Be nice to tie in this with the more analytic Poincaré / Melnikov analysis of this problem. Good. JM.

Simulation of a Chaotic Planar Double Pendulum

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Abstract

In recent years, academic as well as industrial interests on the subject of chaos has been growing as more and more systems from various disciplines exhibit the well known signature of chaos. In this report, the chaotic planar double pendulum is considered. Simulations have been performed on the unactuated system and results verifies the presence of chaos.

1 Introduction

In recent years, academic as well as industrial interests on the subject of chaos has been growing as more and more systems from various disciplines exhibit the well known signature of chaos. From the Lorentz system [6] to applications in lasers [4], disc dynamo [5], and baroclinic waves [7, 8], the importance of the understanding of the theory of chaos is well established and reinforced as the understanding grows deeper.

In this report, the chaotic planar double pendulum is considered. Shinbrot et al [9], Burov [2], and others have considered the problem both from an experimental as well as theoretical perspective. The system is described in Figure 1.

2 Derivation

The Euler-Lagrange approach is used to derive the governing equations of the system. The potential (V) and kinetic energy (K) and the Lagrangian (L) of

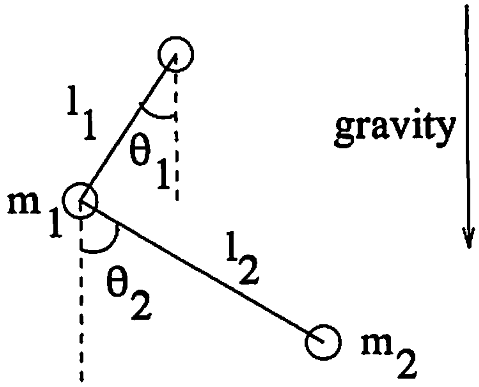

Figure 1: The chaotic planar double pendulum

the system as depicted in Figure 1 are

$$\begin{aligned} V &= l_1(1-\cos\theta_1)m_1g + [l_1(1-\cos\theta_1) + l_2(1-\cos\theta_2)]m_2g \\ K &= \frac{1}{2}(m_1+m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2l_2^2\dot{\theta}_2^2 + m_2l_1l_2\dot{\theta}_1\dot{\theta}_2\cos\theta_1 - \theta_2 \\ L &= K - V \end{aligned} \quad (1)$$

The Euler-Lagrange equations as a system of first order ODEs are thus

$$\dot{\theta}_1 = \gamma_1 \quad (2)$$

$$\dot{\theta}_2 = \gamma_2 \quad (3)$$

$$\dot{\gamma}_1 = \frac{g(\sin\theta_2\cos(\theta_1-\theta_2) - \mu\sin\theta_1)}{l_1(\mu - \cos^2(\theta_1-\theta_2))} - \frac{(l_2\gamma_2^2 + l_1\gamma_1^2\cos(\theta_1-\theta_2))\sin(\theta_1-\theta_2)}{l_1(\mu - \cos^2(\theta_1-\theta_2))} \quad (4)$$

$$\dot{\gamma}_2 = \frac{g\mu(\sin\theta_1\cos(\theta_1-\theta_2) - \sin\theta_2)}{l_2(\mu - \cos^2(\theta_1-\theta_2))} - \frac{(\mu l_1\gamma_1^2 + l_2\gamma_2^2\cos(\theta_1-\theta_2))\sin(\theta_1-\theta_2)}{l_2(\mu - \cos^2(\theta_1-\theta_2))} \quad (5)$$

where $\mu = 1 + \frac{m_1}{m_2}$ and g is the gravitional constant. For this report, the various parameters (m_1, m_2, l_1, l_2, g) are chosen to be $(1.3, 1, 1.2, 1, 9.81)$ respectively.

3 Simulations and Results

The simulations are carried out in C using simulate written by Professor Richard Murray at Caltech and contributed by Sudipto Sur and Robert Behnken. For the unactuated system, twelve simulations are run with each simulation starting from a slightly different set of initial conditions from the others. These initial

run	θ_1	θ_2	γ_1	γ_2
1	90	0	0	0
2	90	5	0	0
3	85	0	0	0
4	85	5	0	0
5	80	0	0	0
6	80	5	0	0
7	75	0	0	0
8	75	5	0	0
9	70	0	0	0
10	70	5	0	0
11	65	0	0	0
12	65	5	0	0

Table 1: Initial conditions for the simulations

conditions are listed in Table 1. Each simulation is carried out for 25 seconds in real time. The presence of chaos is verified as shown in the simulation results with the first two of the initial conditions (Figure 2).

The results of each simulation is made into a movie in Matlab and various frames of each movie are compared. A sample of the comparison plots are included as Figures 3, 4, 5 where simulations with the first four initial conditions in Table 1 are carried out. In each of these plots, the colors represent the time indices of the simulation with yellow as the starting color, dark red the ending, and the colors are interpolated between the starting and ending time indices. Figure 3 shows the simulation of time index 0 to 0.7 seconds, Figure 4 10 to 10.7 seconds, Figure 5 20 to 20.7 seconds, and Figure 6 24.3 to 25 seconds.

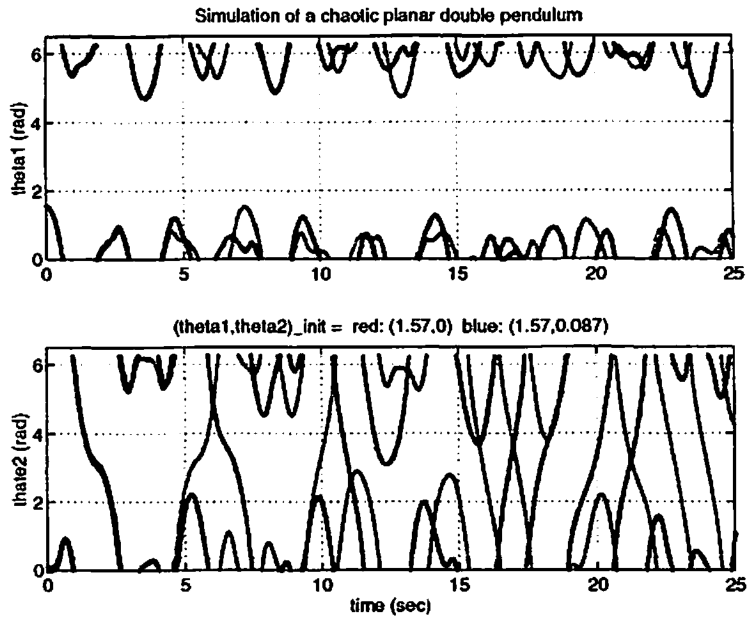

Figure 2: Simulation results with the first two initial conditions

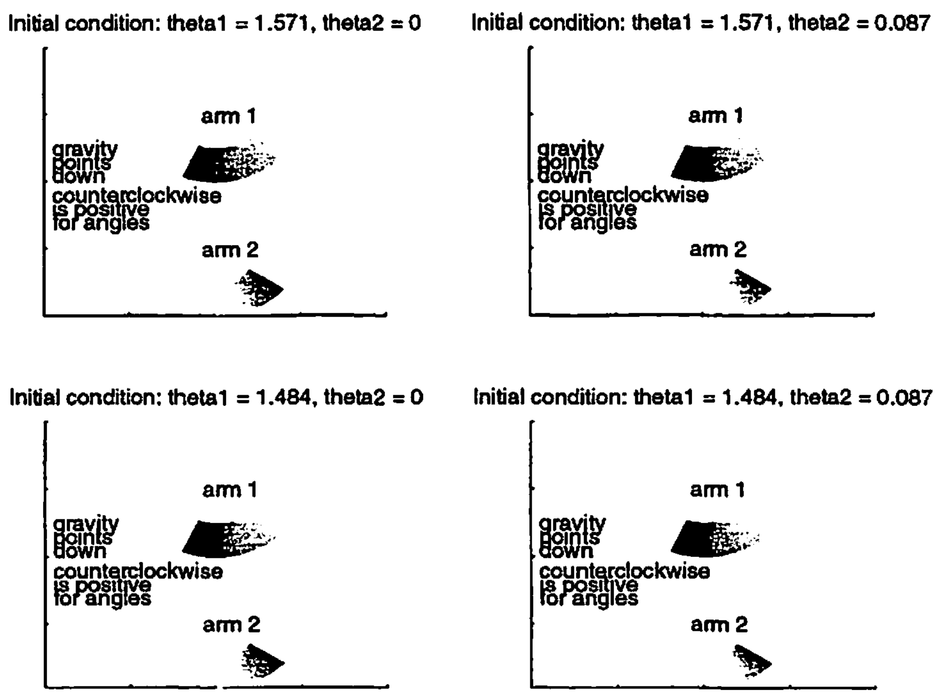

Figure 3: Frames for 0 to 0.7 sec

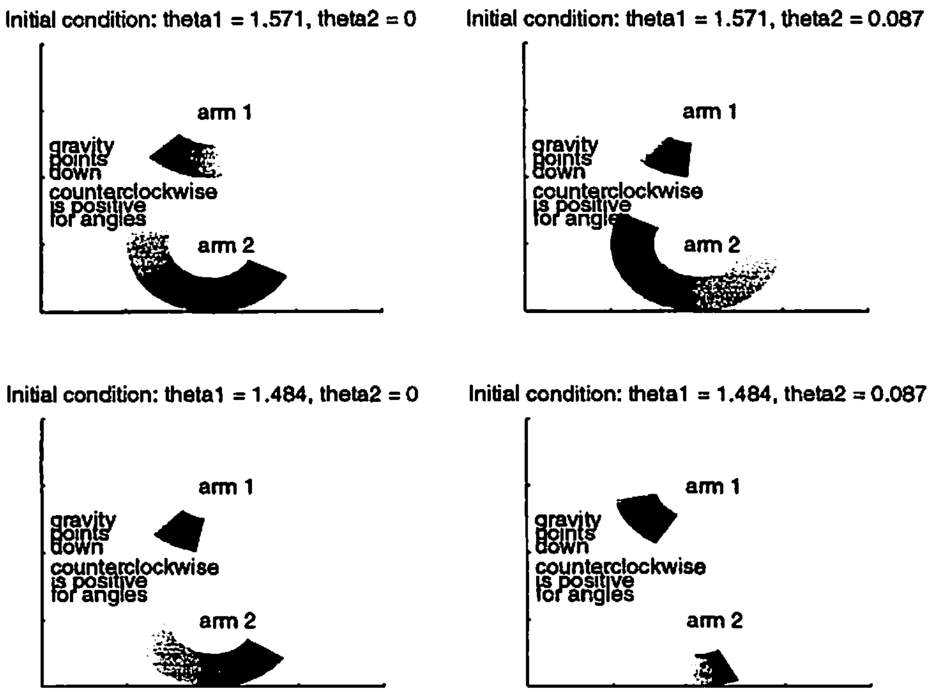

Figure 4: Frames for 10 to 10.7 sec

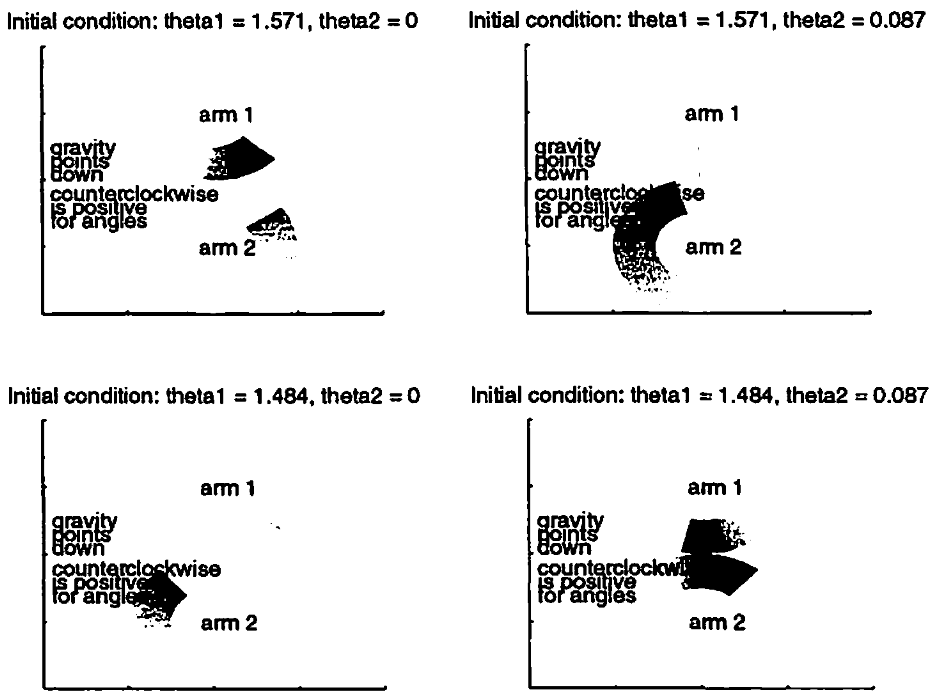

Figure 5: Frames for 20 to 20.7 sec

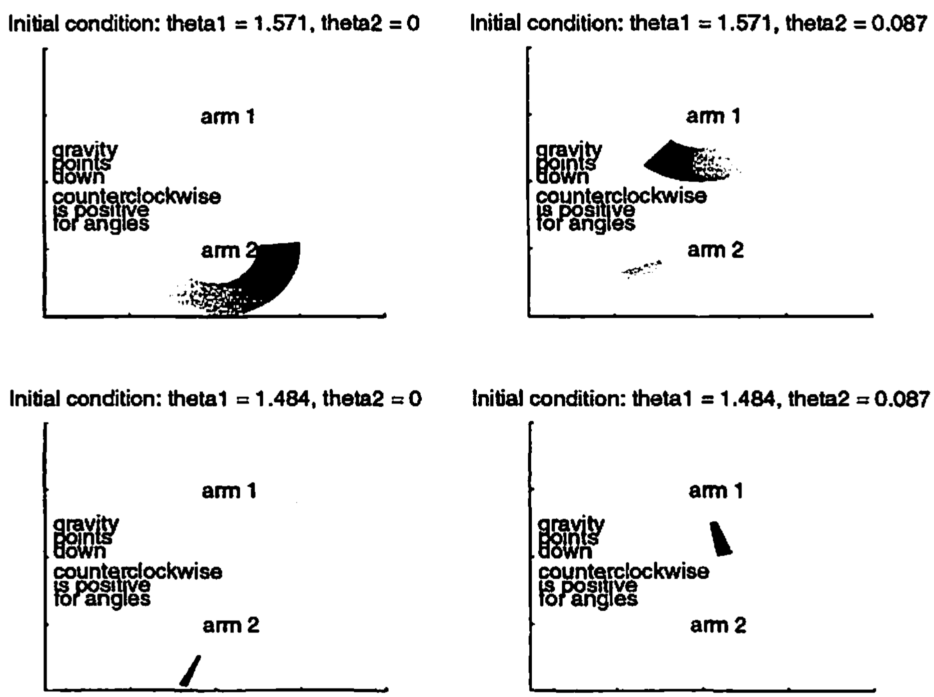

Figure 6: Frames for 24.3 to 25 sec

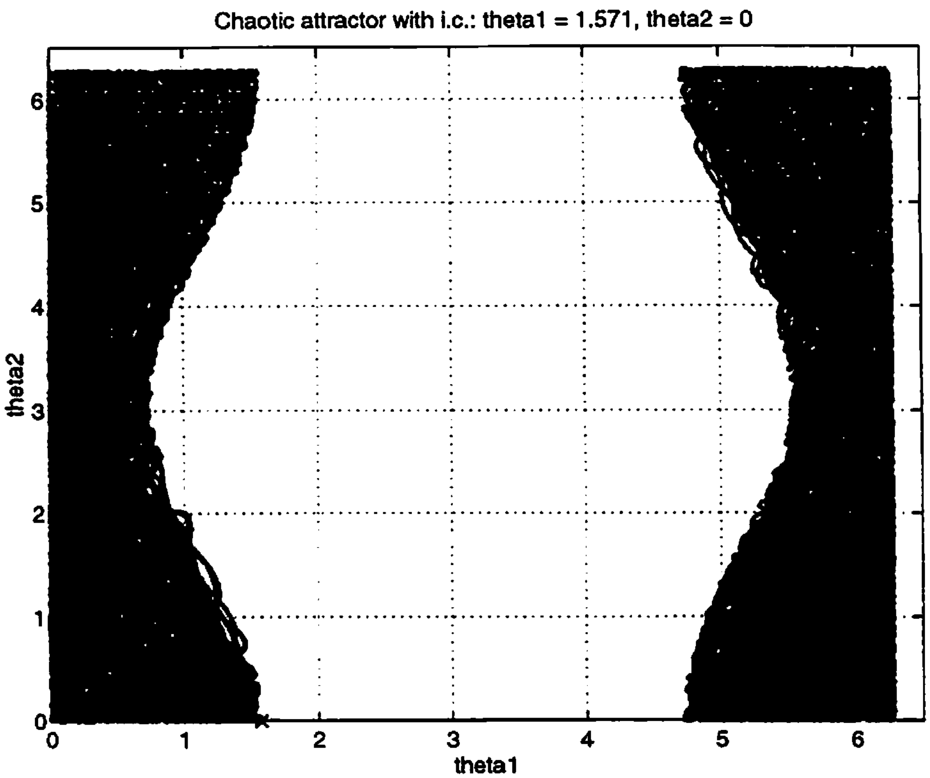

Figure 7: Chaotic attractor from simulating with first i.c.

The simulations of the first four initial conditions are also carried out for a prolonged period of time (1000 seconds) to estimate the chaotic attractor in the (θ_1, θ_2) coordinates. The estimated chaotic attractor resulting from simulating the first initial condition is shown in Figure 7. All the estimated

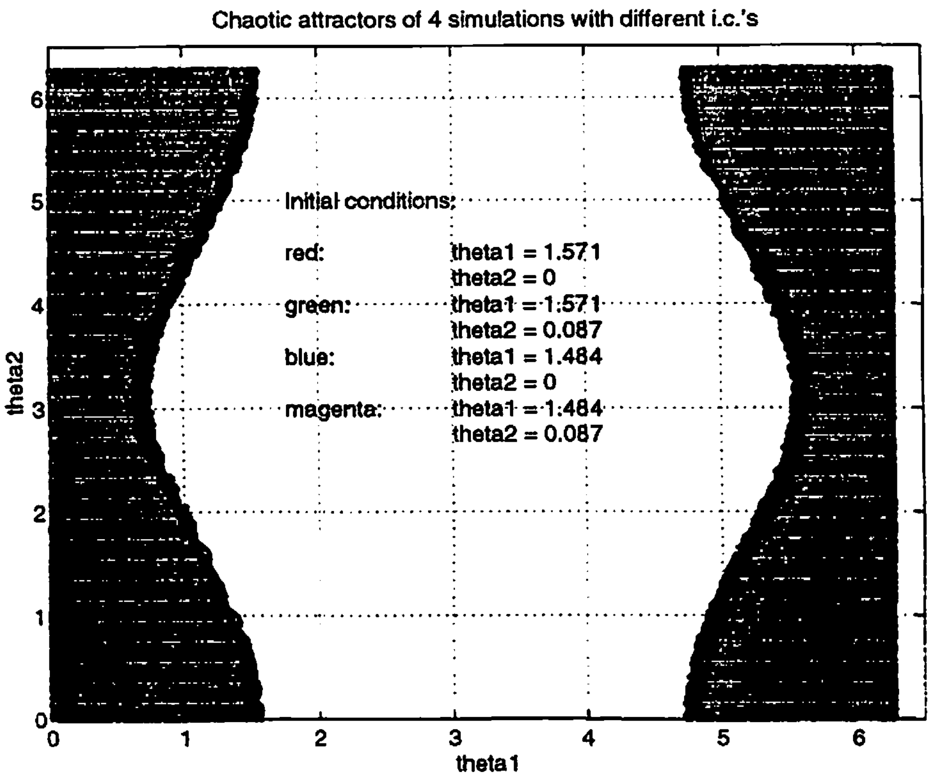

Figure 8: 4 chaotic attractors from simulations

chaotic attractors resulting from simulating the first four initial conditions are plotted together in Figure 8 to verify the validity of the presence of the chaotic attractor [1].

4 Further work

Similar to the Lorentz system [6], the dynamics of the planar double pendulum is chaotic and yet the governing equations are remarkably simple. As mentioned in the Introduction section, systems from different disciplines have been found to resemble the Lorentz system, and controls applications is thus a relevant issue [3, 10, 1, 11]. Similarly, one can consider the controllability of the planar double pendulum and the performance of controllers when applied to the system with actuation entering in various different places.

[1]theoretically speaking, if the simulations were carried out to an inifinite amount of time, these chaotic attractors should be identical as sets on the (θ_1, θ_2) phase space

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