Errata for Elementary Classical Analysis, Second Edition
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What follows are the errata known to the authors in the 2nd edition of Elementary Classical Analysis as of the above date. Please notify one of us if you know of errata not on this list. The book will be reprinted at the next available opportunity with these errata corrected. We have available the beginnings of an instructor's guide with a number of additional solutions, sample exams, and discussions of alternative approaches. Contact us if you would like to receive one.

We would like to thank several readers for taking the trouble to send in errata, especially our own students as well as Collin Bennett, Rob Pratt, Sean Bates, Faan Tone Liu, and Stuart Antman.

In these errata, "line 3↑" means "line 3 from the bottom of the page".

PREFACE

Page ix, line 3↑ "transformation" should be "transformations".

INTRODUCTION

Page 2, line 7 the symbol $\cap$ should be $\cup$.

Page 4, line 10 "Any" should be "Each".

Page 7, note 2 Halmos's book was published in 1960.

Page 15, line 8 should be union and not intersection.

Page 15, line 4↑ "countable" should be "denumerable" or "countably infinite".

Page 16, line 3↑ "... for any set ..." should be "... for every set ...".

Page 21, Exercise 6 should read: "by setting up a one-to-one correspondence between the set $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ and the set $\mathbb{N}$."

Page 22, Exercise 10 $h \circ (f \circ g) = (h \circ f) \circ g$ should be $h \circ (g \circ f) = (h \circ g) \circ f$.

CHAPTER ONE

Page 33, 1.1.9 Proposition the proof requires correction: Replace "One way to do this is:..., avoiding repetitions" with "One way to do this is to consider the points in the plane with integer coordinates, say (p, q) and to assign the fraction p/q (simplified to lowest terms) to this point (leave out assignments when $q = 0$). Now starting at the origin, spiral out, listing all the fractions as you go, omitting any fraction that you have already encountered; that is, avoiding repetitions. In this way, you will count all possible fractions (positive and negative) and thereby set up a one-to-one correspondence between $\mathbb{N}$ and $\mathbb{Q}$."

Page 35, Exercise 4 the words in the parentheses should read, "Do this without using a numerical approximation for $\sqrt{2}$."

Page 41, 1.2.13 Example, Solution the second to last fraction in the first set of equations should be $\frac{\sqrt{2}}{(n-1)!}$.

Page 43, 1.2.16 Theorem there should be a comma after "field".

Page 45, Exercise 5 put the word *strictly* in italics.

Page 45, 1.3.1 Definitions define $\sup(S) = +\infty$ if S is nonempty and is not bounded above. If S is empty, let $\sup(S) = -\infty$.

Page 47, 1.3.3 Proposition add the condition that A and B be non-empty sets.

Page 53, 1.5.3 Definition has some ambiguities in the degenerate cases. We should set:

- $\limsup x_n = +\infty$ if x_n is unbounded above.
- $\limsup x_n = \sup\{\text{cluster points}\}$ if x_n is bounded above (thus, $\limsup x_n = -\infty$ if x_n is bounded above and has no cluster points.)
- $\liminf x_n = -\infty$ if x_n is unbounded below.
- $\liminf x_n = \inf\{\text{cluster points}\}$ if x_n is bounded below (thus, $\liminf x_n = -\infty$ if x_n is bounded below and has no cluster points.)

Page 58, line 3↑ F should be $\mathbb{F}$.

Page 61, first paragraph the Properties should range from i–v.

Page 68, line 1↑ omit λ.

Page 70, Exercise 5 metric should be inner product.

Page 78, Exercise 7c "any" should be "each".

Page 82, line 5↑ should read "$1, 2, 3, \ldots$ is bounded above by $x \ldots$ ".

Page 84, line 1↑ should read $\inf B \leq \inf A \leq \sup A \leq \sup B$.

Page 85, sentence 2 of Method 1 the phrase should be "an amount $1/2^n$" instead of "an amount $1/2$".

Page 88, 1.4.6 Lemma, Proof the second to last $|x_n|$ should be $|x_N|$.

Page 89, line 3 should read "must be larger than N."

Page 89, Proof iii. the statement in the sixth sentence follows not from i., but from the negation of the definition of convergence.

Page 90, line 3↑ "by **1.5.5ib**." should be "by **1.5.5iia**."

Page 94, Proof iii. $d(w - v)$ should be $d(w, v)$.

Page 96, lines 5 and 6 delete comma from x_i^2, z_i^2 (twice).

Page 97, Exercise 3b "any" should be "each".

Page 99, Exercise 20, line 4 replace " where the condition" by "of nonzero orthogonal subspaces for which the condition".

Page 99, Exercise 22a $=$ should be $\leq$.

Page 99, Exercise 23 "$\ldots$ for any integer $\ldots$" should be "$\ldots$ for each integer $\ldots$".

Page 100, Exercise 29 "For any" should be "For each".

Page 100, Exercise 35 the last) should be omitted.

Page 102, Exercise 46 the beginning of the question should read "Prove that each nonempty set S of $\mathbb{R}$ that is bounded $\ldots$".

Page 102, Exercise 47 Delete this exercise; it will be fixed in the next edition.

CHAPTER TWO

Page 105, Figure 2.1-3 the picture on the right should have the label $\mathbb{R}^2$ instead of $\mathbb{R}$.

Page 109, 2.2.3 Example, Solution insert a backslash (set theoretic difference) before the $\{1\}$ in the second line.

Page 112, Exercise 5 R should be $\mathbb{R}$.

Page 115, Figure 2.4-2 The top most disk should not be shaded.

Page 123, 2.8.2 Definition should read "...if there is a number B such that $||x_k|| \leq B$ for every k. In a metric space we require that there be a point x_0 such that $d(x_k, x_o) \leq B$ for all k."

Page 125, Exercise 1 change the problem to "Suppose N is a complete subset of a metric space (M, d). Show that N is closed."

Page 125, Exercise 5 "at least one cluster point" should read "at least one cluster point in M".

Page 128, line 4 the term $\leq \sum_{k=0}^{n} \frac{1}{2^k}$ should read $\leq \sum_{k=0}^{\infty} \frac{1}{2^k}$.

Page 134, line 4 $D(x, r)$ should read $D(x, R)$.

Page 134, line 7↑ the word "Proposition" is missing an "o".

Page 137, Proof iii. in the fourth line of equations regarding s_{2^k-1}, the last term is $\frac{1}{(2p-1)^{p-1}}$ and should be $\frac{1}{(2^{k-1})^{p-1}}$.

Page 140, Example 2.1, Solution the first equation should have "1 or" after the $>$ sign.

Page 147, Exercise 30 "... any open set ..." should be "... each open set ...."

Page 148, Exercise 46a should be "If $f = o(g)$ and if $g(x) \to \infty$ as $x \to \infty$, then show that $e^{f(x)} = o(e^{g(x)})$ as $x \to \infty$."

CHAPTER THREE

Page 156, line 7↑ should read $1/(N - 1)$ rather than $1/(1 - N)$.

Page 163, line 6↑ there should be a period at the end of the line.

Page 166, line 2 add "The number r is called a *Lebesgue number* for the covering. The infimum of all such r is called *the* Lebesgue number for the covering." at the end of Lemma 3.

Page 170, Proof of Theorem 3.5.2 (1) C should be defined as $C = \phi^{-1}(U \cap A)$, and D should be defined as $D = \phi^{-1}(V \cap A)$. (2) The statement that C and D are nonempty since $a \in C$ and $b \in D$ should read $\phi^{-1}(x) \in C$ and $\phi^{-1}(y) \in D$.

Page 174, Exercise 20 should read "Prove that a compact subset of a metric space must be closed as follows: Let x be in the complement of A. For each $y \in A$, choose disjoint neighborhoods U_y of y and V_y of x. Consider the open cover $\{U_y\}_{y \in A}$ of A to show the complement of A is open."

Page 175, Exercise 26 should read "Show that the completeness property of $\mathbb{R}$ may be replaced by the *Nested Interval Property*. If $\{F_n\}_1^{\infty}$ is a sequence of closed bounded intervals in $\mathbb{R}$ such that $F_{n+1} \subset F_n$ for all $n = 1, 2, 3, \ldots$, then there is at least one point in $\bigcap_{n=1}^{\infty} F_n$."

Page 175, Exercise 33 " ... for any ... " should be "... for each ..." and "... $\text{int}(\text{cl}(S)) \neq \varnothing$." should be "... $\text{int}(\text{cl}(S)) = \varnothing$."

Page 176, Exercise 34 "any" should be "each".

CHAPTER FOUR

Page 184, Exercise 5 should read "Let A and B be subsets of $\mathbb{R}$ with B not empty. If $A \times B \subseteq \mathbb{R}^2$ is open, must A be open?"

Page 191, Exercise 4 should specify that c is a continuous function.

Page 191, §4.5, line 3 replace "non-continuous" with "discontinuous".

Page 208, line 8 the integral should be underlined: "Hence $\underline{\int_0^1}$."

Page 212, 4.2.1 Theorem, Proof change A to K.

Page 217 the second sentence after equation (3) should read "Then the right-hand side of (3) is bounded above by...".

Page 232, Exercise 9 $[a, b[$ should be $[a, b]$.

Page 233, Exercise 14c hint should read "Consider the function given in polar coordinates by $r \tan(\theta/4)$, $0 \le r < \infty$, $0 \le \theta < 2\pi$."

Page 234, Exercise 22 the d should be ρ.

Page 236, line 2 The $k^2 =$ should come before the fraction, not in the numerator.

CHAPTER FIVE

Page 238, line 3↑ the second x should be $f(x)$.

Page 239 in the two paragraphs after 5.1.2, the large number N should be L. There are eight of them.

Page 242, Solution a. the last line should read $n > (\log(\epsilon|1 - x|)/\log x) - 1$.

Page 242, Solution b. the error should be smaller for $-x$ than it is for x rather than the absolute values of these quantities, and the numerator of the RHS fraction should be $nx^n(1 - x) + x^{n+1}$.

Page 250, Figure 5.3-1 the caption should read "This sequence converges to zero pointwise, but the corresponding sequence of integrals does not converge to zero."

Page 251, line 7 should read $\sum_{n=0}^{N} \int_0^x t^n dt = \ldots = -\log(1 - x) - \int_0^x \frac{t^{N+1}}{1-t} dt$.

Page 263, line 3↑ $2k!$ should be $(2k)!$.

Page 273, line 3 B should be $\mathcal{B}$.

Page 286, Exercise 4 x, y should be (x, y).

Page 289, Exercise 3 the second $\le$ should be $<$.

Page 296, line 1↑ should read $\int_a^b f_n(x)dx - \epsilon(b - a) \le \underline{\int_a^b} f(x)dx$.

Page 300, line 1 Diagrammatically is misspelled.

Page 301, line 9↑ $x_n + 1$ should be x_{n+1}.

Page 302, line 5↑ should read $\mathcal{C}([t_0 - \delta, t_0 + \delta], \mathbb{R})$.

Page 315, line 5↑ "... $\sum_{k=1}^{\infty} g_\sigma(k) \ldots$" should be "... $\sum_{k=1}^{\infty} g_{\sigma(k)} \ldots$".

Page 317, Exercise 5 in the second line, $\mathbb{R}$ should be $\mathbb{R}^m$, and in the next line, $||f(x)||$ should just be $|f(x)|$.

Page 317, Exercise 11b "for all $x, y \in X$." should be "for all $x, y \in X$ such that $x \neq y$."

Page 318, Exercise 13 omit the hint.

Page 319, Exercise 25 g should be f (twice).

Page 319, Exercise 27 after the comma, the sentence should be, "continuous, and that $x_n \in \,]a, b[$ with $x_n \to b \ldots$ ".

Page 320, Exercise 29d $f(x)$ should be $x\sin(1/x)$.

Page 321, Exercise 45a the word "equicontinuous" should come before "sequence" and f_n should be f_k. Same correction in Exercise 47, page 322.

Page 324, Exercise 57 a, b, c, and d should be i, ii, iii, and iv, respectively.

Page 324, Exercise 59c the first c should be omitted from the equation.

Page 325, Exercise 66 the last line of the exercise should read "that $t_n \leq Cn^{-1/\beta}$.]"

Page 326, Exercise 68 $\mathbb{R}^n$ should be $\mathbb{R}^m$.

CHAPTER SIX

Page 329, Figure 6.1-2 in the figure on the left, the axes should be labeled x and y instead of x_1 and x_2.

Page 332, line 8↑ $\mathbf{D}L = L$ should be $\mathbf{D}L(x_0) = L$.

Page 345, 6.5.1 Chain Rule in the second line, $\mathbb{R}^n$ should be $\mathbb{R}^m$.

Page 370, line 7↑ ∂x_1 should be ∂x_i.

Page 379, line 9 $h = 2x$ should be $h = \lambda x$

CHAPTER SEVEN

Page 393, line 7↑ "exist a" should be "exists an open".

Page 398, line 8↑ "a neighborhood" should be "an open neighborhood".

Page 399, 7.2.2 Corollary the partial derivatives in the second matrix (the matrix whose inverse is taken) should be with respect to $y_i, i = 1, ..., m$, not x_i.

Page 403 and 427 in Theorem 7.4.1, replace the last line by "*class C^r and a neighborhood W of x_0 in $\mathbb{R}^p$ such that $g \circ f(x_1, \ldots, x_p) = (x_1, \ldots, x_p, 0, \ldots, 0)$ for all $(x_1, \ldots, x_p) \in W$.*"

Page 439, line 8↑ replace $\cos\theta$ by $\cos\varphi$.

CHAPTER EIGHT

Page 457, line 1↑ g should be f.

Page 464, 8.5.6 Example, Solution the first line should read "If f were absolutely integrable on $[1, \infty[$, then $|f|$ would be integrable ... ".

Page 468, 8.6.6 Example in the third line, $[a_{i+1}, a_i]$ should be $[a_{i+1}, a_i[$, and in the next line, the sum should be $\sum_{j=1}^{\infty} f_j(a_{j+1} - a_j)$.

Page 468, line 2↑ the sum should be $\sum_{j=1}^{n-1} f_j(a_{j+1} - a_j)$.

Page 477, Step 1, lines 5 and 6 change f to g three times.

Page 478, line 8↑ omit the prime ′ after $S \in C$ under the summation sign.

Page 479, line 5 $1_A(x) - 1_A(y)$ should be $|1_A(x) - 1_A(y)|$.

Page 479, 8.3.4 Theorem, ii. part of the sentence should be " ... *for all x in A and* ... ".

Page 490, Exercise 11 $v(S)$ should be $v(A)$.

Page 492, Exercise 22 $\int_1^\infty$ should be $\int_0^\infty$.

Page 494, Exercise 39 the last term in the sum should be $1/(2n)$, not $1/(2^n)$.

CHAPTER NINE

Page 500, line 2↑ $dydy$ should be dy.

Page 513, lines 3 and 4↑ The sentence should read, "Let $0 = y_0 < y_1 < \ldots y_n$ be a partition of an interval $[0, Y]$ with $Y > \sup \{f(x) \mid x \in [a, b]\}$ containing the range of f."

Page 536, Exercise 8a all the ν should be v. Also, $f(u, v)$ denotes the function f after the change of variable is made. Part of the exercise is to figure out the notation.

Page 538, Exercise 22 $\max(f(x, g(x)))$ should be $\max(f(x), g(x))$.

CHAPTER TEN

Page 547, line 4↑ replace ∞ by n.

Page 554 in the formula for $P_n(x)$, $2n + 1$ should be $\sqrt{2n + 1}$.

APPENDIX A

Page 664, Exercise 9b The integral of f over B should be the integral of g over A.

Page 665, Exercise 20a "coordinates" is missing an "r".

Page 676, Exercise 66b "linear" is missing an "e".

APPENDIX C

Page 685, 1.5 (3) should be "Use **1.5.5** to show that there are points $x_{N(n)}$ within $\frac{1}{n}$ of a (or b) ... ".

Page 686, 1.7 (1) "The sup norm is ..." should be "The distance given by the sup norm ..." .

Page 686, 1.8 (1b) should be $(11/5) + 2i$.

Page 688, (39a) the denominators $(x^2 + y^2)$ should be $(x^2 + y^2)^2$ in the real and imaginary cases.

Page 688, 2.1 (5) should be "No in general; yes if B is open or $0 \notin B$."

Page 689, 2.5 (1) cl(S) should be cl(A).

Page 689, 2.5 (5) capital D should be lowercase d, and lowercase b should be capital B.

Page 691, (3) omit "If B is closed and $B \subset A$, then $B \subset cl(A)$."

Page 691, (17) should be "The series $\sum_m ((\sin m)x_m)$ converges ... ".

Page 693, (1c) should be compact, not connected if $n = 1$, connected and compact if $n \geq 2$.

Page 694, (11b) the suggestion given is for part c.

Page 694, (15a) in the first line, "form" should be "from".

Page 695, (31) should read "The set A must be either not closed or not bounded or both. Treat these cases separately. If A is not closed, there must be an accumulation point of A which is not in A."

Page 696, 4.1 (5a) should read $f(x) = 1$, $U = $ any non-empty open set.

Page 696, 4.1 (5b) the last line should read "1, if $x \geq 1$."

Page 696, 4.2 (3) B_n should be B.

Page 698, 4.8 (7) the last sentence should read "Show that inf $U(f, P) \leq$" rather than "Show that $U(f, P) \leq$".

Page 698, (3a) k should be K.

Page 698, (5) the last $=$ should be $\geq$.

Page 698, (9) $f^{-1}(F) \cap f^{-1}(F)$ should be replaced by $f^{-1}(F) \cup g^{-1}(F)$.

Page 698, (13) sup$(F(V))$ should be sup$(f(V))$.

Page 701, 5.4 (7) the three capital E's should be replaced by e's.

Page 701, 5.6 (3a) the reference to **5.6.4** should be to **5.6.6**.

Page 701, 5.6 (5) the reference to Example 1 should be to Example **5.6.4**.

Page 702, (3a) the answer should be "Converges pointwise on $\mathbb{R}$. Uniformly on $]-\infty, b]$. Discontinuities at the positive integers."

Page 702, (3d) the answer should be "Converges uniformly on any set bounded away from odd multiples of π where the sum has discontinuities."

Page 702, (5) the answer should be "The key inequality is

$$\|f_k(x)g_k(x) - f(x)g(x)\| \leq |f_k(x)|\|g_k(x) - g(x)\| + |f_k(x) - f(x)|\|g(x)\| \ldots"$$

Page 702, (7) there is a period missing before "So ... ".

Page 703, (29b) the interval $[1, \infty]$ should be $[1, \infty[$, and $[0, \infty]$ should be $[0, \infty[$.

Page 703, (31) "$|(a_1) + \ldots$" should be "$|(a_1 - a) + \ldots$" in the fourth and fifth lines of the solution.

Page 704, (47) "countably" should be "countable".

Page 704, (53b) the answer should be e^2.

Page 704, (57) in the third term of the last line, "f_n" should be "f".

Page 705, (63) the beginning of the solution should say, "Let $u_k = (\alpha(\alpha - 1) \ldots$".

Page 706, 6.4 (5) should be "Study Example **6.3.4**."

Page 714, 8.5 In Exercise **1**, Example **2b** should be Example **2(ii)**.

Page 714, 8.5 Exercise **3** should be Exercise **5**.

Page 716, 9.3(3) a factor of 2 is missing. The 2 should either be outside the integrals or should be multiplied by the integrand.

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