CDS 140a: Homework Set 1

Due: Friday, October 9th, 2009.

1. Consider the following planar system for $(x, v) \in \mathbb{R}^2$:

$$\dot{x} = v$$
$$\dot{v} = -x^3$$

   (a) Find the equilibrium points for the system
   (b) Find a conserved energy for the system
   (c) Draw the phase portrait
   (d) Argue informally that all the trajectories outside the origin are periodic

2. Draw the phase portrait for the system

$$\ddot{x} = -x^3 - \dot{x}$$

   and comment on its structure.

3. Consider the following second order equation for $x \in \mathbb{R}$:

$$\ddot{x} = 2x + x^2 - x^3$$

   (a) Find the equilibrium points for the system
   (b) Find a conserved energy for the system
   (c) Draw the phase portrait and comment on the periodic orbit structure

4. Draw the phase portrait for the system

$$\ddot{x} = 2x + x^2 - x^3 - 2\dot{x}$$

   and comment on its structure.

5. Discuss symmetry and reversibility properties (if any) of the equations in problems 1 and 2

6. Discuss symmetry and reversibility properties (if any) of the equations in problems 3 and 4

7. Consider the following dynamical system in the plane that depends on the real parameter μ.

$$\dot{x} = -x(x^2 + y^2 - \mu) - y(x^2 + y^2)$$
$$\dot{y} = -y(x^2 + y^2 - \mu) + x(x^2 + y^2)$$

   (a) Show that the system has a periodic orbit for $\mu > 0$.

(b) Is it stable?

8. Discuss the evolution of the phase portraits in the preceding question as μ varies from negative to positive.

9. Consider the system

$$\ddot{x} = \alpha x - x^3.$$

Study the evolution of the phase portraits of this system as α varies from negative to positive.

10. Add a fixed amount of dissipation to the preceding system and repeat the question. Include a study of symmetries as well.