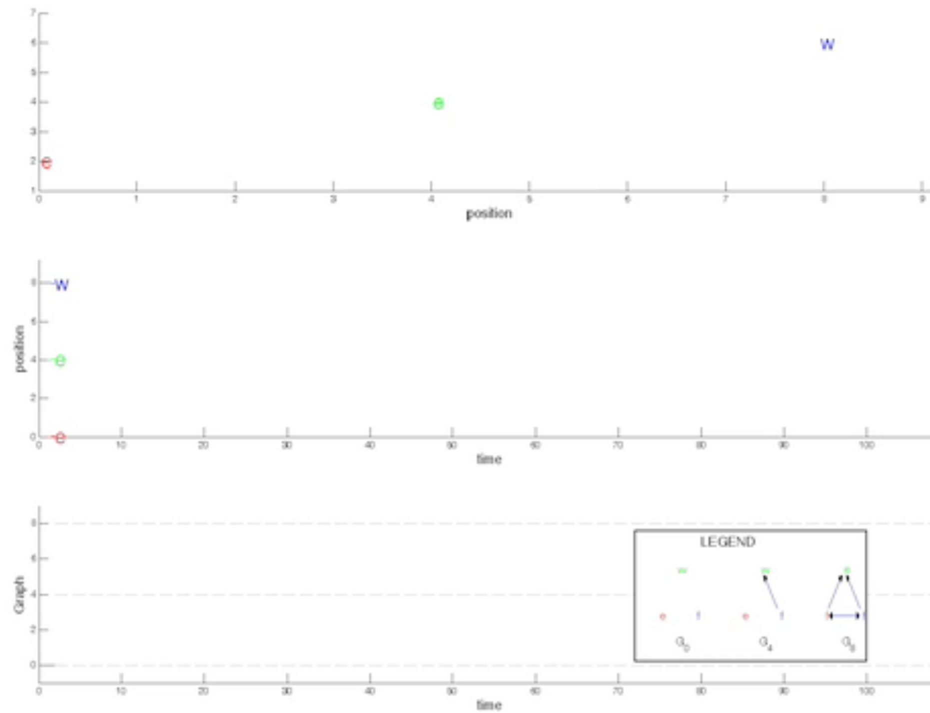


Example: Embedded Graph Grammars

$$\mathbf{u}_i(\gamma) = \begin{cases} 1 & \text{if } i.mode = \mathtt{east} \\ -1 & \text{if } i.mode = \mathtt{west} \\ \sum_{j \in N(i)} \mathbf{x}_j - \mathbf{x}_i & \text{if } i.mode = \mathtt{follow} \end{cases}$$

Φ:

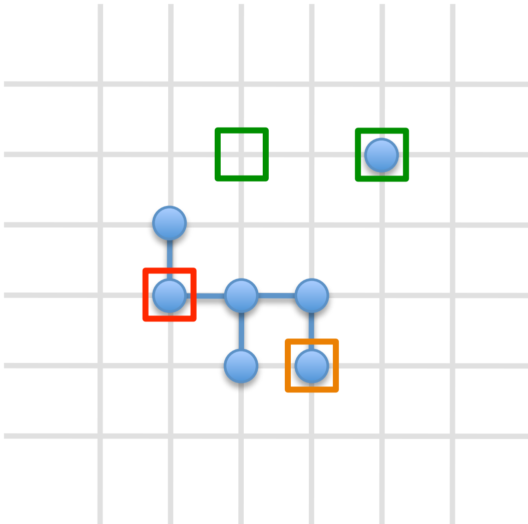
Rule r_1 (Join)	Rule r_2 (Close Cycle)
Precondition: $\|\|x_i - x_j\|\| < 3$, $e \quad w$ **Effect:** $w \leftarrow f$	**Precondition:** $\|\|x_i - x_j\|\| < 5 \wedge \|\|x_k - x_j\|\| < 5 \wedge \|\|x_i - x_k\|\| < 5$ e $w \leftarrow f$ **Effect:** $f \rightarrow e$, $f \rightarrow f$, $e \leftarrow f$



Animation

J. M. McNew. Ph.D. Thesis. 2008.
J. M. McNew, E. Klavins and M. Egerstedt. Solving Coverage Problems with Embedded Graph Grammars. HSCC 2007.

Example: Metabolobotics



NSF EFRI: Build it and program it!
MURI: Verification tools should apply!

- Bond
- Part
- Welder
- Breaker
- Carrier

Action name: $\text{manipulate}(i, j):$

Least Specific Guard: $(r_i = CARRIER \vee r_i = BONDER \vee r_i = BREAKER) \wedge (x_i, y_i) = (x_j, y_j)$

Update: $m_i' = m_i \cup j$

Other actions: move, weld, break, label, communicate

Example: Metabolobotics

role == Carrier ∧ (¬ EjiMj ∧ ¬ Efree0) : MoveU[i]
role == Carrier ∧ (¬ EjiMj ∧ ¬ Efree0) : MoveD[i]
role == Carrier ∧ (¬ EjiMj ∧ ¬ Efree0) : MoveL[i]
role == Carrier ∧ (¬ EjiMj ∧ ¬ Efree0) : MoveR[i]

role == Carrier ∧ (¬ EjiMj ∧ Efree0) :
  Manipulate[i, iFree] ∧
  WriteSublabel[Last[PartsGetManipulating[i]], idxLabelName, nameCarried]

role == Carrier ∧ (EjiMj ∧ ¬ EfinalD1) : MoveU[i]
role == Carrier ∧ (EjiMj ∧ ¬ EfinalD1) : MoveD[i]
role == Carrier ∧ (EjiMj ∧ ¬ EfinalD1) : MoveL[i]
role == Carrier ∧ (EjiMj ∧ ¬ EfinalD1) : MoveR[i]

role == Carrier ∧ (EjiMj ∧ EfinalD1) :
  WriteSublabel[Last[PartsGetManipulating[i]], idxLabelName, nameFinal] ∧
  Release[i]

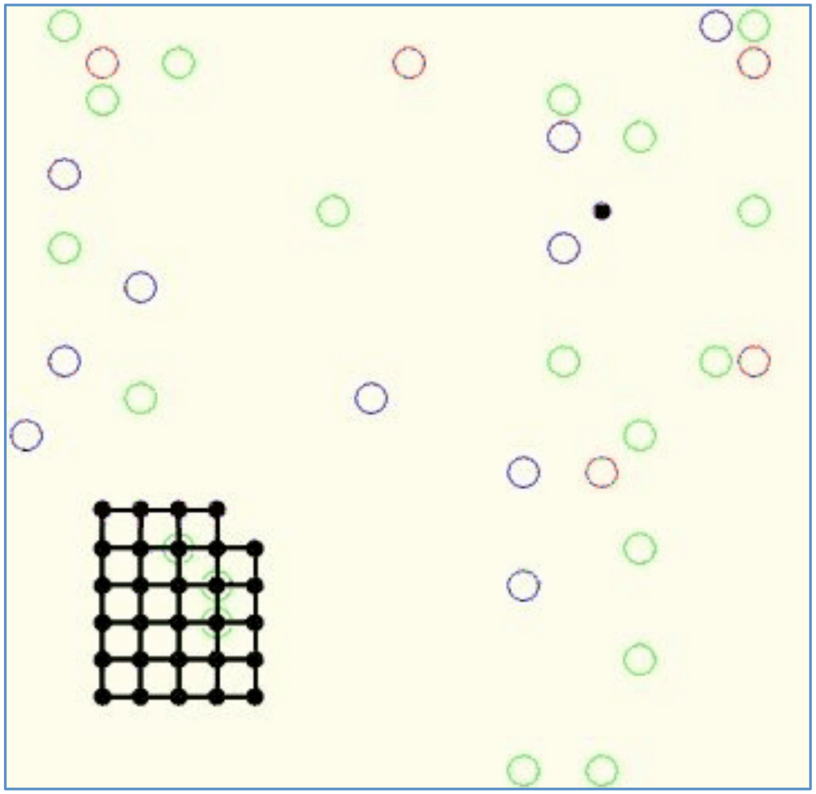
role == Welder ∧ (¬ EfinalD0 ∨ ¬ EfinalD1 ∨ BondedQ[iFinal0, iFinal1]) : MoveU[i]
role == Welder ∧ (¬ EfinalD0 ∨ ¬ EfinalD1 ∨ BondedQ[iFinal0, iFinal1]) : MoveD[i]
role == Welder ∧ (¬ EfinalD0 ∨ ¬ EfinalD1 ∨ BondedQ[iFinal0, iFinal1]) : MoveL[i]
role == Welder ∧ (¬ EfinalD0 ∨ ¬ EfinalD1 ∨ BondedQ[iFinal0, iFinal1]) : MoveR[i]

role == Welder ∧ EfinalD0 ∧ EfinalD1 ∧ ¬ BondedQ[iFinal0, iFinal1] :
  Manipulate[i, iFinal0] ∧
  Bond[i, iFinal0, iFinal1] ∧
  Release[i]

role == Breaker ∧ ¬ Einitial0 : MoveU[i]
role == Breaker ∧ ¬ Einitial0 : MoveD[i]
role == Breaker ∧ ¬ Einitial0 : MoveL[i]
role == Breaker ∧ ¬ Einitial0 : MoveR[i]

role == Breaker ∧ Einitial0Unbonded :
  Manipulate[i, iInitial0Unbonded] ∧
  WriteSublabel[iInitial0Unbonded, idxLabelName, nameFree] ∧
  Release[i, iInitial0Unbonded]

role == Breaker ∧ Einitial0Bonded :
  Manipulate[i, iBondedPair[[1]]] ∧
  Break[i, iBondedPair[[1]], iBondedPair[[2]]] ∧
  Release[i, iBondedPair[[1]]]

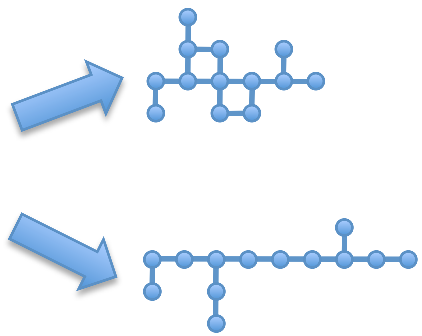


Animation

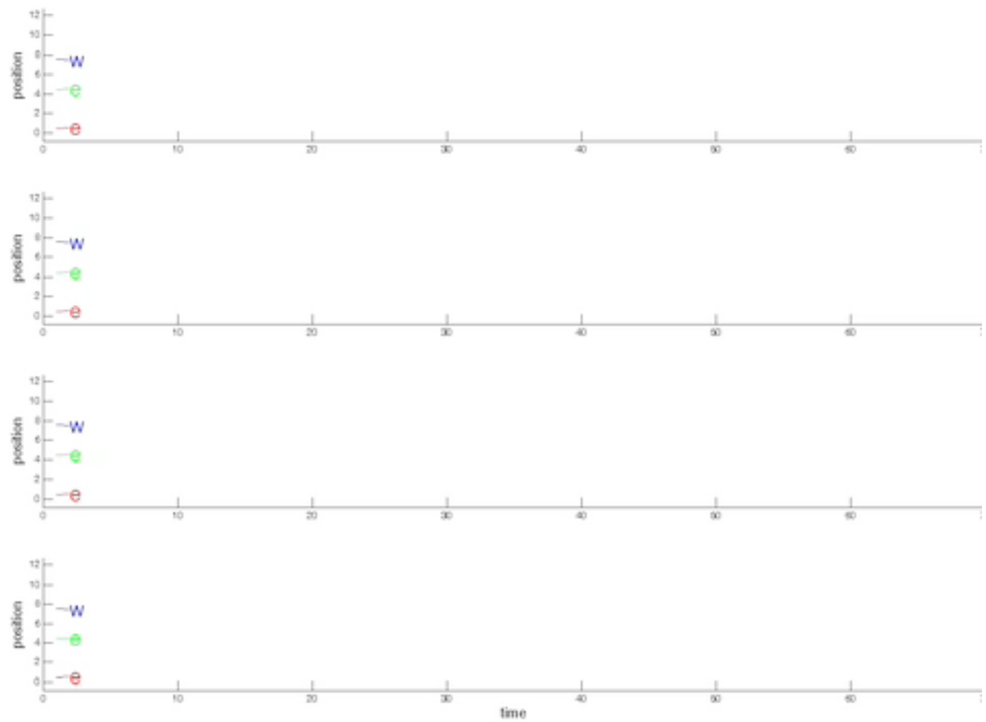
Sources of Nondeterminism (Due to Abstraction)

- Rule application order
- Rule application time
- Rule application place

Initial condition



Metabolobotics example produces many different structures (in many different ways) – but they are (a) connected and (b) contain all parts.



EGG example produces different results depending on timing and order.

Animation

Specification: Make statements about desired <u>sets</u> of possible trajectories.
Verification: Check whether our programs always produce trajectories contained within those sets.

Verification: States, Updates and Trajectories

$\sigma = \langle s_0, s_1, ... \rangle$ (or $\sigma : \mathbb{R}^+ \cup \{0\} \rightarrow S$)

$s[\![f]\!] \Leftrightarrow$ *s* is a state contained in the set of states specified by the formula *f*.

$s[\![a]\!]t \Leftrightarrow$ *(s,t)* is a pair of states contained in the set of pairs specified by the action *a*.

$\sigma[\![F]\!] \Leftrightarrow$ σ is a trajectory contained in the set of trajectories specified by the formula F.

Lamport’s Temporal Logic of Actions

(Lamport, ACM Toplas 16, 3 (May 1994) 872-923)

Semantics

$$s[\![f]\!] \triangleq f(\forall \text{`}v\text{'} : s[\![v]\!]/v) \qquad \sigma[\![F \wedge G]\!] \triangleq \sigma[\![F]\!] \wedge \sigma[\![G]\!]$$

$$s[\![\mathcal{A}]\!]t \triangleq \mathcal{A}(\forall \text{`}v\text{'} : s[\![v]\!]/v,\ t[\![v]\!]/v') \qquad \sigma[\![\neg F]\!] \triangleq \neg\sigma[\![F]\!]$$

$$\models \mathcal{A} \triangleq \forall s, t \in \mathbf{St} : s[\![\mathcal{A}]\!]t \qquad \models F \triangleq \forall \sigma \in \mathbf{St}^{\infty} : \sigma[\![F]\!]$$

$$s[\![Enabled\ \mathcal{A}]\!] \triangleq \exists t \in \mathbf{St} : s[\![\mathcal{A}]\!]t$$

$$\langle s_0, s_1, \ldots \rangle[\![\Box F]\!] \triangleq \forall n \in \mathsf{Nat} : \langle s_n, s_{n+1}, \ldots \rangle[\![F]\!]$$

$$\langle s_0, s_1, \ldots \rangle[\![\mathcal{A}]\!] \triangleq s_0[\![\mathcal{A}]\!]s_1$$

Additional notation

$$p' \triangleq p(\forall \text{`}v\text{'} : v'/v) \qquad \Diamond F \triangleq \neg\Box\neg F$$

$$[\mathcal{A}]_f \triangleq \mathcal{A} \vee (f' = f) \qquad F \leadsto G \triangleq \Box(F \Rightarrow \Diamond G)$$

$$\langle \mathcal{A} \rangle_f \triangleq \mathcal{A} \wedge (f' \neq f) \qquad \mathrm{WF}_f(\mathcal{A}) \triangleq \Box\Diamond\langle \mathcal{A} \rangle_f \vee \Box\Diamond\neg Enabled\ \langle \mathcal{A} \rangle_f$$

$$Unchanged\ f \triangleq f' = f \qquad \mathrm{SF}_f(\mathcal{A}) \triangleq \Box\Diamond\langle \mathcal{A} \rangle_f \vee \Diamond\Box\neg Enabled\ \langle \mathcal{A} \rangle_f$$

Guard:Rule pairs, programs, refinements, implementations and specification can all be written in TLA.

Verification: Progress via Lyapunov Co-Design

Lamport's TLA inference rule for progress uses a *discrete Lyapunov function* H_c.

Initial Condition **Discrete Lyapunov Function** **Goal Condition**

$$\frac{F \wedge (\mathsf{c} \in \mathsf{S}) \;\Rightarrow\; (H_{\mathsf{c}} \leadsto (G \vee \exists \mathsf{d} \in \mathsf{S} : (\mathsf{c} \succ \mathsf{d}) \wedge H_{\mathsf{d}}))}{F \;\Rightarrow\; ((\exists \mathsf{c} \in \mathsf{S} : H_{\mathsf{c}}) \leadsto G)}$$

McNew's Lyapunov Codesign Method:

1) Build individual behaviors, each decreasing it's own LF U_i
2) Compose behaviors, noting priority
3) Verify that U_i increasing -> U_j decreases for some j<I
4) $U=(U_1,\ldots,U_k)$ is a Lyapunov function under the lexicographic ordering

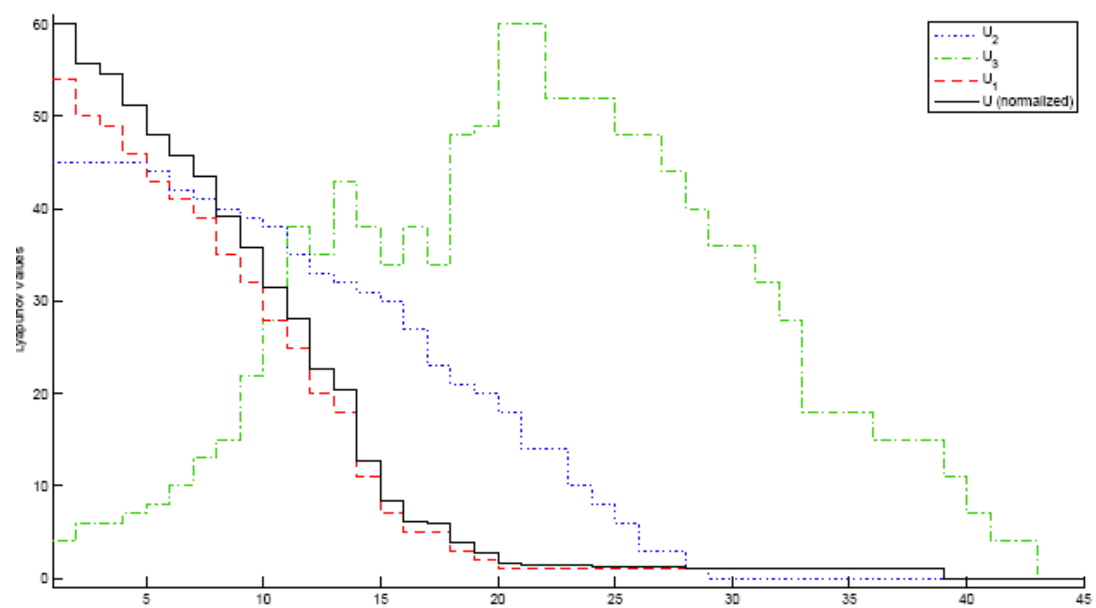
Verification: Progress via LLFs

$$\Phi = \begin{cases} (w,x) \quad (y,z) \rightharpoonup (w-1,x) \quad (y,z+x) \\ (0,x) \quad (y,z) \rightharpoonup (0,x) - (y,z) \\ (0,x) - (y,z) \rightharpoonup (0,x) - (\max(y-1,0), max(z-1,0)) \end{cases}$$

Lexicographic
Lyapunov
Function

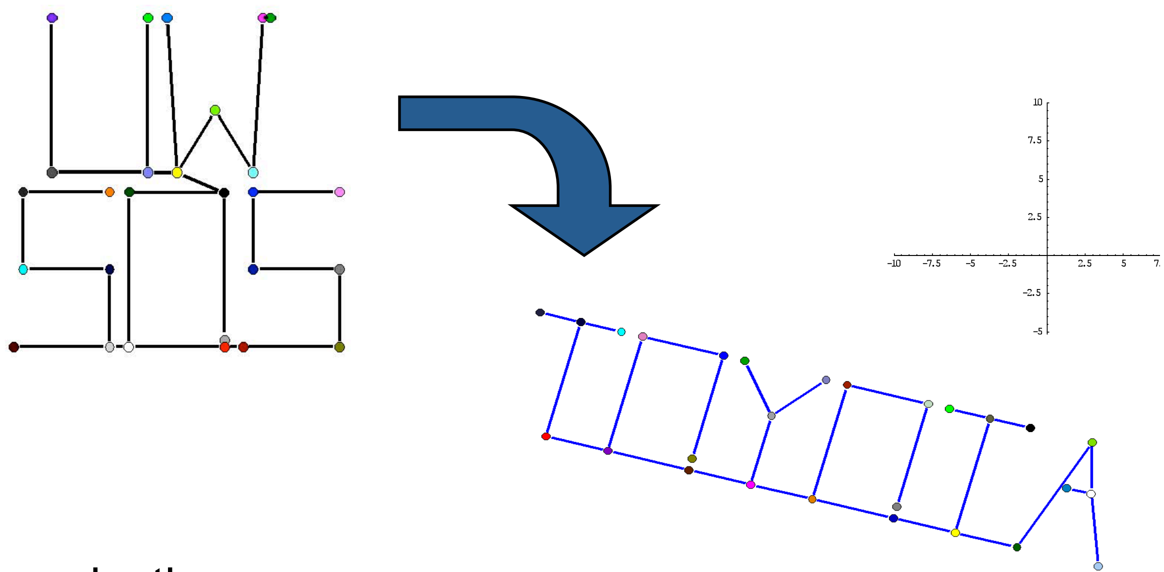
Let $\mathbf{U} = (\mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3)$ where

1. $\mathbf{U}_1 = \sum_{i \in V} i.a$
2. $\mathbf{U}_2 = (|V|^2 - |V|)/2 - |\mathrm{E}|$
3. $\mathbf{U}_3 = \sum_{i \in V} i.b$



This grammar results in a fully connected graph all of whose vertices are labeled (0,0).

Tree-tree Reconfiguration



Assumptions:

- Local communication.
- Unknown initial state.

Goals:

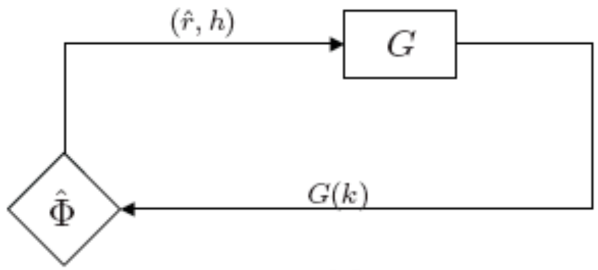
- Convergence to an isomorphism of the desired tree formation (progress).
- All intermediate states are tree formations (safety).
- Correct behavior from <u>all</u> initial conditions.

J. M. McNew and E. Klavins. Non-deterministic Reconfiguration of Tree Formations. American Control Conference, June 2008, pp. 690-697.

Verification: Separation of Discrete and Continuous

$$\begin{array}{c} f \\ \downarrow \\ \hat{f} \end{array}$$

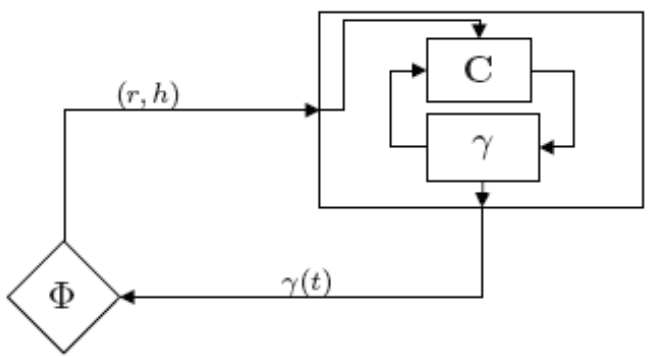
$(G_0, \hat{\Phi}) \models \hat{f}$



1. Write a program for only the discrete part of the problem (e.g. network reconfiguration).

$$\begin{array}{c} (f, \Phi) \\ \uparrow \\ (\hat{f}, \hat{\Phi}) \end{array}$$

$(\gamma_0, \Phi, C) \models f$

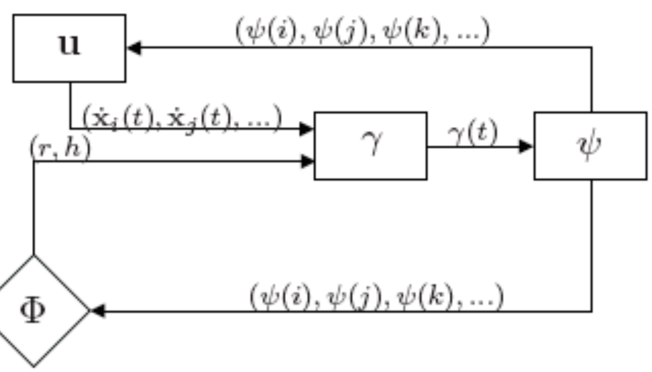


2a) Lift the discrete solution to the hybrid (discrete/continuous state).

2b) Determine a specification, C, that if met by u and ψ guarantees safety and progress in discrete solution.

$(\psi, \mathbf{u}) \models C$

$(\gamma_0, \Phi, \psi, \mathbf{u}) \models f$



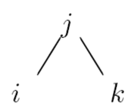
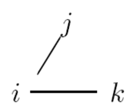
3) Design u and ψ to meet the specification C.

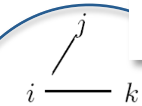
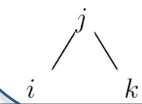
Tree-tree Reconfiguration

(Discrete Part)

Rule r_1
Vertices : i, j
Precondition:
$i.mode = t \wedge j.mode = a$
Denote $i.role$ by v. $\exists w \in N_{i.tree}(v)$ such that $(w.mode = a \wedge vw.order(w) = ij.order(j))$
$G[\{i,j\}] = i - j$
Effect:
$w.mode := t, \quad j.tree := i.tree$
$j.mode := t, \quad j.role := w$
$ij.offset := vw.offset, \quad ij.head := j$

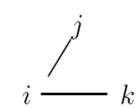
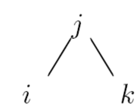
Each robot maintains its role and an estimate of the tree

Rule r_2
Vertices : i, j, k
Precondition:
$i.mode = t \wedge j.mode = a \wedge k.mode = a$
$ij.order(j) > b_i$
$jk.order(k) \geq b_i \vee ij.order(j) - jk.order(k) \geq b_i$
$G[\{i,j,k\}] =$ 
Effect:
$G[\{i,j,k\}] :=$ 

Rule r_3
Vertices : i, j, k
Precondition:
$i.mode = t \wedge j.mode = a \wedge k.mode = a$
$ij.order(j) < b_i \wedge ik.order($
$G[\{i,j,k\}] =$ 
Effect:
$G[\{i,j,k\}] :=$ 

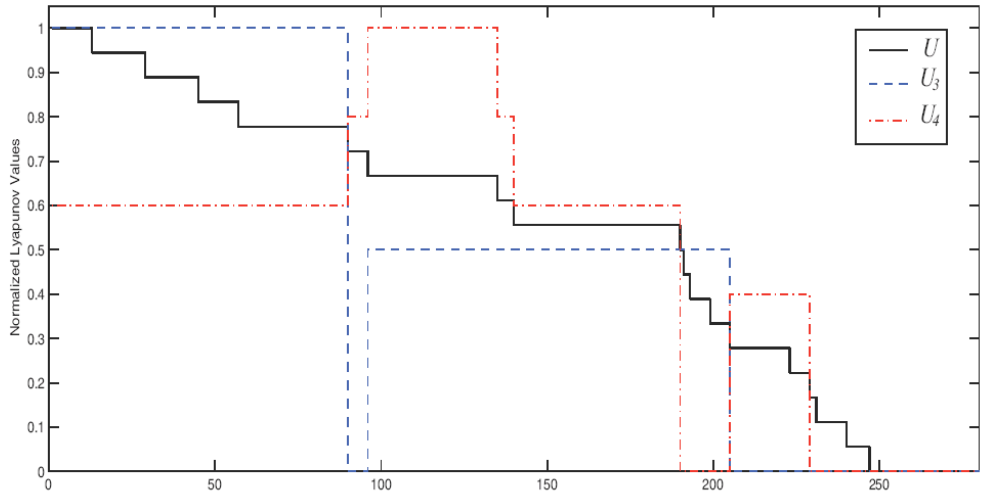
Local network reconfigurations are guaranteed to preserve tree

Rule r_4
Vertices : i, j, k
Precondition:
$i.mode = t \wedge j.mode = a \wedge k.mode = a$
$ij.order(j) > b_i$
$jk.order(k) < b_i \wedge ij.order(j) - jk.order(k) < b_i$
$G[\{i,j,k\}] =$ 
Effect:
$i.mode := m$

Rule r_5
Vertices : i, j, k
Precondition:
$i.mode = m \wedge j.mode = a \wedge k.mode = a$
$G[\{i,j,k\}] =$ 
Effect:
$G[\{i,j,k\}] :=$ 
$i.mode := a$

J. M. McNew and E. Klavins. Non-deterministic Reconfiguration of Tree Formations. American Control Conference, June 2008, pp. 690-697.

Verification: Progress



U_1—The number of vertices yet to be matched to the target graph.

U_2—The number of t-a edges to which r_2 might apply.

U_3—The average distance from b_i on t-a edges to which r_2 might apply.

U_4—The summed distance from b_i on t-a edges to which r_3 applies.

U_5—The number of subgraphs to which r_4 or r_5 apply.

U_6—The number of subgraphs to which r_5 applies.

J. M. McNew and E. Klavins. Non-deterministic Reconfiguration of Tree Formations. American Control Conference, June 2008, pp. 690-697.

Tree-tree Reconfiguration: Continuous Part

Intermediate specification on u and ψ

Safety: $\Box(ij \in E \rightarrow ij \in E_\psi)$

Progress: $\dot{x} = u \leadsto (\dot{x} \neq u \vee \exists r.x \in \text{cont\_guard}(r))$

Rule r_5 *(Merge Ahead)*

Vertices : i, j, k

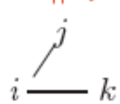
Variables : b_i

Precondition:

$i.mode = m \wedge j.mode = a \wedge k.mode = a$

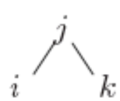
$||\mathbf{x}_i - \mathbf{x}_j|| < \Delta \wedge ||\mathbf{x}_i - \mathbf{x}_k|| < \Delta \wedge ||\mathbf{x}_k - \mathbf{x}_j|| < \Delta$

$E \ =$



Effect:

$E \ :=$



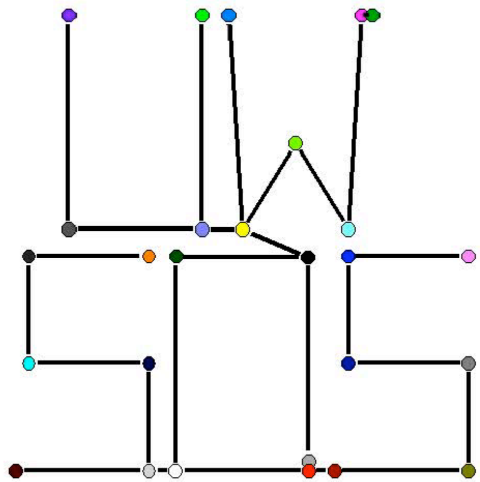
$i.mode := a$

$$\dot{\mathbf{x}}_i = -\sum_{j \in N_\psi(i)} \frac{2(\Delta - ||\alpha||) - ||\mathbf{x}_i - \mathbf{x}_j - \alpha||}{(\Delta - ||\alpha|| - ||\mathbf{x}_i - \mathbf{x}_j - \alpha||)^2}(x_i - x_j - \alpha)$$

Simple implementation (can prove it satisfies C)

Continuous predicate updates to rules.

Simulation



The Separation of Verification and Performance Tuning

Code for a *non-deterministic walk*

$$
\begin{aligned}
&mode = search : move\_lt \\
&mode = search : move\_rt \\
&mode = search : move\_up \\
&mode = search : move\_dn
\end{aligned}
$$

TLA spec with fairness on movement

$$
\begin{aligned}
\Pi \quad = \quad & \Box(x_{min} \leq x \leq x_{max} \wedge y_{min} \leq y \leq y_{max}) \\
= \quad & \bigwedge_{x_{min} < w \leq x_{max}} \mathrm{SF}(x = w \wedge move\_lt) \\
= \quad & \bigwedge_{x_{min} \leq w < x_{max}} \mathrm{SF}(x = w \wedge move\_rt) \\
= \quad & \bigwedge_{y_{min} < z \leq y_{max}} \mathrm{SF}(y = z \wedge move\_dn) \\
= \quad & \bigwedge_{y_{min} \leq w < y_{max}} \mathrm{SF}(y = z \wedge move\_up) \\
& \vdash \Diamond(x = x^* \wedge y = y^*)
\end{aligned}
$$

Inference rule

$$
\frac{\text{true}}{\text{mode} = \text{search} \leadsto (\text{mode} \neq \text{search} \vee (x = x^* \wedge y = y^*))}
$$

Safarik, Napp and Klavins, In progress.

The Separation of Verification and Performance Tuning

Code for a *feedback controlled random walk*

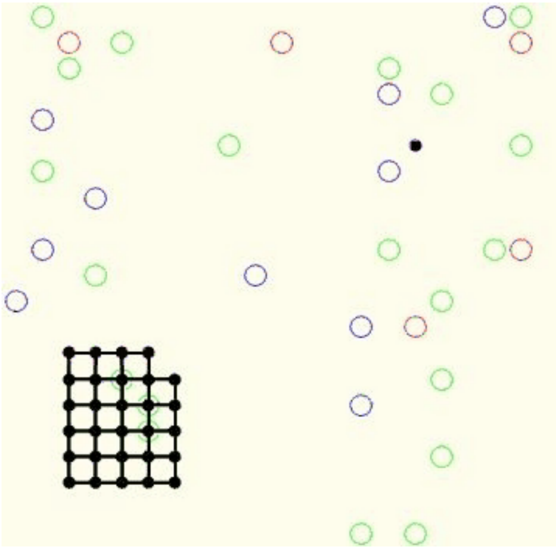
$$\begin{array}{ll} mode = search : move\_lt & u_l(x,y,t)dt \\ mode = search : move\_rt & u_r(x,y,t)dt \\ mode = search : move\_up & u_u(x,y,t)dt \\ mode = search : move\_dn & u_d(x,y,t)dt \end{array}$$

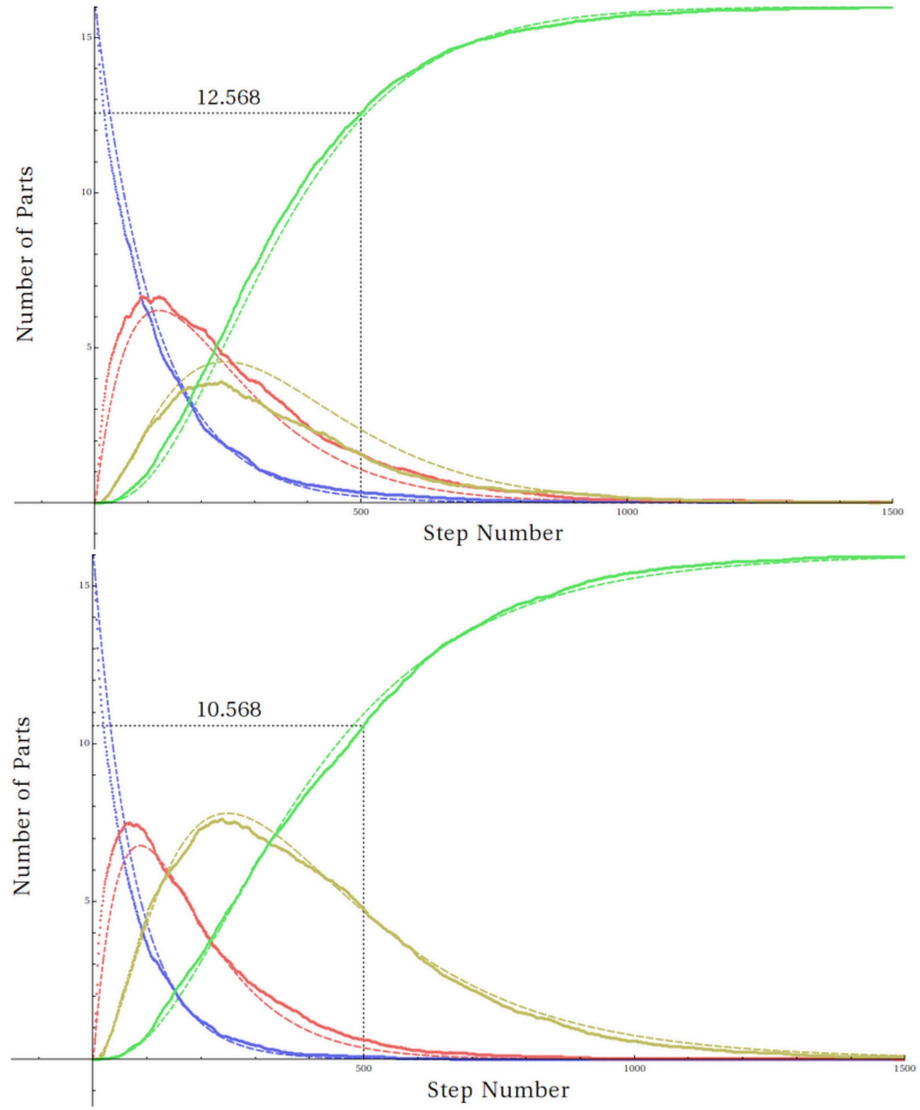
Probability that the rule will fire in the next dt seconds given the state is (x,y)

Thm: Set of unfair trajectories of the non-deterministic system has measure zero in any probabilistic implementation in which $u_*(x,y,t) > 0$ for all x, y and t.

Thus, even an (almost) deterministic search algorithm is guaranteed to be correct.

An Example: Tuning Metabolism While Preserving Correctness





Reduced model (k_i depends on allocation of welders, breakers, carriers).

$$B \xrightarrow{k_1} F$$
$$F \xrightarrow{k_2} P$$
$$P \xrightarrow{k_3} D$$

Safarik, Napp and Klavins, In progress.

Summary

- Examples/Testbeds
  – Next: EFRI testbed available / simulator enhanced
- Lyapunov Co-Design
  – Next: Lyapunov co-design in stochastic systems (e.g. Parrilo)
- Separation of continuous and discrete design
  – Next: Formalize in TLA
- Separation of correctness and performance
  – Next: Formalize in TLA
  – Next: Feedback controlled performance that does not affect correctness
- Stochastic diagnosis, model reduction
  – David's talk