

# Foundations of Mechanics

Second Edition

Revised, enlarged, and reset

*A mathematical exposition of classical mechanics  
with an introduction to  
the qualitative theory of dynamical systems  
and applications to the three-body problem*

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*University of California*

*Santa Cruz and Berkeley*

with the assistance of

**Tudor Ratiu and Richard Cushman**

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# Contents

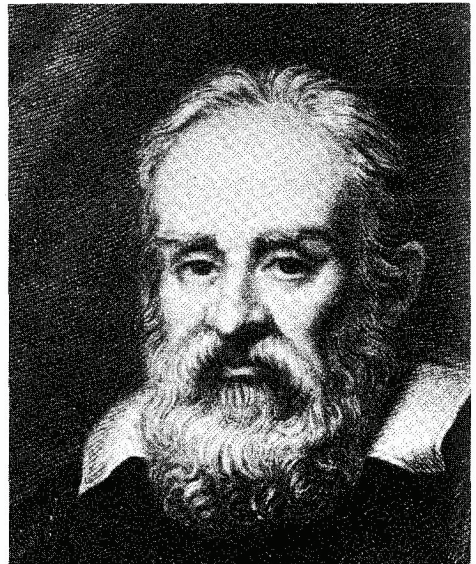
	<i>Page</i>
Archimedes	m-iii
Nicholas Copernicus	m-iv
Galileo Galilei	m-iv
Johannes Kepler	m-v
Isaac Newton	m-v
Gottfried Wilhelm Leibnitz	m-v
Pierre Louis Moreau de Maupertuis	m-vi
Leonhard Euler	m-vi
Joseph-Louis Lagrange	m-vii
Pierre-Simon de Laplace	m-vii
Adrien Marie Legendre	m-vii
Simeon-Denis Poisson	m-viii
Karl Gustav Jakob Jacobi	m-viii
William Rowan Hamilton	m-ix
Joseph Liouville	m-ix
Georg Friedrich Bernhard Riemann	m-ix
Gaston Darboux	m-x
Marius Sophus Lie	m-x
Sonya Kovalevsky	m-xi
Henri Poincaré	m-xi
Aleksandr Mikhailovich Liapounov	m-xi
Constantin Carathéodory	m-xii
Edmond Taylor Whittaker	m-xii
Albert Einstein	m-xiii
George David Birkhoff	m-xiv
Elie Cartan	m-xiv
Amalie Emmy Noether	m-xv
Carl Ludwig Siegel	m-xv
Andrei Nikolaevic Kolmogorov	m-xv
Jürgen Moser	m-xvi
Stephen Smale	m-xvi
Vladimir I. Arnold	m-xvi



Archimedes, 287 B.C.-212 B.C. *Courtesy of the Library of Congress, Washington, D.C., U.S.A.*



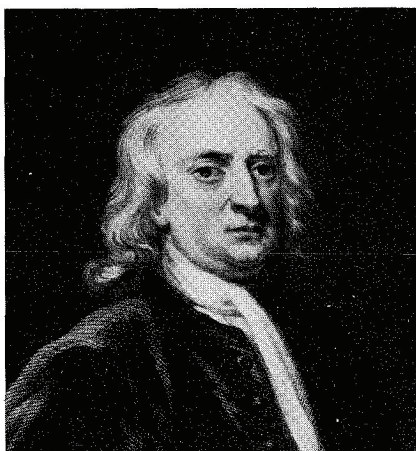
Nicholas Copernicus, 1473-1543.  
*Courtesy of the Library of Congress,  
Washington, D.C., U.S.A.*



Galileo Galilei, 1564-1642.  
*D. J. Struik, A Concise History of  
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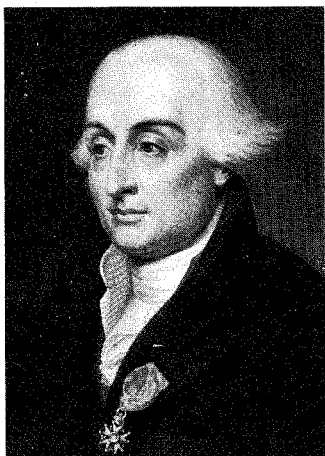
Gottfried Wilhelm Leibniz, 1646-1716.  
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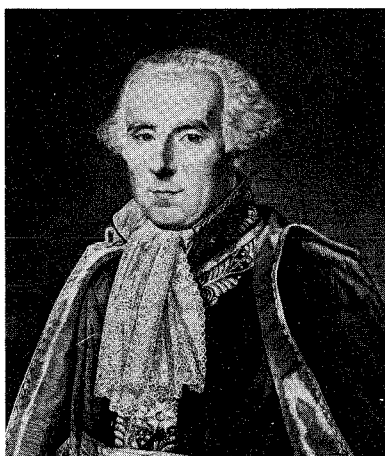
Pierre Louis Moreau de Maupertuis,  
1698-1759. *Courtesy of the  
Bibliothèque Nationale,  
Paris, France.*



Leonhard Euler, 1707-1783.  
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New York (1937).*



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*Courtesy of the Bibliothèque Nationale,  
Paris, France.*



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*Courtesy of the Bibliothèque Nationale,  
Paris, France.*

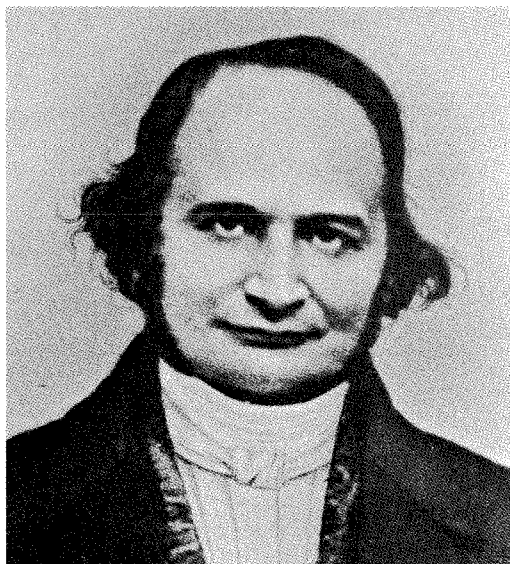


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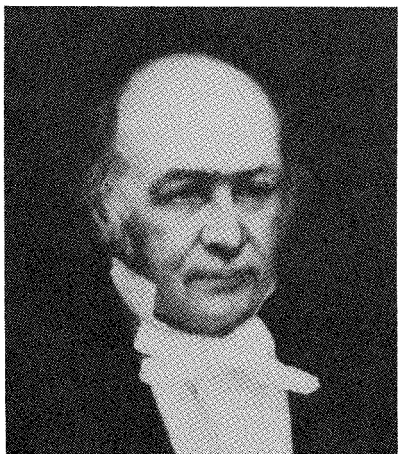




Simeon-Denis Poisson, 1781-1840. *Courtesy of the Bibliothèque Nationale, Paris, France.*



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et Discours Librairie Scientifique.  
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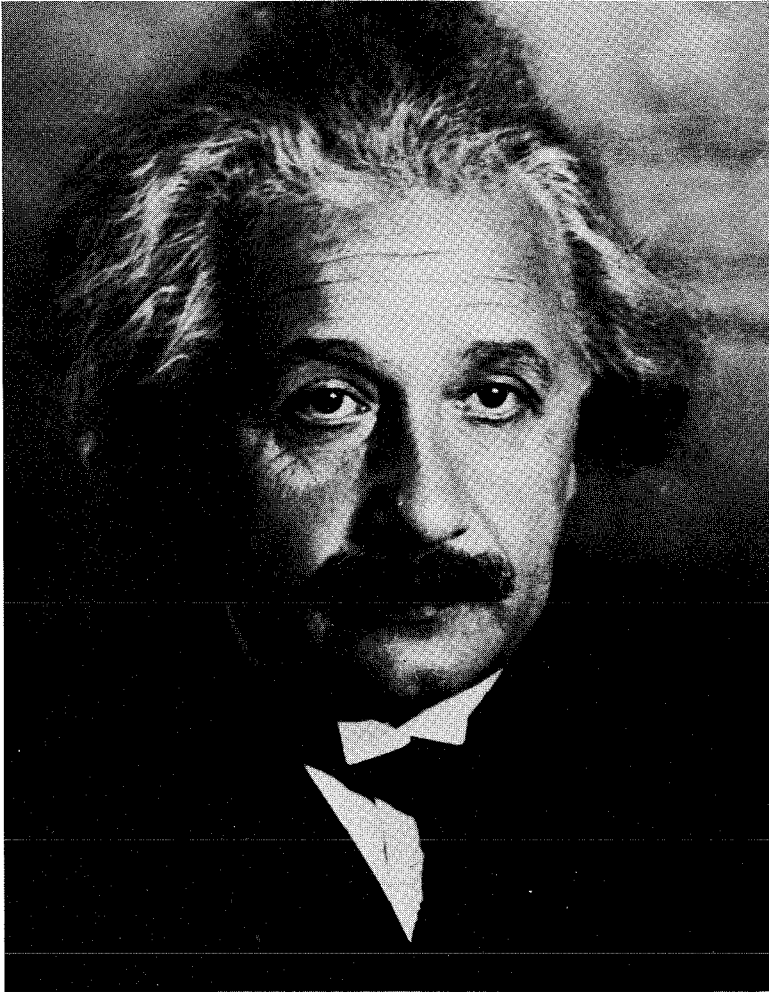
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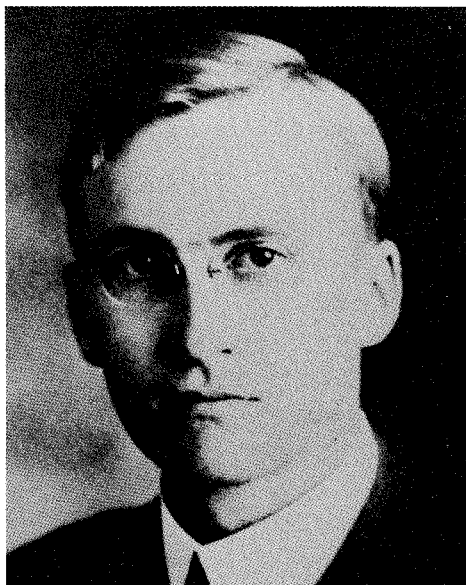


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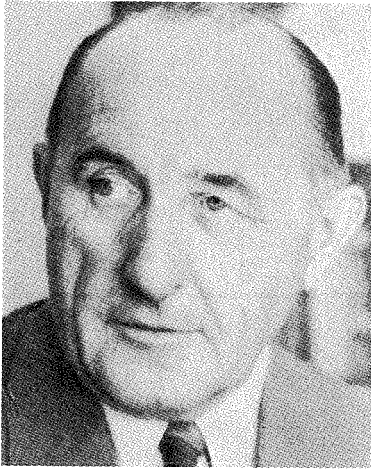
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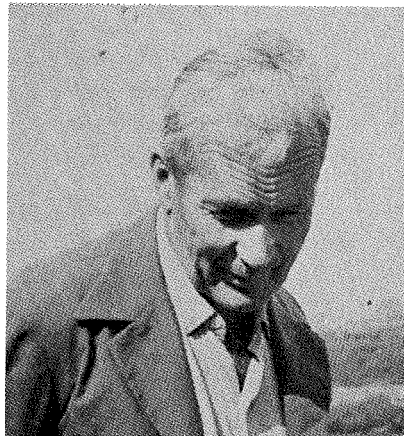


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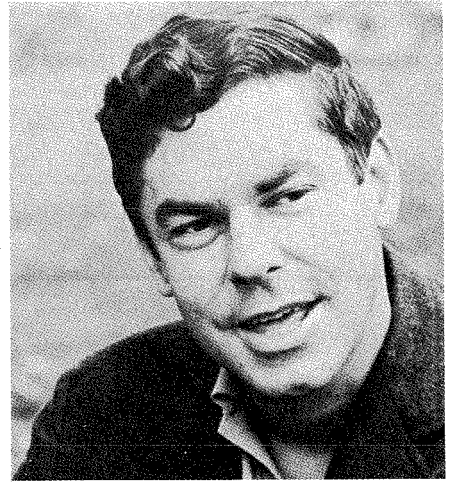


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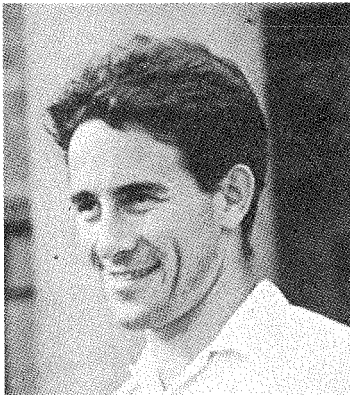




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