Optimization of Space Trajectories: Invariant Manifolds + DMOC

Ashley Moore

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Introduction

- Objective: Design a low energy space trajectory
  - Use Invariant Manifold techniques to determine initial trajectory
  - Apply DMOC to generate an optimal solution
- “Shoot the Moon”
  - Test method by designing trajectory from Earth to Moon
    - Split problem into two coupled planar circular restricted 3-body systems and patch them together
      - Sun – Earth – Spacecraft (SE)
      - Earth – Moon – Spacecraft (EM)
  - Based on PhD thesis of Shane Ross and “Shoot the Moon” paper by Koon, Lo, Marsden, and Ross

DMOC
Overview

- DMOC is based on a direct discretization of the Lagrange-d'Alembert principle for a dynamical system
  - Produces the forced discrete Euler-Lagrange equations
    - Serve as optimization constraints given a cost function
- Need good initial guess that obeys dynamics to work successfully

Junge, O., Marsden, J.E., and Ober-Blöbaum. "*Discrete Mechanics and Optimal Control.*"

DMOC
Motivating Example

- Orbit Problem
  - Goal: Optimally move a spacecraft from circular orbit r = 5 to r = 10 with 2 revolutions around the earth.
  - Minimize the control effort
- Lagrangian

$$L(q,\dot{q}) = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\varphi}^2\right) + \frac{GMm}{r}$$

- Force

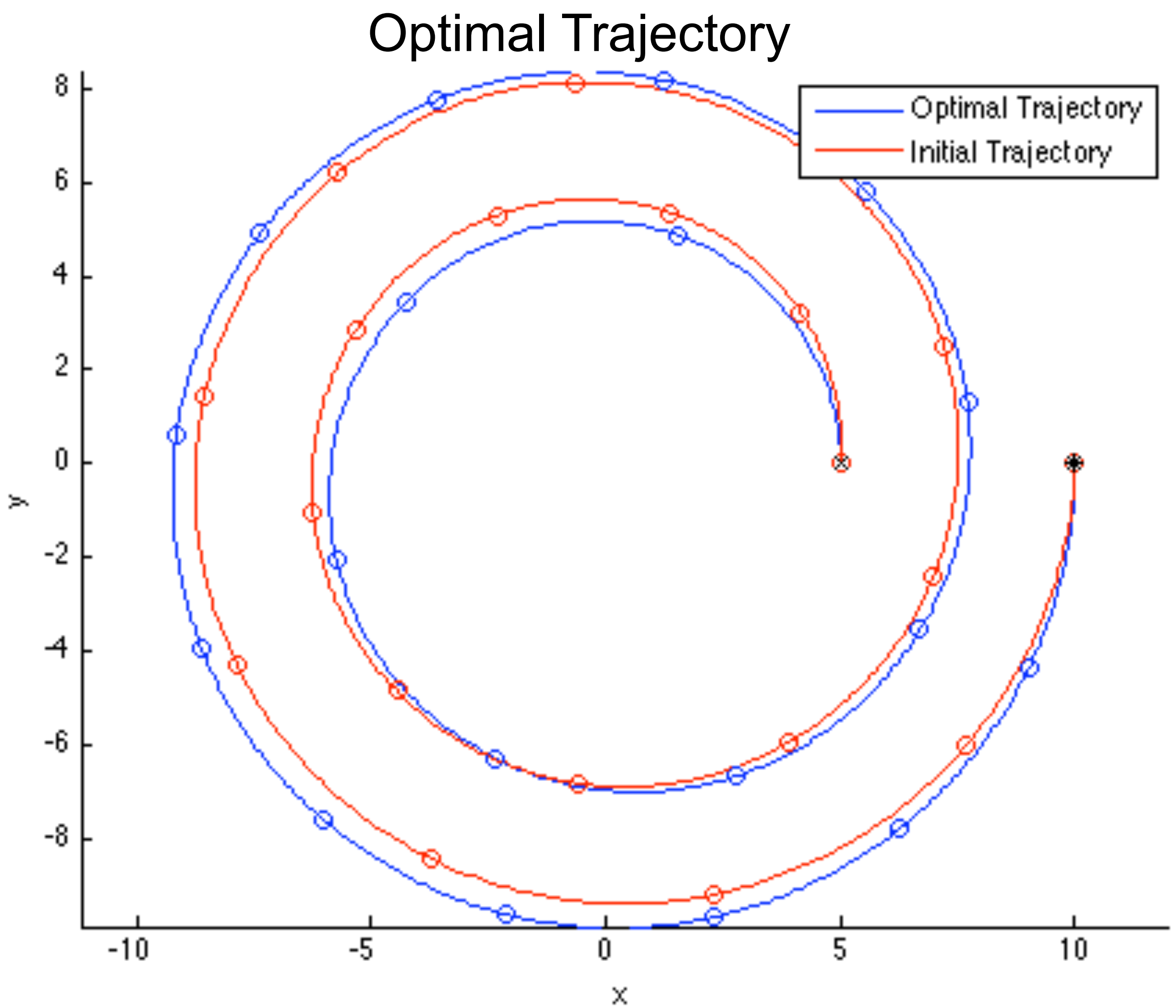
$$f = \begin{pmatrix} 0 \\ ru \end{pmatrix}$$

- Cost function

$$J(q,u) = \int_0^T u(t)^2\,dt$$

Junge, O., Marsden, J.E., and Ober-Blöbaum. "*Discrete Mechanics and Optimal Control.*"

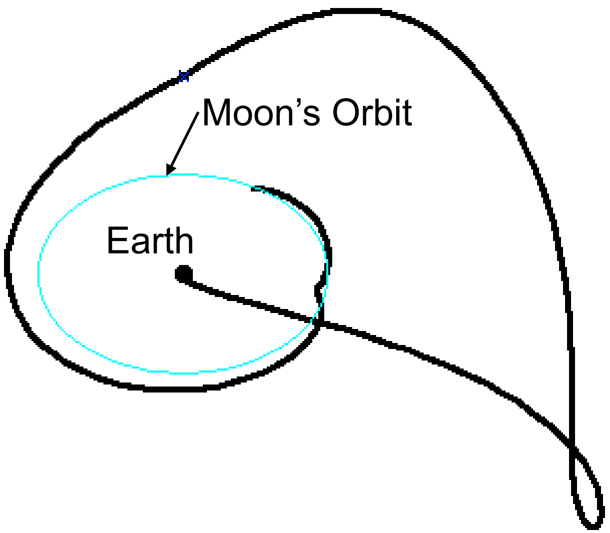
DMOC



Optimal Trajectory
Optimal Trajectory
Initial Trajectory
8
6
4
2
0
-2
-4
-6
-8
y
-10
-5
0
5
10
x

DMOC
Motivating Example

- What if the desired trajectory looks like this:



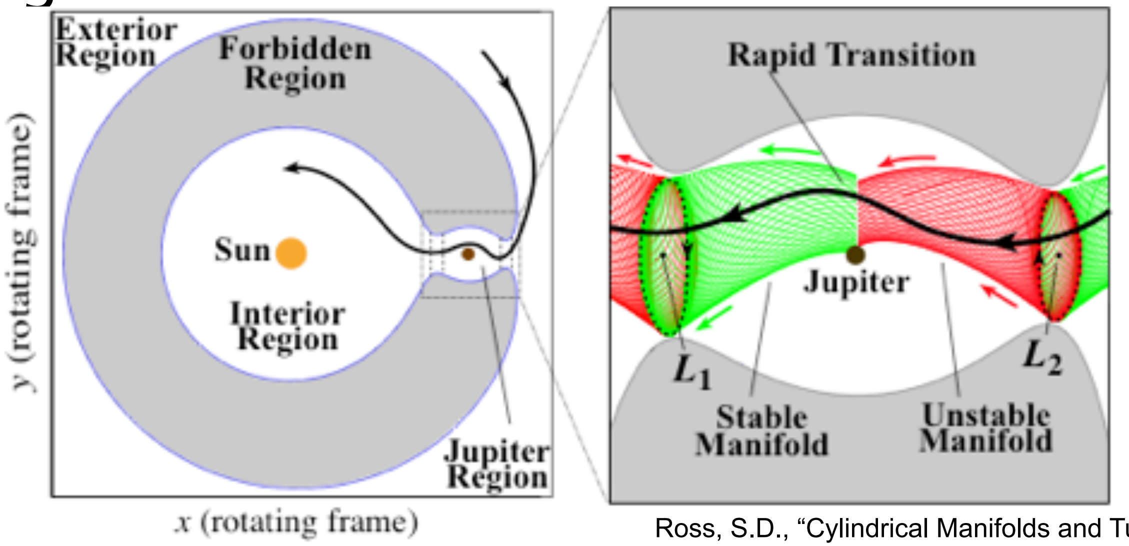
- DMOC will need an excellent initial guess

DMOC + Invariant Manifolds

- Invariant Manifold method generates initial condition (patch point)
- Integrate patch point in Bicircular 4 body model for initial trajectory
- Apply initial trajectory to DMOC using same model
- What should be minimized?
  - Depends on payload
    - If people – minimize time or distance
    - If supplies/robotics – minimize fuel
- Constraints
  - Euler-Lagrange equations
  - Initial position and momentum
  - Final position and momentum
- What do we expect?
  - Perhaps DMOC will generate trajectory with gradual ΔV instead of concentrated ΔV at patch point
  - Shorter flight time or distance

Invariant Manifolds
Basic Idea

- Stable and unstable manifolds emanate from the periodic orbits of Lagrange points of the PCR3BP
- Manifold tubes connect regions of space
  - Spacecraft may travel from one region to another through tubes



Ross, S.D., "Cylindrical Manifolds and Tube Dynamics in the Restricted Three-Body Problem" (PhD Thesis, California Institute of Technology, 2004), pp. 121.

Invariant Manifolds
Details

- Use rotating coordinate system centered on barycenter of m_1 and m_2.
- Normalize system using mass parameter

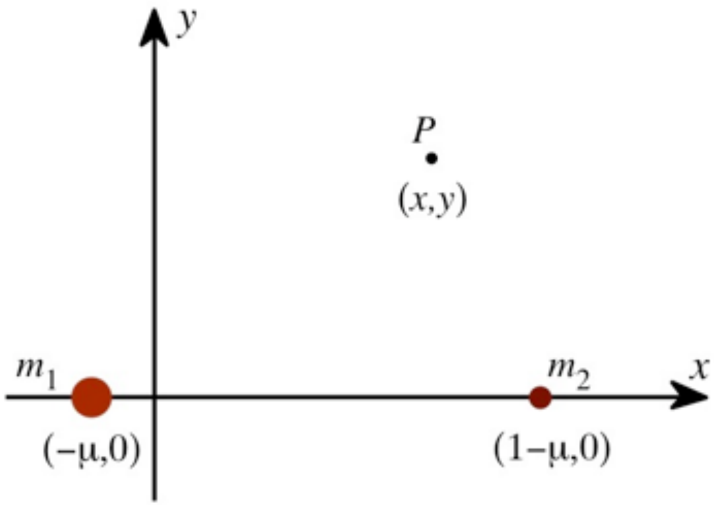
$$\mu = \frac{m_2}{m_1 + m_2} \quad \text{where } m_1 > m_2$$

- Neglect spacecraft mass
- PCR3BP equations

$$\ddot{x} - 2\dot{y} = \Omega_x$$

$$\ddot{y} + 2\dot{x} = \Omega_y$$

$$\Omega = \frac{x^2 + y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$



Ross, S.D., "Cylindrical Manifolds and Tube Dynamics in the Restricted Three-Body Problem" (PhD Thesis, California Institute of Technology, 2004), pp. 8.

Invariant Manifolds
Details

- Energy Integral

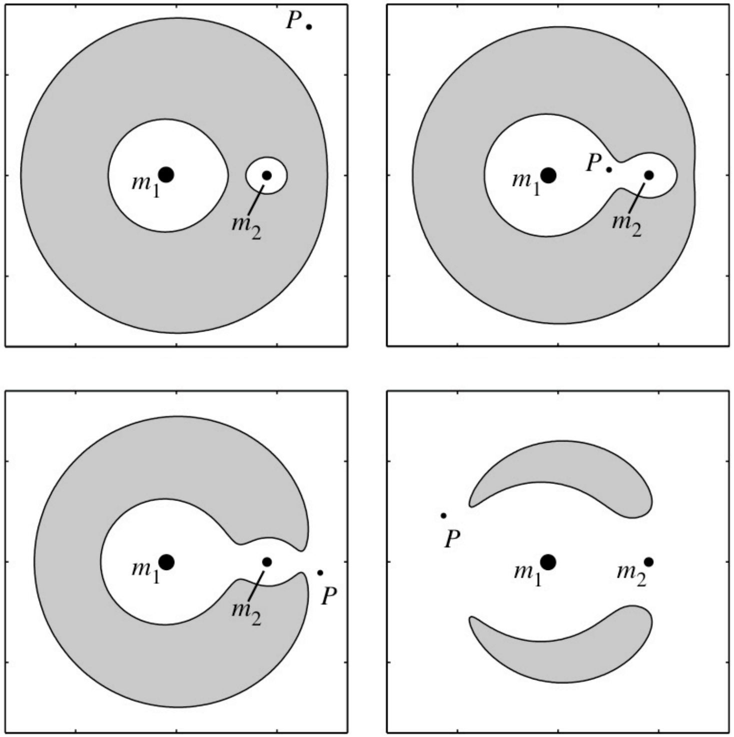
$$E(x,y,\dot{x},\dot{y}) = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \overline{U}(x,y)$$

$$\overline{U}(x,y) = -\frac{1}{2}\left(\mu_1 r_1^2 + \mu_2 r_2^2\right) - \frac{\mu_1}{r_1} - \frac{\mu_2}{r_2}$$

$$\mu_1 = 1 - \mu, \quad \mu_2 = \mu$$

- Energy divides the phase space into regions
  - The energy restricts the motion of a spacecraft

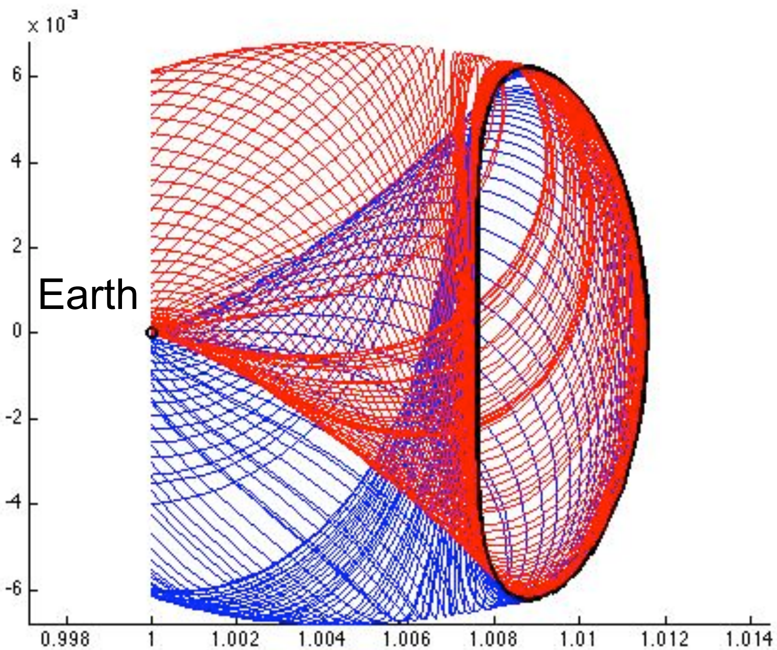
Hill's Regions



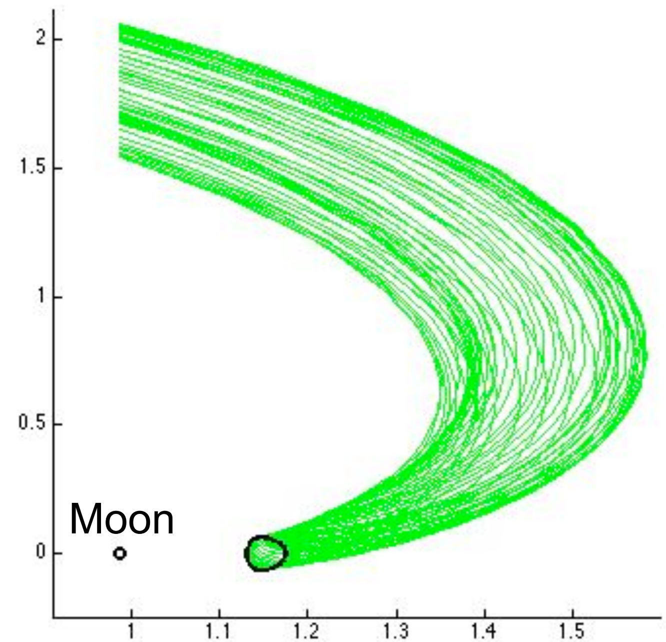
Ross, S.D., "Cylindrical Manifolds and Tube Dynamics in the Restricted Three-Body Problem" (PhD Thesis, California Institute of Technology, 2004), pp. 14.

Invariant Manifolds
"Shoot the Moon"

- Locate L2 Lagrange point for the SE and EM systems
- Compute periodic orbit and 'grow' manifolds



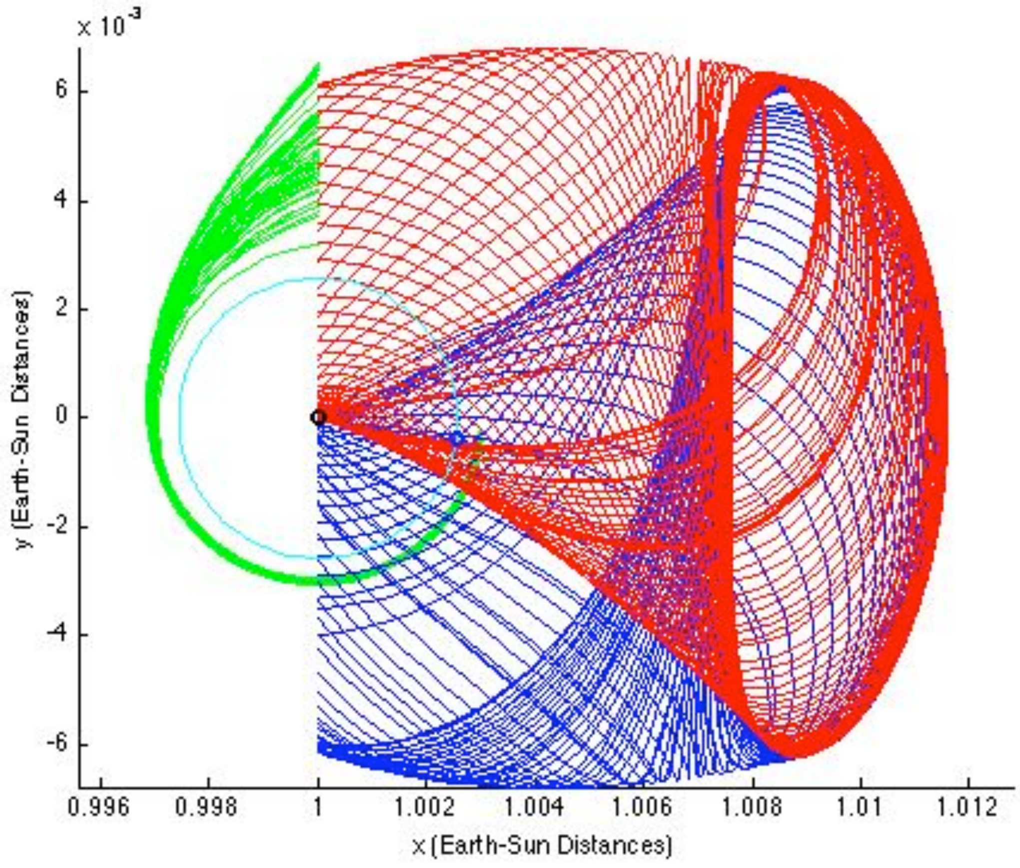
Sun-Earth Manifolds



Earth-Moon Stable Manifold

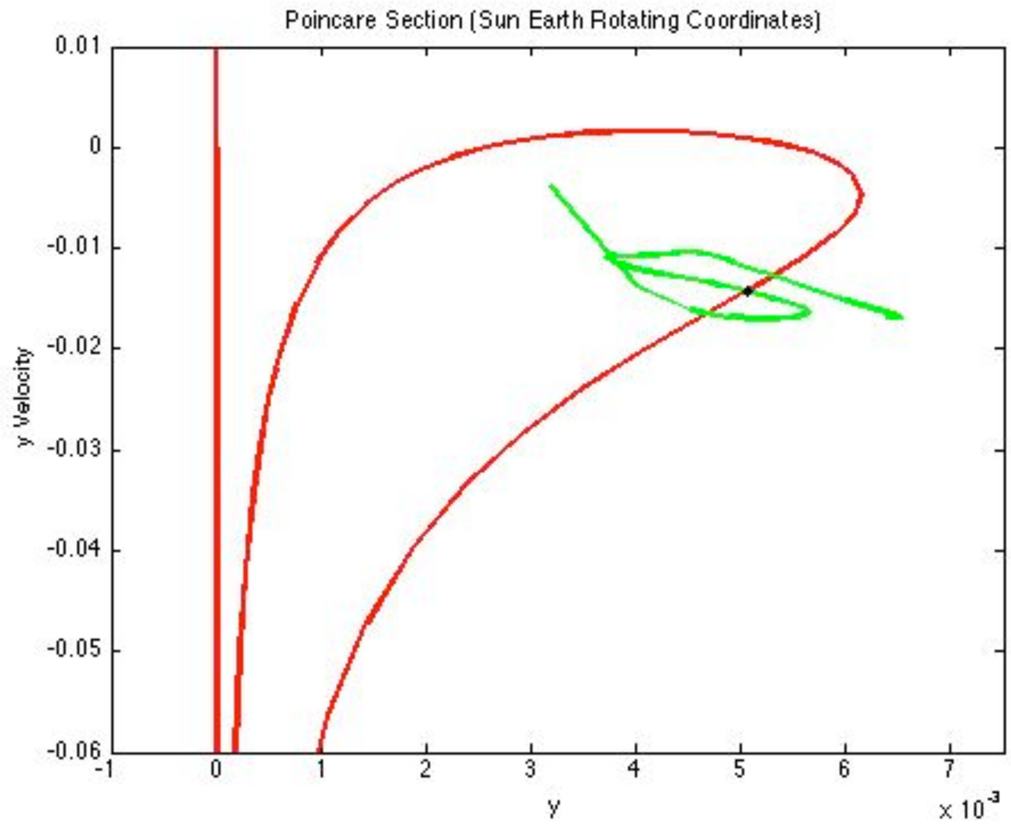
Invariant Manifolds
"Shoot the Moon"

- Transform EM manifold into SE rotating coordinates and plot manifolds together



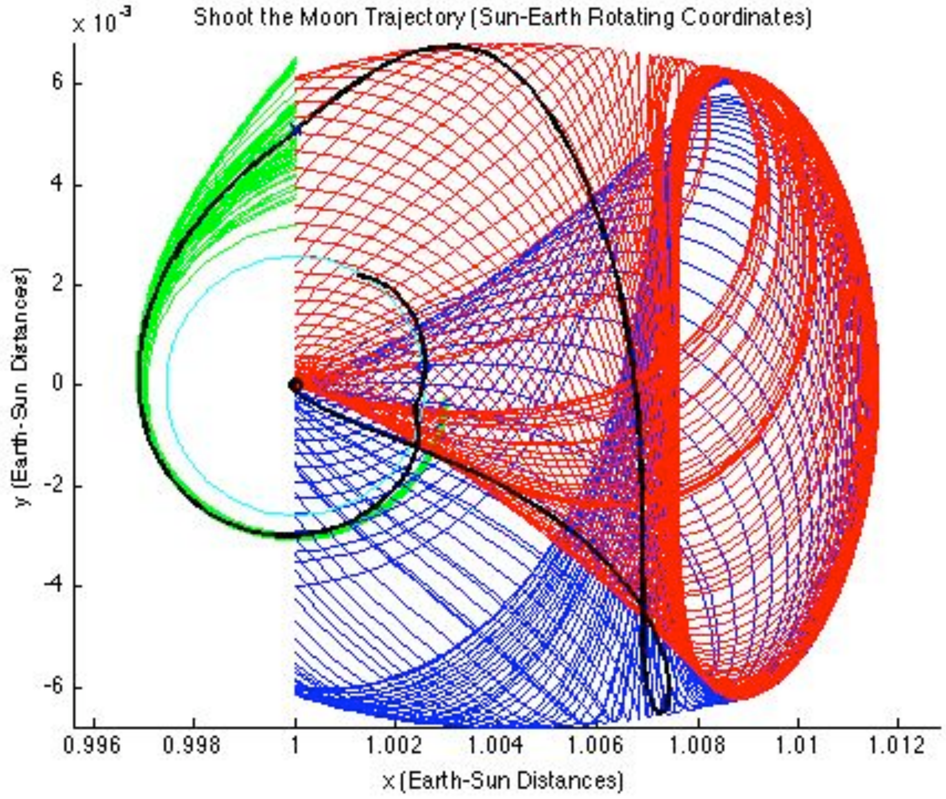
Invariant Manifolds
"Shoot the Moon"

- Compute Poincaré Sections and select 'patch' point
  - Select point just outside Sun–Earth manifold and inside Earth–Moon manifold



Invariant Manifolds
"Shoot the Moon"

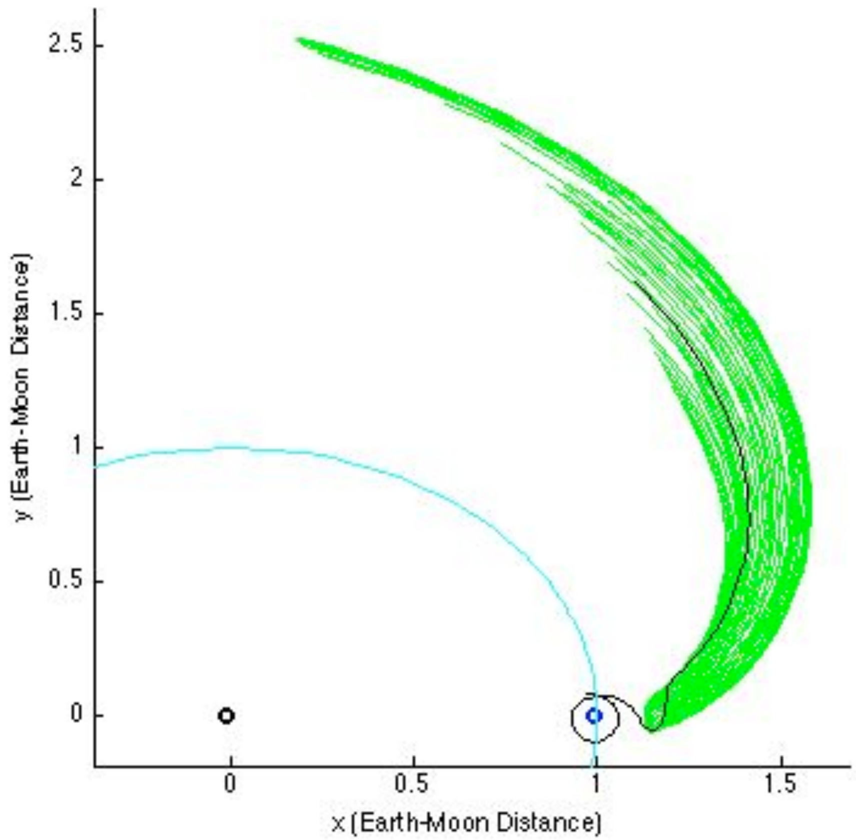
- Use selected point as initial condition
  - Integrate forwards on Earth-Moon stable manifold
  - Integrate backwards on Sun-Earth unstable manifold



Invariant Manifolds
"Shoot the Moon"

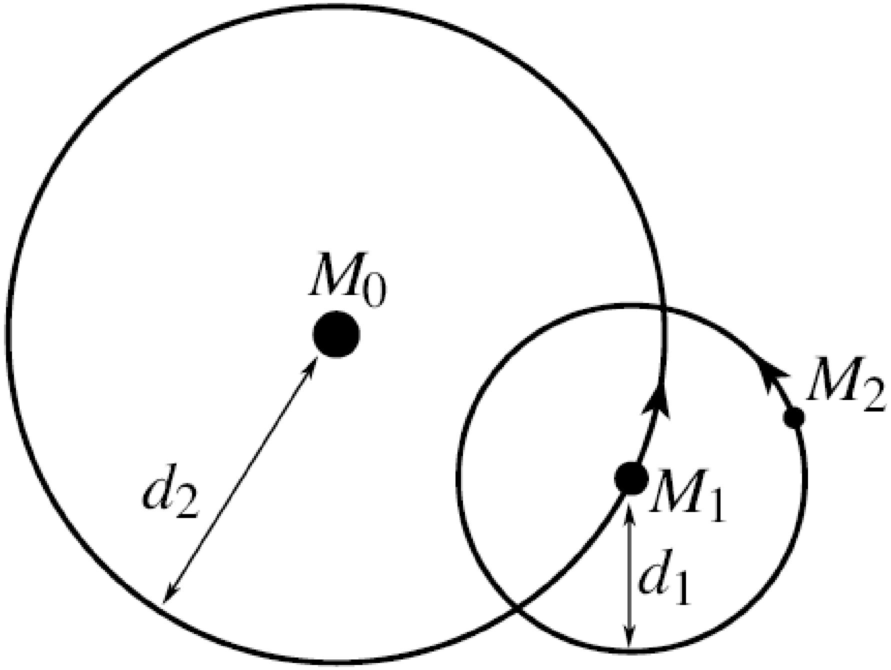
- Capture at Moon occurs naturally

EM Trajectory in EM Rotating Coordinates



Bicircular Model

- Create similar trajectory using the Bicircular Model of the four body problem (BCM4)
- M_1 and M_2 rotate in circular motion about their barycenter
- M_0 and M_1–M_2 barycenter rotate in circular motion about their common center of mass



Bicircular Model

- Sun Earth Rotating system:

$$\dot{x} = u$$

$$\dot{y} = v$$

$$\dot{u} = x + 2v - \frac{\mu_E (x - x_E)}{\left((x - x_E)^2 + y^2\right)^{3/2}} - \frac{\mu_S (x - x_S)}{\left((x - x_S)^2 + y^2\right)^{3/2}} - \frac{\mu_M (x - x_M)}{\left((x - x_M)^2 + (y - y_M)^2\right)^{3/2}}$$

$$\dot{v} = y + 2u - \frac{\mu_E y}{\left((x - x_E)^2 + y^2\right)^{3/2}} - \frac{\mu_S y}{\left((x - x_S)^2 + y^2\right)^{3/2}} - \frac{\mu_M (y - y_M)}{\left((x - x_M)^2 + (y - y_M)^2\right)^{3/2}}$$

$$\mu = \frac{M_E}{M_E + M_S} = 3.0035 \times 10^{-6}$$

$$\mu_S = 1 - \mu$$

$$\mu_E = -\mu$$

$$\mu_M = 3.734 \times 10^{-8}$$

$$x_S = -\mu$$

$$x_E = 1 - \mu$$

$$a_M = 2.573 \times 10^{-3}$$

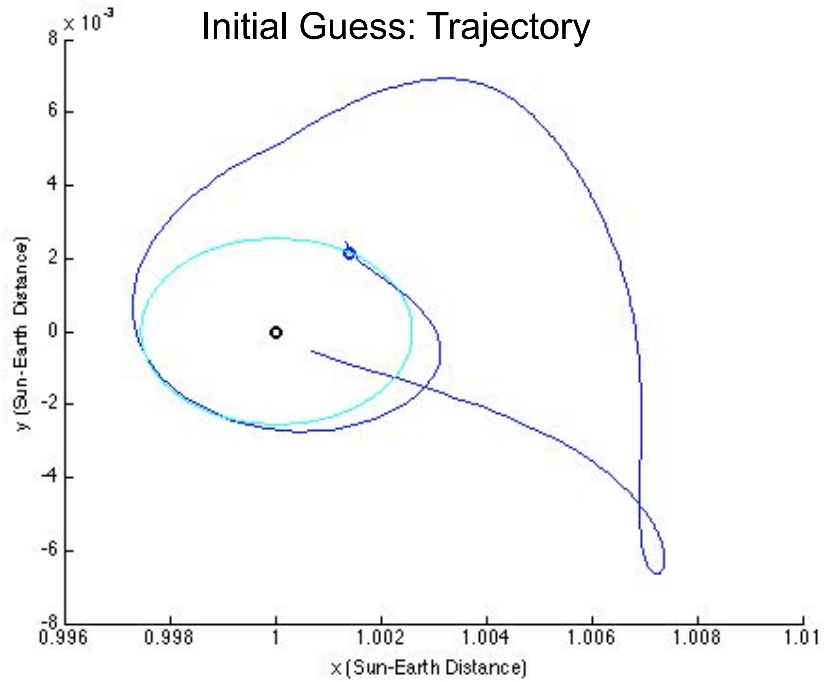
$$\omega_M = 12.369$$

$$\theta_M = \omega_M t + \theta_{M0}$$

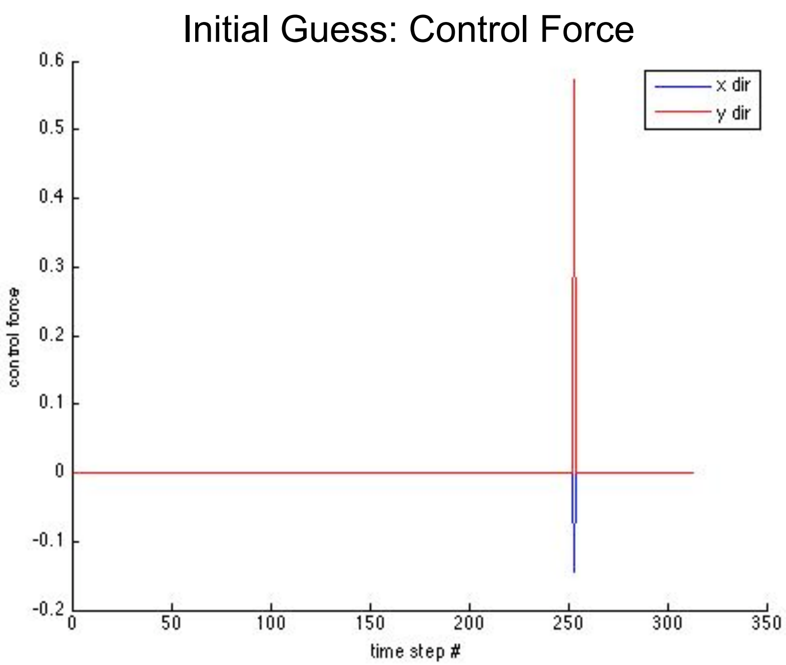
$$x_M = a_M \cos(\theta_M)$$

$$y_M = a_M \sin(\theta_M)$$

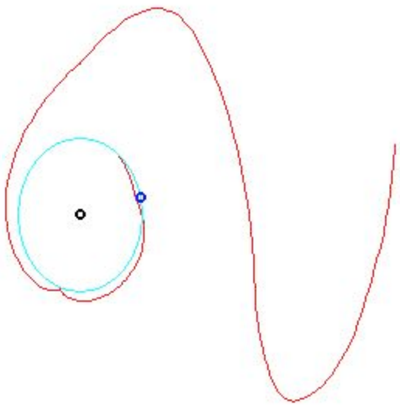
Bicircular Model



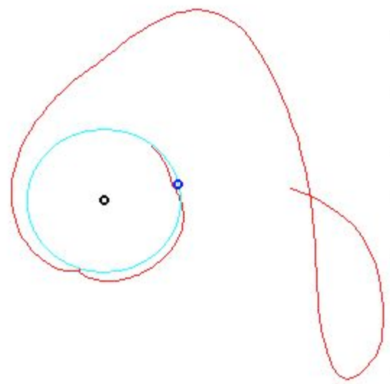
- Trajectory
  - Start at 800 km circular Earth orbit
  - ΔV =175.8 m/s



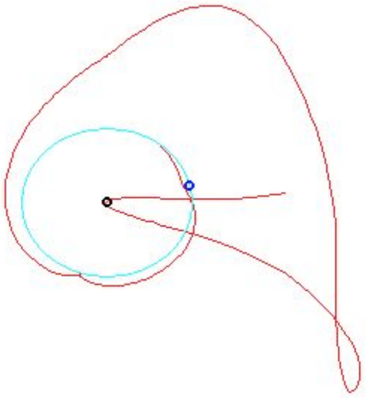
Trajectory Sensitivity



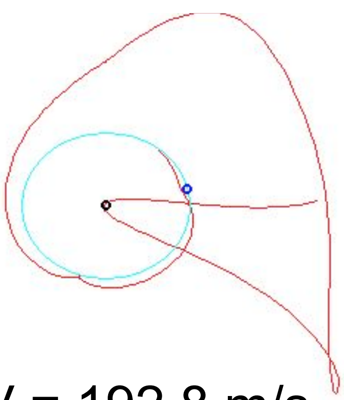
ΔV = 207 m/s



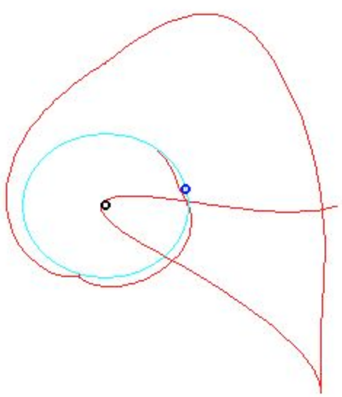
ΔV = 196 m/s



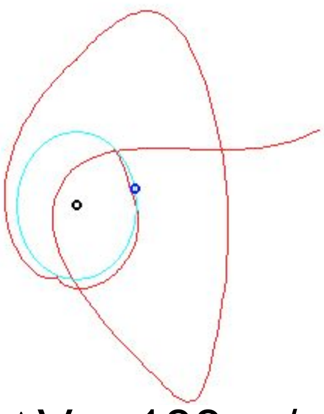
ΔV = 193 m/s



ΔV = 192.8 m/s



ΔV = 191 m/s



ΔV = 188 m/s

DMOC+IM

- Lagrangian is derived from BCM4 in SE rotating coordinates

$$L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2}\left(x^2 + y^2\right) + x\dot{y} - y\dot{x} + \frac{\mu_E}{\sqrt{\left(x - x_E\right)^2 + y^2}} + \frac{\mu_M}{\sqrt{\left(x - x_M\right)^2 + \left(y - y_M\right)^2}} + \frac{\mu_S}{\sqrt{\left(x - x_S\right)^2 + y^2}}$$

- DMOC equations

$$q_0 = q^0 \qquad q_N = q^1$$

$$D_2L\left(q_0, \dot{q}_0\right) + D_1L_d\left(q_0, q_1\right) + f_0^- = 0$$

$$D_2L_d\left(q_{k-1}, q_k\right) + D_1L_d\left(q_k, q_{k+1}\right) + f_{k-1}^+ + f_k^- = 0 \quad \text{for } k = 1, \ldots, N-1$$

$$-D_2L\left(q_N, \dot{q}_N\right) + D_2L_d\left(q_{N-1}, \dot{q}_N\right) + f_{N-1}^+ = 0$$

- Minimize control effort $J(q,u) = \int_0^T u_x(t)^2 + u_y(t)^2 \, dt$

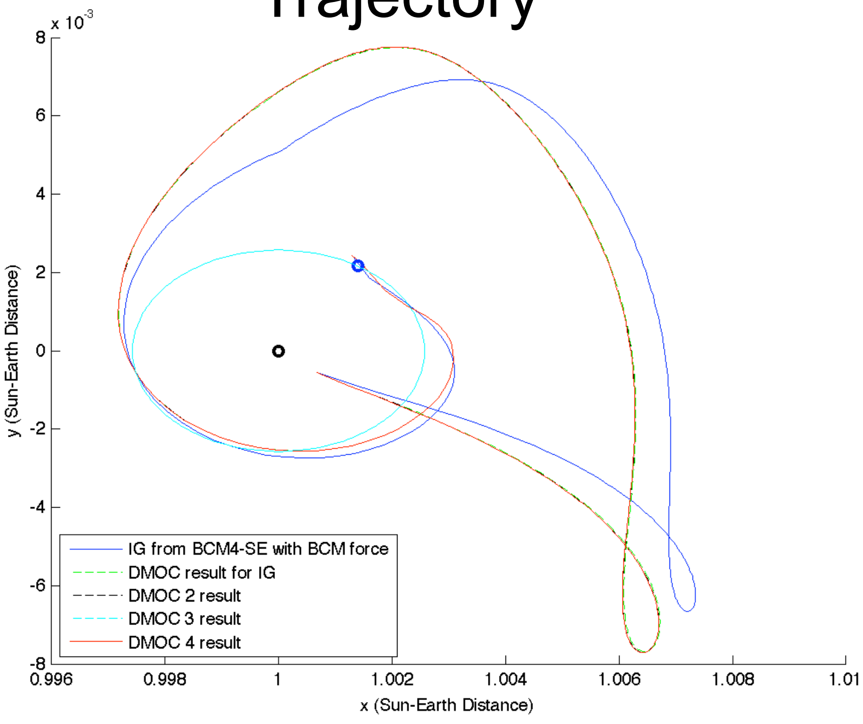
- Control Force

$$u_x = \frac{\Delta V_x}{\Delta t}, \qquad u_y = \frac{\Delta V_y}{\Delta t}$$

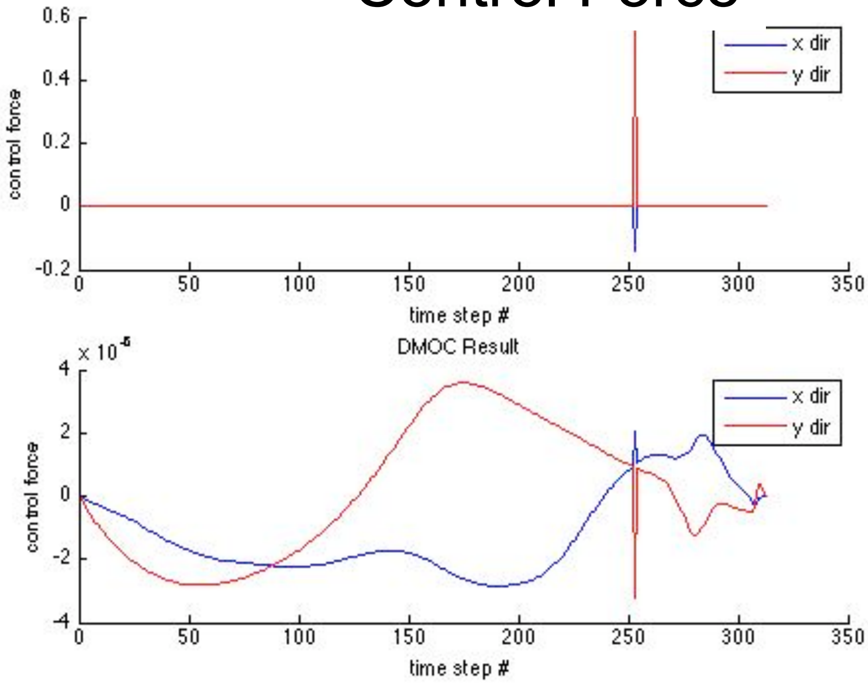
DMOC Results

DeltaV (m/s)	
IG	175.8273
DMOC 1	2.1374
DMOC 2	0.6105
DMOC 3	0.2342
DMOC 4	0.2331

Trajectory



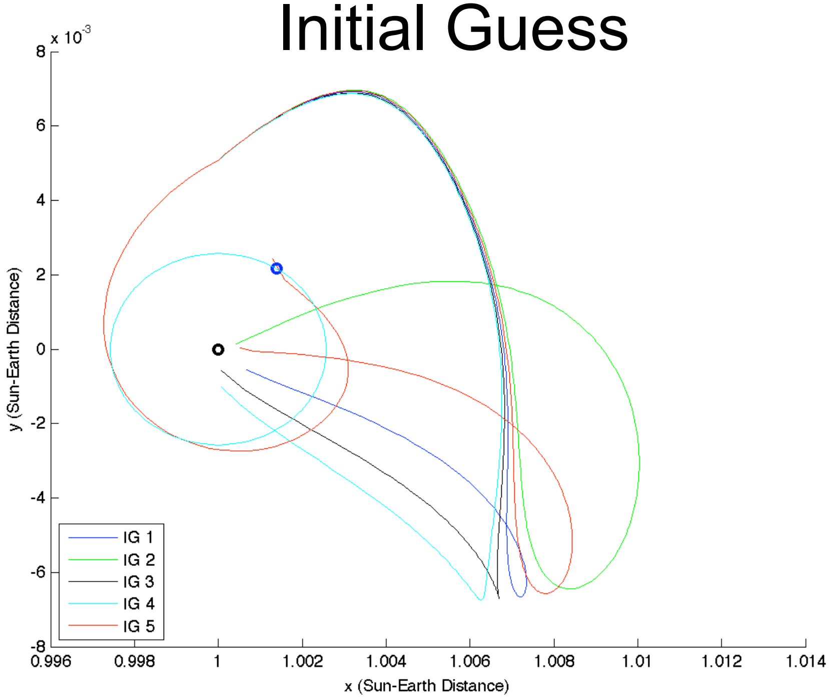
Control Force



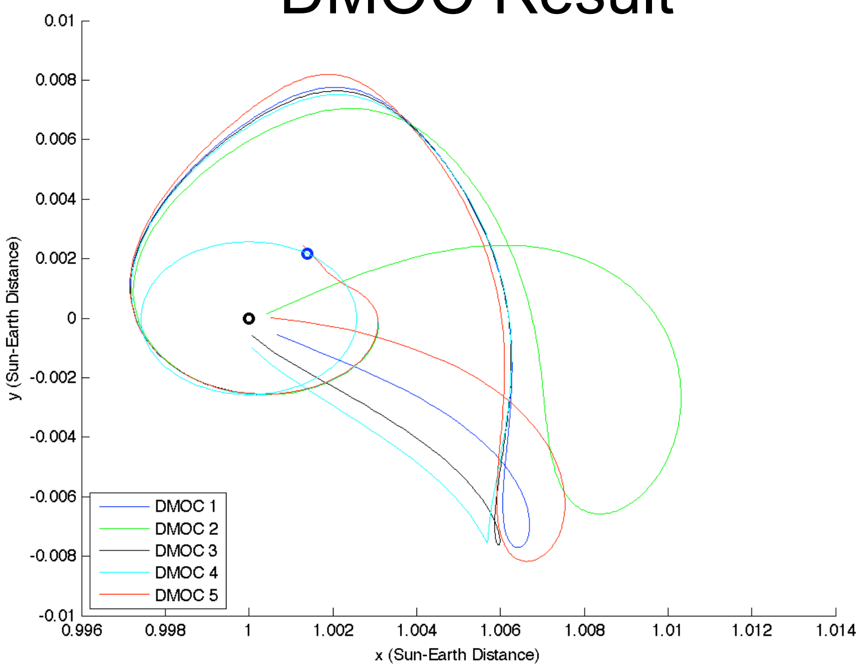
DMOC Results

Delta V (m/s)		
	Initial Guess	DMOC
case 1	175.8273	0.2331
case 2	178.5763	0.4452
case 3	172.7951	0.0672
case 4	171.3516	0.0902
case 5	177.8498	0.4386

Initial Guess



DMOC Result



Comparison

- How does this compare with a Hohmann Transfer?
  - Case 1: trajectory begins in ~800 km altitude circular orbit.
    - Starting velocity of trajectory = 6.24 km/s
    - circular velocity of parking orbit = 7.4 km/s
      - Initial ΔV = 1.17 km/s
    - ΔV = 0.2331 m/s for trajectory portion
    - Total ΔV = 1170.23 m/s
  - Hohmann Transfer from 800 km circular orbit to Moon
    - Total ΔV = 3812.6 m/s

DMOC + Invariant Manifolds
Future Work

- Optimize for time and control
- Enforce momentum boundary conditions to ensure capture
- Solve same problem using JPL’s MYSTIC
  - compare with DMOC+IM method
- Use method to generate trajectory to Titan
  - Also include fly-by of Enceladus
    - May require additional maneuvers

References

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Questions?