Oberwolfach Meeting, July 20 -July 26, 2008
Applied Dynamics and Geometric Mechanics

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Summary and Rationale. In many of our past Oberwolfach meetings we have stressed specific topics, some of them focused on applied subjects. For example, these topics included water waves, robotics, oceanography, and atmospheric physics. This meeting will "return to the core" in the sense that each speaker and invitee should present a new mathematical or computational tool in the area of geometric mechanics or dynamical systems. They will also be expected to show how it is useful in applications, or how it bridges the gap between these areas and some interesting aspect of the pure mathematical side of dynamical systems or geometric mechanics. We would not be restricted to any particular application area, but would attempt to cross fertilize a number of different topics in Engineering and the Sciences

This meeting philosophy is based on the fact that the number of interesting new applicable basic tools and techniques in these areas continues to grow and be developed in interesting ways, both from the point of view of fundamental theory as well as from their applicability. Some examples of specific areas are decomposition and model reduction methods for complex interconnected systems, new optimization methods, continued progress in the development of discrete mechanics and related geometric integrators, etc. Of course we are not strictly limited to these areas.

Specific Topics.

Core Dynamical Systems. The basic theory of dynamical systems continues to develop with numerous new ideas that have importance in applications. For example, the concept of Finite Time (or Space) Liapunov Exponents and their derivative, Lagrangian Coherent Structures are tools that extend invariant manifold theory in a nice way to the time dependent case. These ideas have found application in, for example, the detection of recirculation zones in the heart. Another example is the extension of classical concepts from bifurcation and stability theory to time dependent systems with applications to phase transitions and materials with memory. Another area is the basic theory of resonances and their role in the development of coherent (large scale) modes.

Complex Interconnected Systems. The computational limitations of dealing with complex interconnected systems, such as fuel cells, aircraft, etc have already been reached. New methods are being developed that move away from the monolithic approach (that is, thinking of systems of thousands of coupled equations, be they ode's or pde's to the idea of many computations running in parallel with massive message passing and information exchange. But this is an area where dynamical systems theory is playing a key role; for example, methods are being developed that can determine, automatically, which subsystems are strongly coupled and which are weakly coupled and how to arrange the dynamical computations accordingly.

Structured Model Reduction. Related to the previous topic is that of forming dimensionally reduced models for computational feasibility. This is important in, for example, complex fluid flows, where resolvability of the Navier-Stokes equations is simply not possible and where models such as LES (Large Eddy Simulation) and LANS-alpha (Lagrangian averaged Navier-Stokes) models have been developed. POD (proper orthogonal decomposition) methods are also undergoing continued development and their limitations being better understood. The introduction of hierarchical and wavelet methods are also quite promising. In carrying out such reductions, it is of special interest for this proposal to do so in a way that preserves structure (such as symmetry structures, mechanical structures, etc). Such methods that preserve structure have already proven useful in some pilot studies.

Uncertainty and Stochastic Methods. Another area in which dynamical systems can play a key role is how to deal with systems whose very models are uncertain; for example, think of modeling an asteroid moving mainly in the field of the sun and Jupiter. How much error is introduced by neglecting the effects of Saturn? Is this more important than errors in initial conditions? How do error balls propagate under the dynamics? Of course in many systems, such as laboratory based mechanical systems, there are sources of uncertainty due to noise as well and one needs techniques to distinguish and deal with this from the other sources of uncertainty and from numerical uncertainties.

Core Geometric Mechanics. Despite its maturity, especially over the last few decades, the basic theory of geometric mechanics continues to thrive and develop. For example, the reduction of mechanical systems with symmetry continues to grow and find applications in, for instance, computation and control. Areas that are currently undergoing particularly interesting growth that have links with applications are reduction by stages (for example, applied to locomotion in fluids), singular cotangent bundle reduction (relevant for instance, to the dynamics of multiple pendula), and the Gromov symplectic squeezing theorem (relevant for uncertainty propagation). Also, the theory of integrable systems continues to be a valuable link between geometric mechanics and pure mathematics.

Geometric Integrators. The area of structured integrators for mechanical systems continues to undergo strong growth for problems in which preserving structure is important, such as symplectic structures for backward error analysis, in fluid systems in which it is important to conserve circulation and in the development of asynchronous integrators. New insight into the development of methods that are robust to uncertainty are also quite promising.

Optimization and Control of Mechanical Systems. New methods for optimization for mechanical systems such as DMOC (discrete mechanics and optimal control) are showing promise for the optimization of, for example, complex systems of vehicles—for instance a swarm of micro-air vehicles that is sent to investigate a biohazard. In these methods, the use of techniques that are successful already in internet congestion control as well as the use of parallel computation are quite attractive. Other types of control, such as stabilization continue to benefit from basic advances in the theory and to make strong links with, for example, geometric integrators and discrete mechanics.