Design of a Multi-Moon Orbiter

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Stanford, December 4, 2002

Mission to Europa

- *Motivation: Oceans and life on Europa?*

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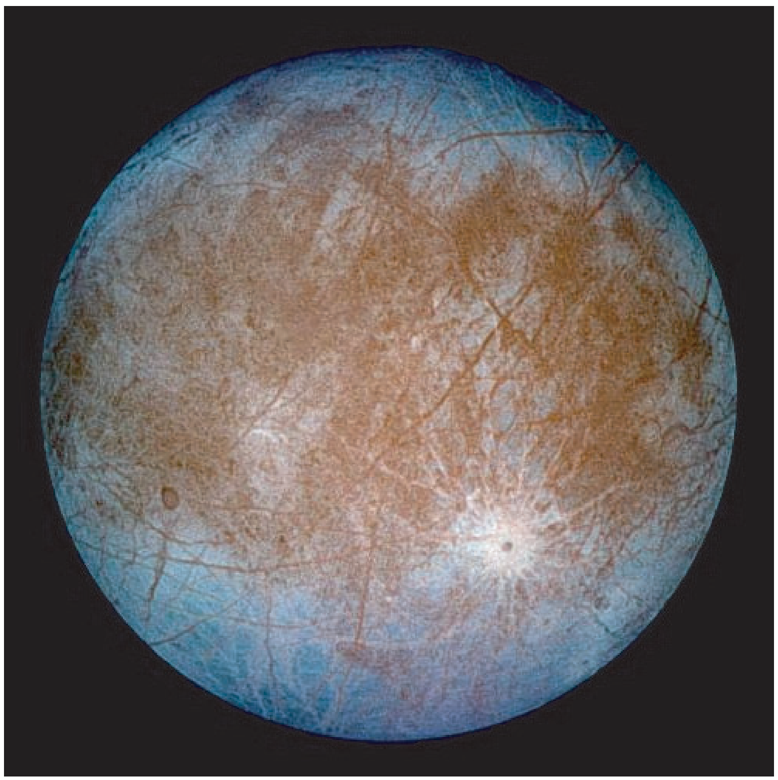
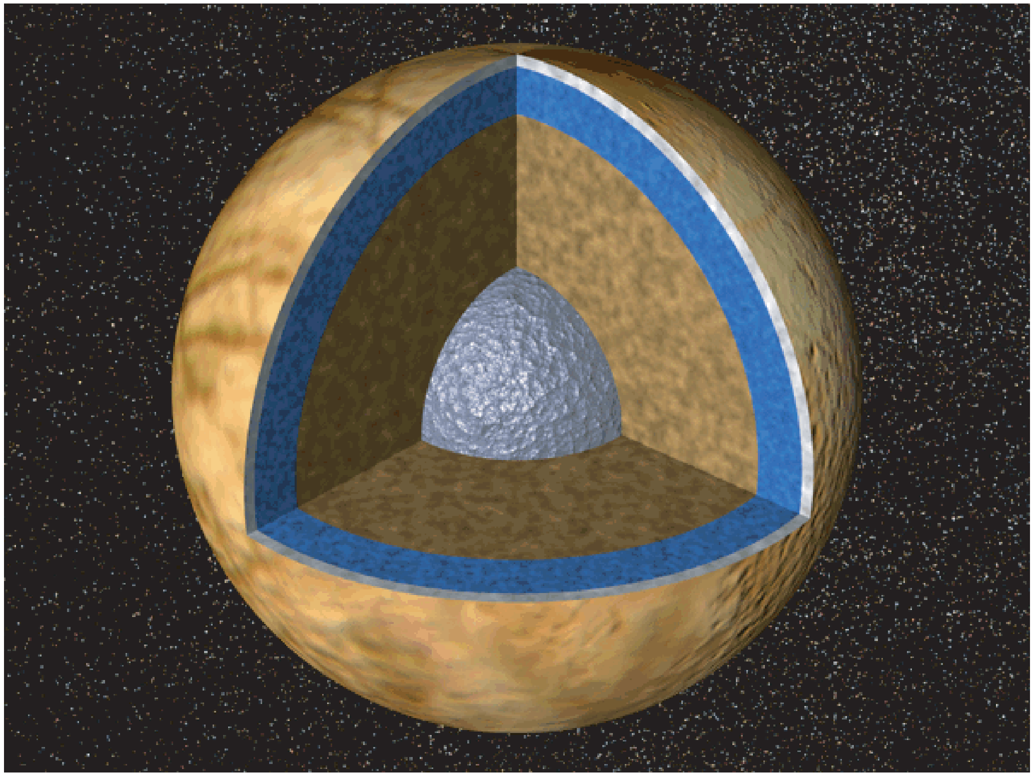
  - There is international interest in sending a scientific spacecraft to orbit and study ***Europa***





NASA's Europa Orbiter

- Original plans canceled due to budget constraints

Europa orbiter - courtesy NASA

Multi-Moon Orbiter

- *Orbit each moon in a single mission*
  - Other Jovian moons are also worthy of study
    - Evidence from *Galileo* suggests all may have oceans,

Multi-Moon Orbiter

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Multi-Moon Orbiter

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- Each moon is orbited for much longer than the quick flybys of previous missions
- Using a standard "patched-conics" approach, the ΔV necessary would be prohibitively high
- By decomposing the N-body problem into 3-body problems and using the natural dynamics of the 3-body problem, the ΔV can be lowered significantly

What is Involved

- 3-body problem dynamics

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- coupling different 3-body systems

Some History

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- *1965-present*: Research in the 3 and 4 body problems continues by *many people*.
- *1970-present*: Concrete missions (such as ICEE-3, SOHO and Hiten) begin to use dynamical systems methods in interesting ways. Especially the pioneering work of Farquhar, Simo (the Barcelona Group), Miller and Belbruno

General Three Body Problem

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- New periodic solutions found recently (Montgomery, Chenciner, Simo...)

Special 3-Body Solutions

figure 8 orbit

Special 3-Body Solutions

Exotic A

Special 3-Body Solutions

Exotic B

Restricted Circular Problem

- Two primaries move in circles; the smaller third body moves in the field of the primaries (without affecting them); view the motion in a ***rotating frame***

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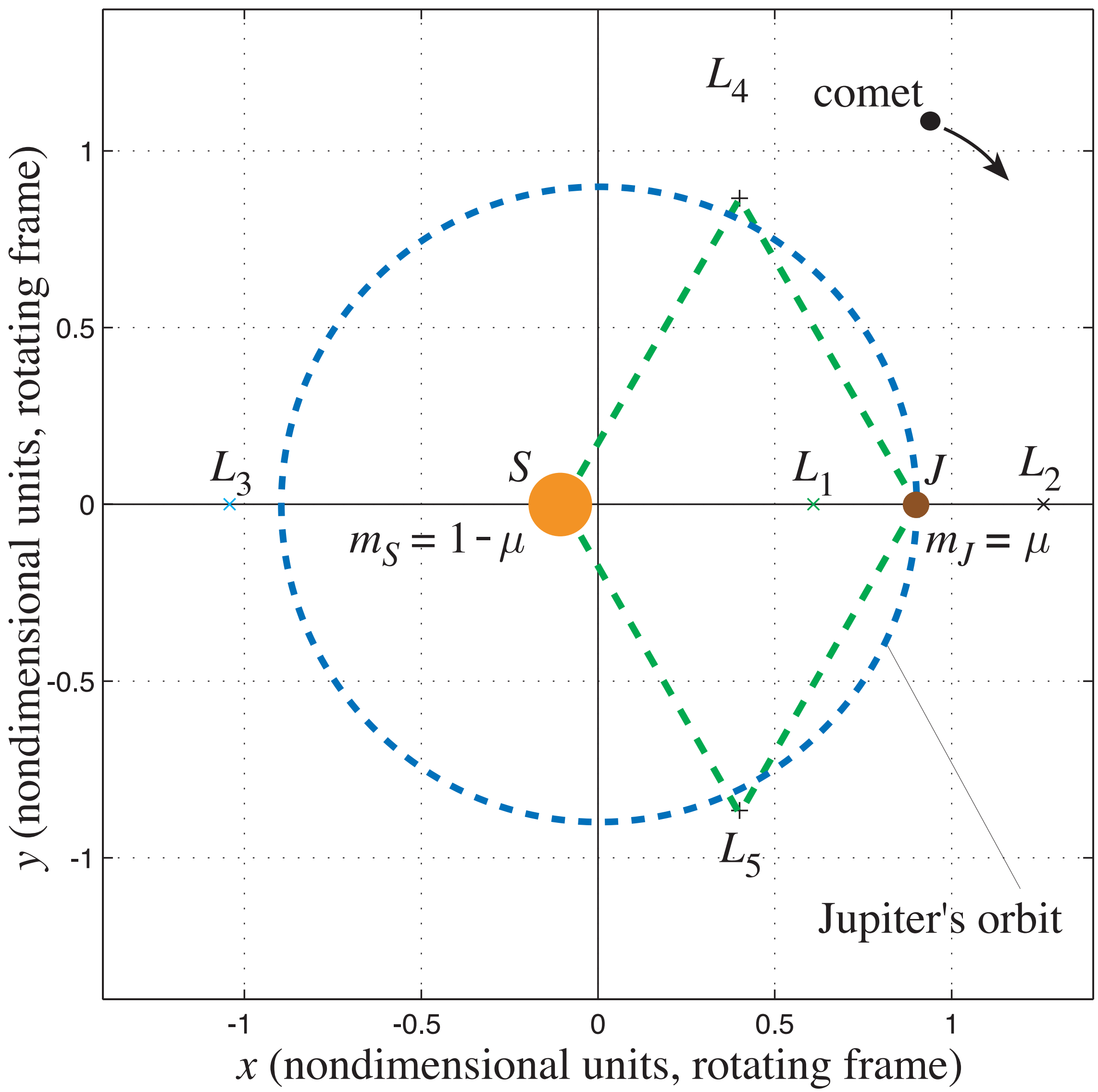
- There are five such ***equilibrium points***:
  - Three ***collinear*** (Euler,1750) on the x-axis—L_1, L_2, L_3
  - Two ***equilateral points*** (Lagrange,1760)—L_4, L_5

Restricted Circular Problem



Equilibrium points for the three body problem

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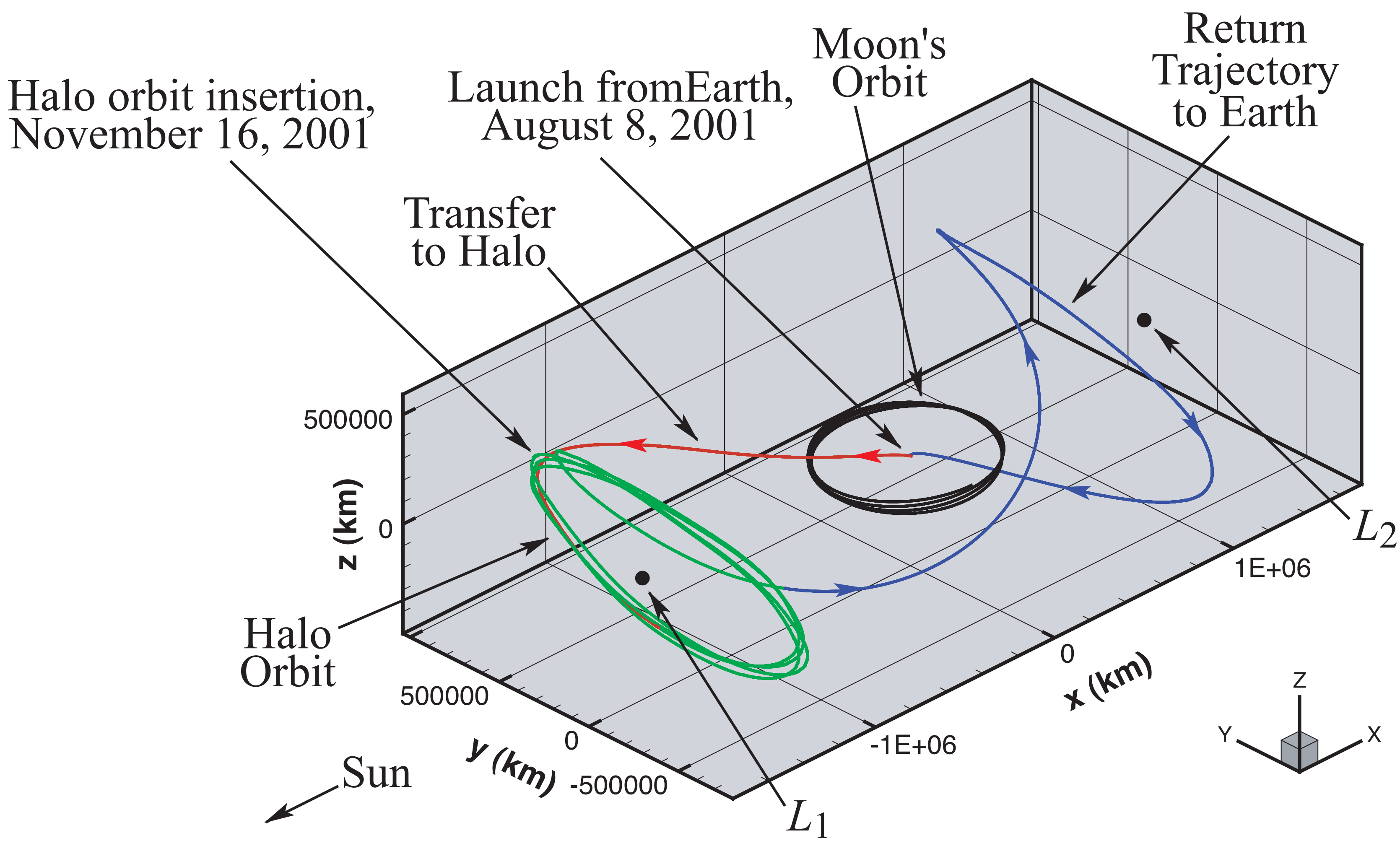
- Genesis is interesting from a dynamical systems perspective: it has a *heteroclinic return orbit* to Earth

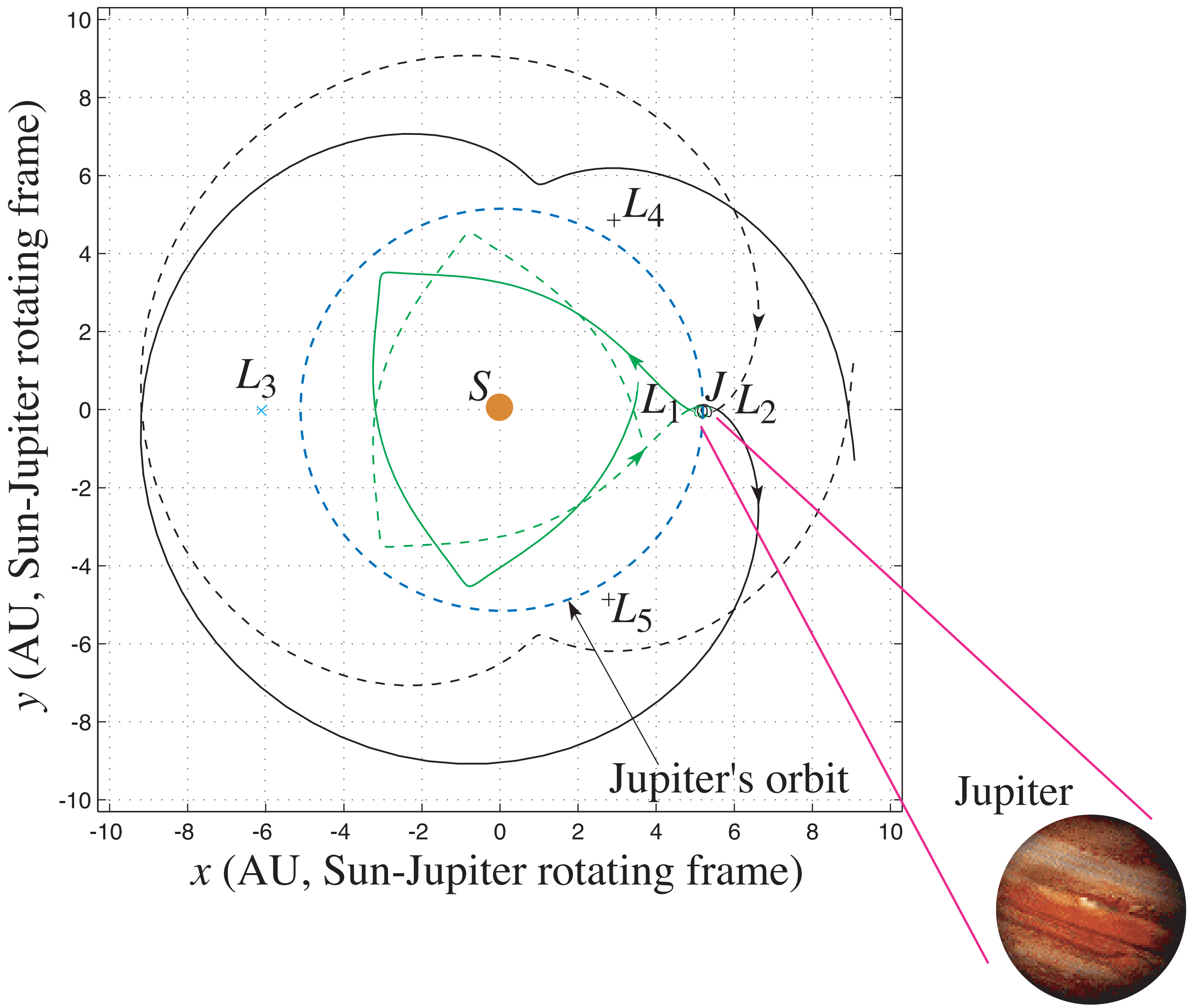
Genesis Launch–Aug 8, 2001

Genesis Spacecraft

Genesis Orbit

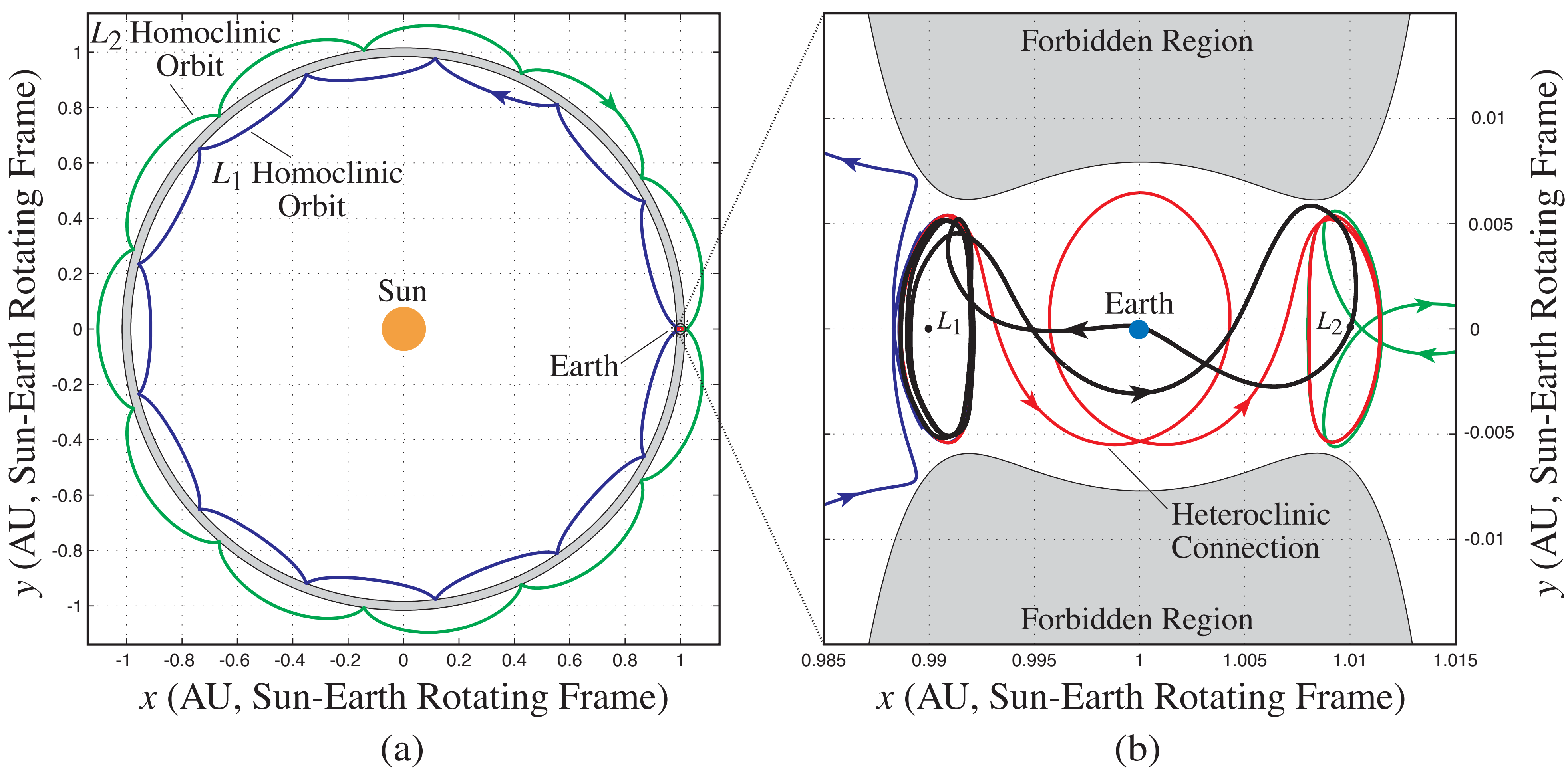


Invariant Manifolds in the 3-Body Problem



Invariant manifolds for the *Sun-Jupiter-spacecraft* 3-body problem

Invariant Manifolds–Genesis Overlay



Invariant manifolds for the *Sun-Earth-spacecraft* 3-body problem

Spatial Problem

- The dynamics of the 3-body problem relevant to the motion of comets and Genesis-type spacecraft; orbit structure and heteroclinic connections between periodic orbits now fairly well understood[1]
- In the 3D problem, connections are between tori instead of periodic orbits—one can extend most of the preceding picture to guarantee, for instance, lots of interesting high inclination orbits[2]

[1] Koon, Lo, Marsden, & Ross [2001]
[2] Gómez, Koon, Lo, Marsden, Masdemont & Ross [2001]

Equations of Motion

- consider the *planar case*—the *spatial case* is similar
- *Kinetic energy* (wrt inertial frame) in rotating coordinates:

$$K(x, y, \dot{x}, \dot{y}) = \frac{1}{2}\left[(\dot{x} - \omega y)^2 + (\dot{y} + \omega x)^2\right]$$

- *Lagrangian* is K.E. − P.E., given by

$$L(x, y, \dot{x}, \dot{y}) = K(x, y, \dot{x}, \dot{y}) - V(x, y); \quad V(x, y) = -\frac{1-\mu}{r_1} - \frac{\mu}{r_2}$$

- *Euler–Lagrange equations*:

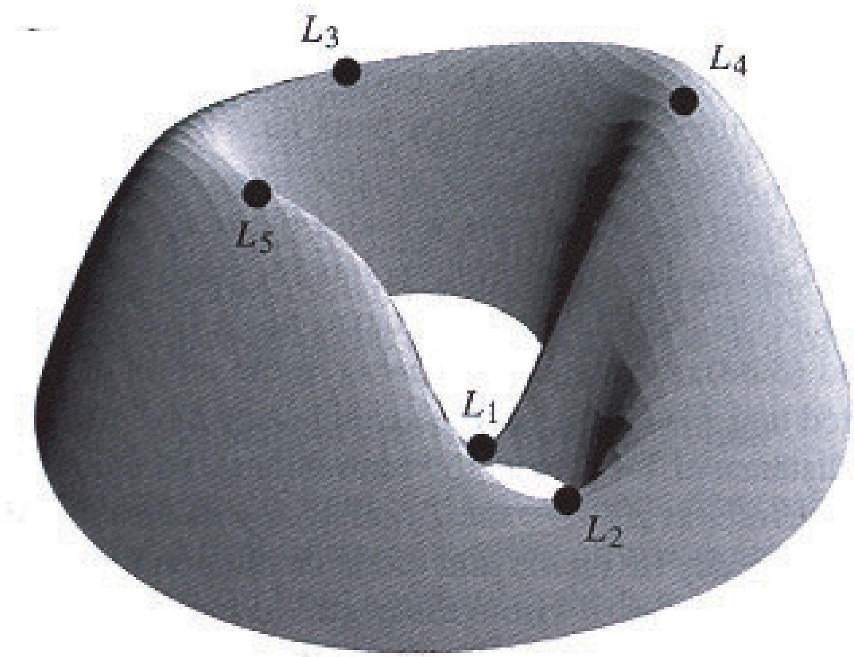
$$\boxed{\ddot{x} - 2\omega\dot{y} = -\frac{\partial V_\omega}{\partial x}, \qquad \ddot{y} + 2\omega\dot{x} = -\frac{\partial V_\omega}{\partial y}}$$

where the *effective potential* is

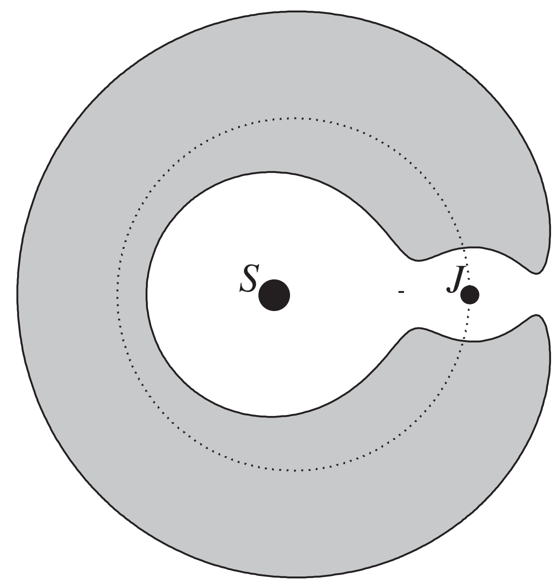
$$V_\omega = V - \frac{\omega^2(x^2 + y^2)}{2}$$

Effective potential

- In the circular planar restricted three body problem, and in a rotating frame, the equations for the third body are those of a *particle moving in an effective potential plus a magnetic field* (results of Jacobi, Hill, etc)



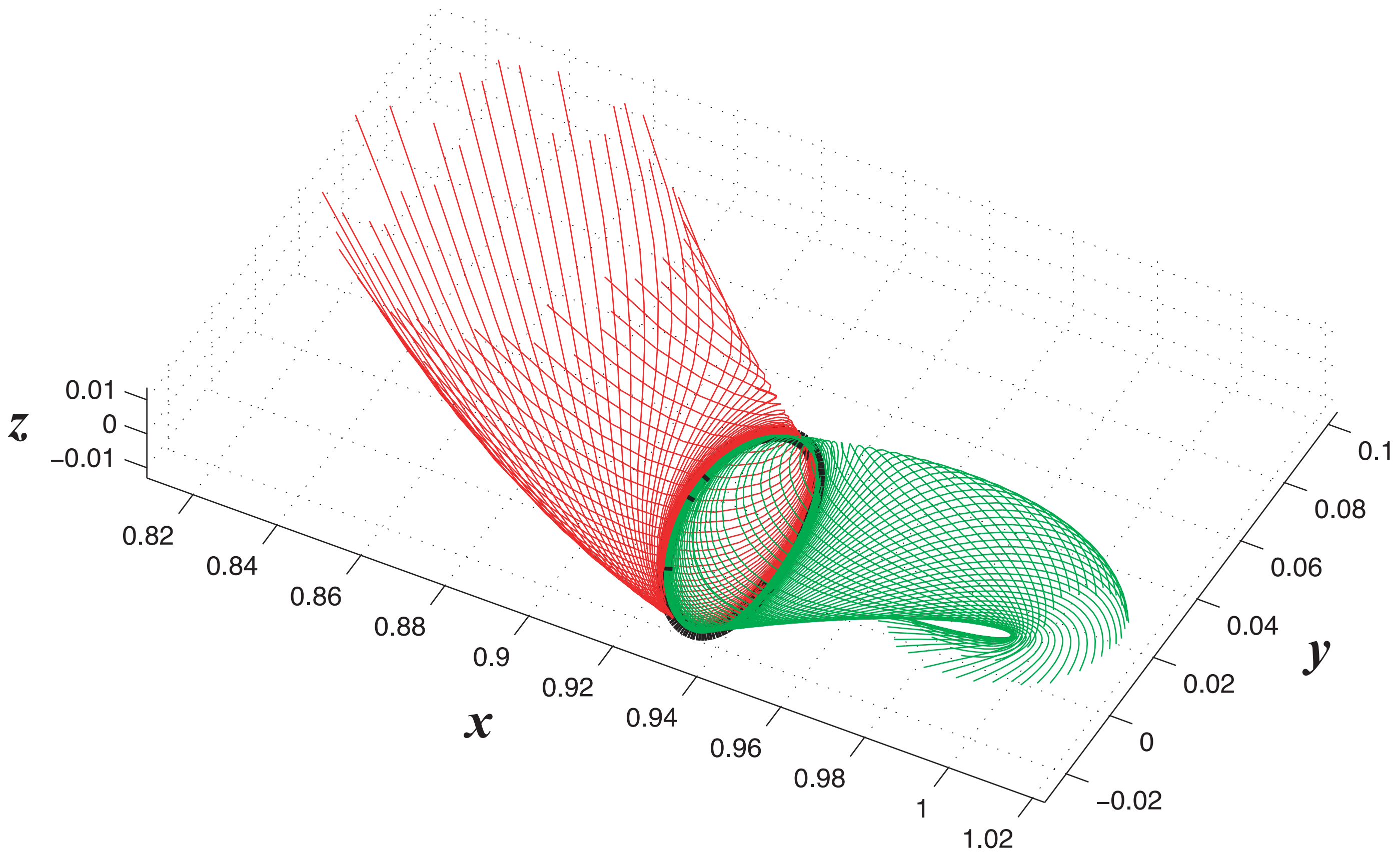
Effective Potential



Level set shows the Hill region

Invariant Manifold Tubes

- invariant manifolds of a halo orbit (projected to configuration space) for illustration
- red = unstable, green = stable



Invariant Manifold Tubes

- These manifold tubes play a crucial role in what ***passes through*** the resonance (transit orbits)
- and what ***bounces back*** (non-transit orbits)
- transit possible if you are "inside" the tube, otherwise nontransit—important for ***transport issues***

Idea of Tube-Hopping

LunarL1GatewayService.mov

Back to Jupiter's Moons

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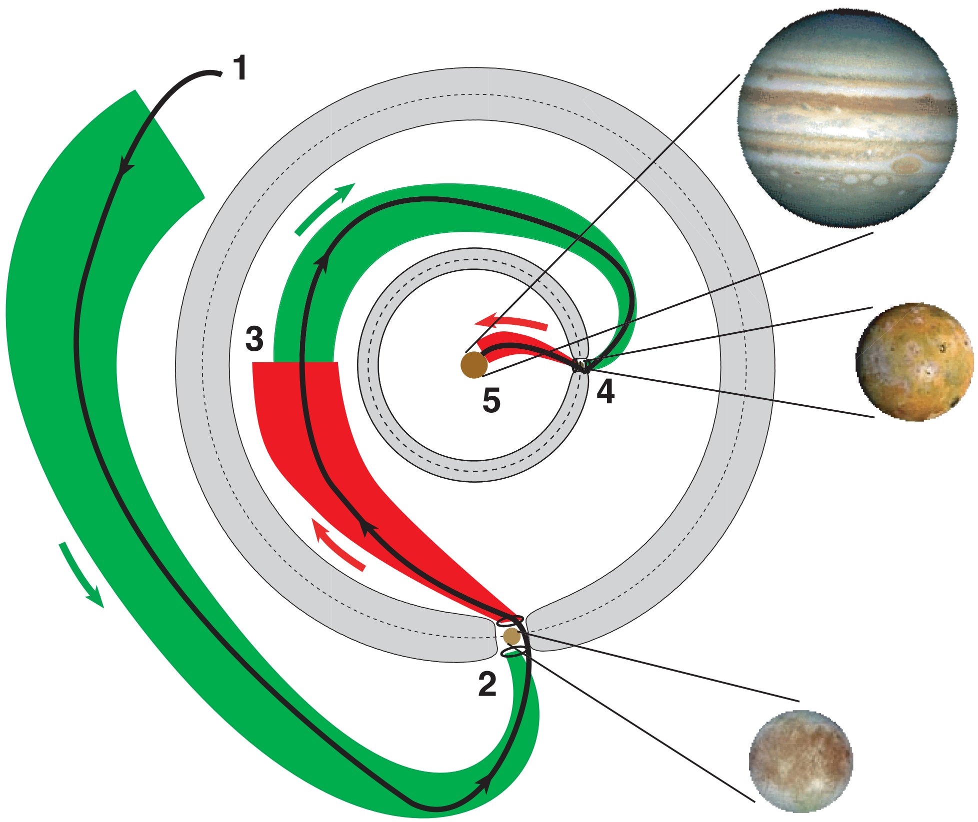
- Apply these ideas to find a trajectory that orbits multiple moons of Jupiter
- Example 1: Europa → Io → Jupiter collision

1. Begin tour
2. Europa encounter
3. Jump between tubes
4. Io encounter
5. Collide with Jupiter



Construction Strategy

- use burns (controls) that enable a transfer from one 3-body system to another via *direct McGehee-Conley tube hopping*
  - from the *Jupiter-Europa-spacecraft* system to
  - the *Jupiter-Io-spacecraft* system

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- here is a close-up of the Io encounter

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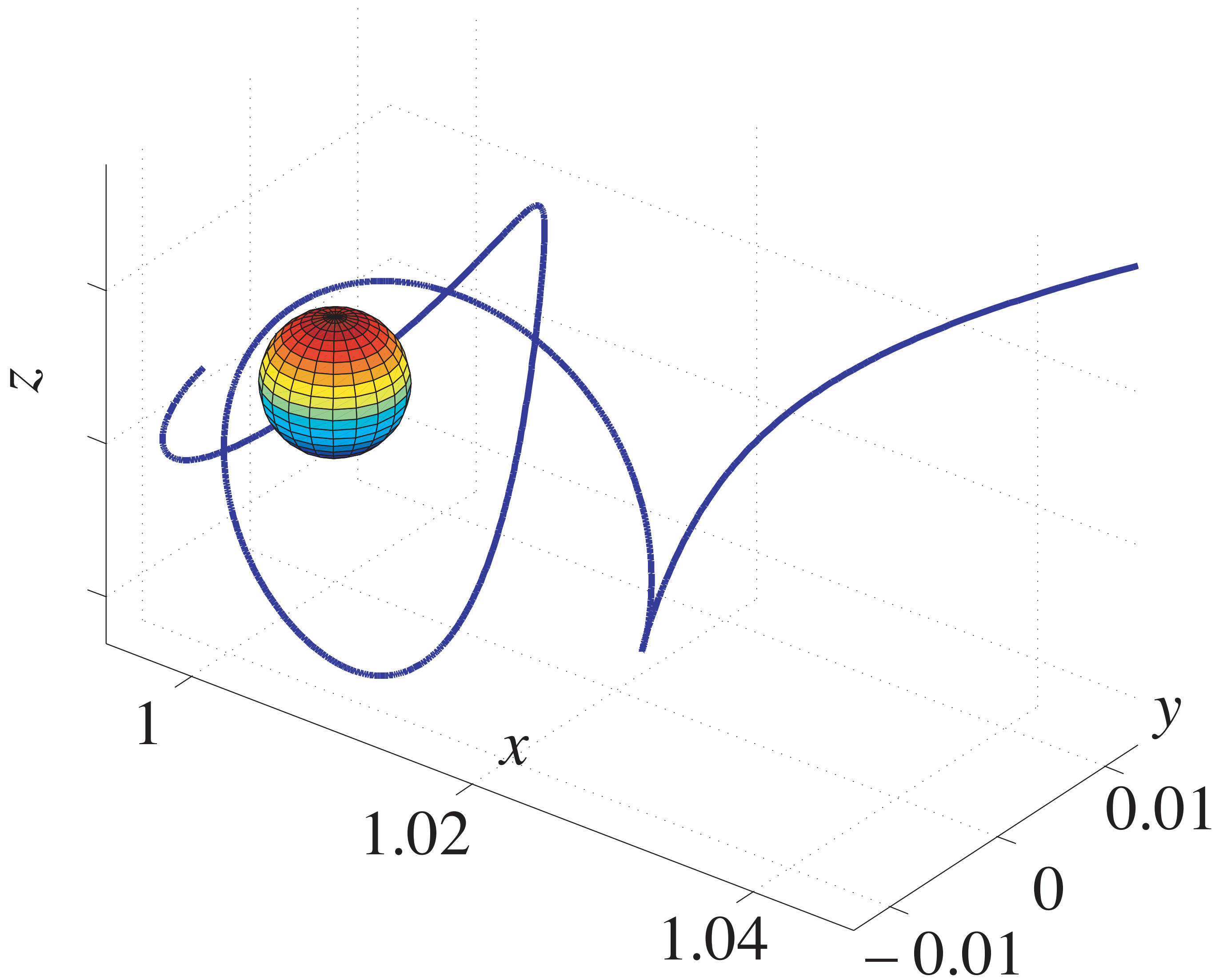


Close-up of the Io encounter

Construction Strategy

- **Example 2:** *Ganymede → Europa → orbit injection around Europa*

pgt-3d-movie-inertial.qt

Construction Strategy

pgt-3d-movie-ga.qt

Construction Strategy

pgt-3d-movie-eu.qt

Multi-Moon Orbiter—Refinement

- Preceding ΔV of 1400 m/s for the Ganymede-Europa orbiter was half the Hohmann transfer (that is, using patched conics, as in manned moon missions)
- Desirable to decrease ΔV further—one now does not *directly* "tube-hop", but rather makes more refined use of the phase space structure
- New things: *resonant gravity assists* with the moons
- Interesting: still fits well with the tube-hopping method

Some History

- *Resonance Gravity Assists*
  - *1890s*: Poincaré: repeated close encounters of a particle with the second primary in the 3-body problem can change its orbit from one Keplerian ellipse to another, termed “second species solutions”
  - *1960s–1980s*: Arenstorf, Perko, Breakwell, Guillaume, and Henrard consider periodic second species solutions
  - *1990s*: Bollt, Meiss, Schroer, and Ott contruct Earth to Moon trajectories using lunar resonances; Sweetser et al. use resonance hopping for initial Europa Orbiter trajectory; Belbruno, Brian Marsden, Lo, and Ross consider resonance hopping of comets
  - *1999–2001*: Schoenmaekers, Horas, and Pulido use lunar resonances to reach moon in design of ESA’s SMART-1, to launch in March 2003

Some History

- *2000-present*: Barrabés, Font, Gómez, Nunes, and Simó (part of the Barcelona group) systematically study jumping between resonant orbits; Koon, Lo, Marsden, and Ross at Caltech systematically study jumping between interior and exterior resonances and its application to space mission trajectory design

Some History

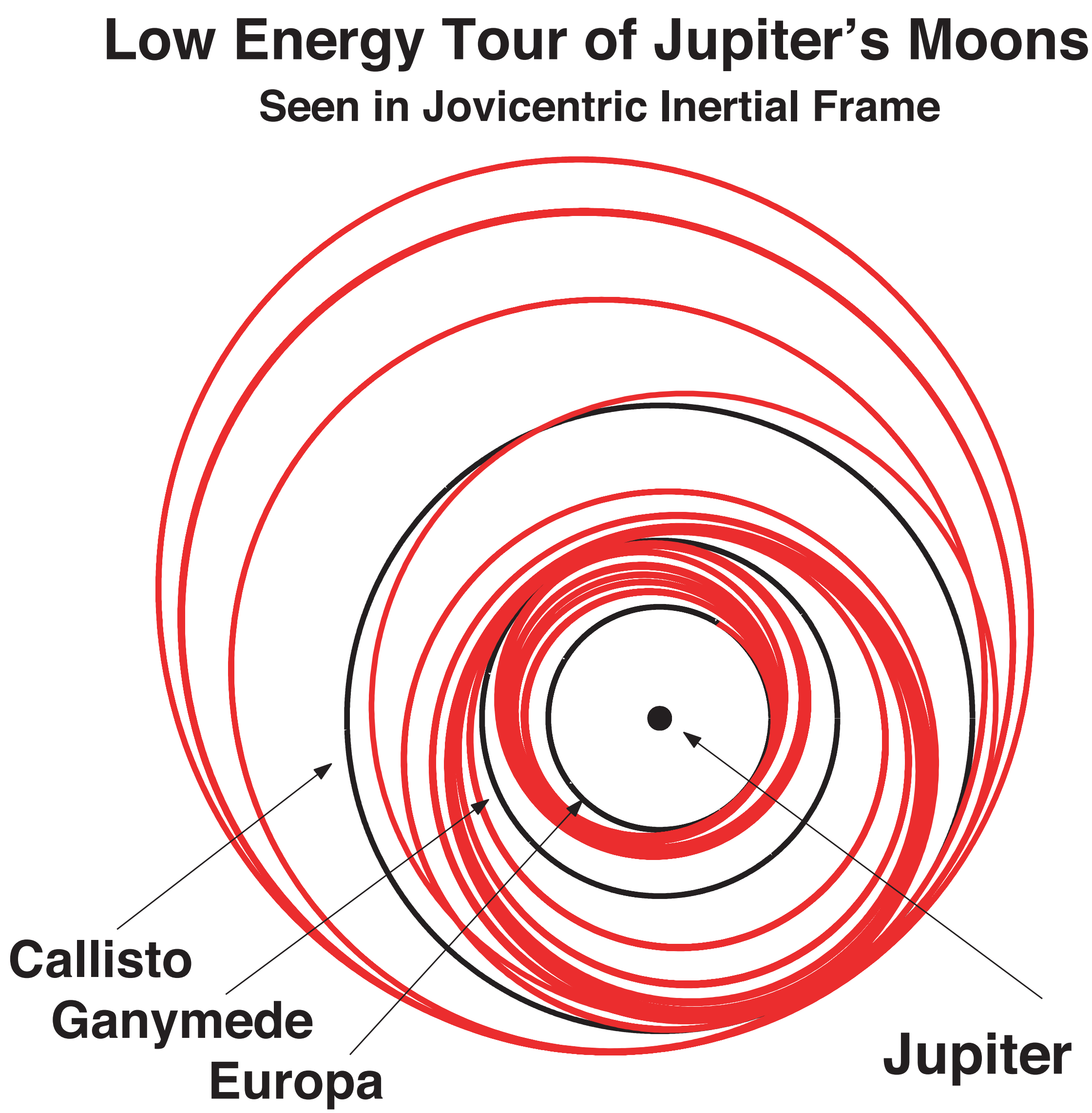
- *Ballistic Capture/Escape & Patched Three-Body Model*
    - *1950s-1960s*: Moser, McGehee, Conley, at al. make fundamental contributions to the 3-body problem
    - *1990s*: Belbruno and Miller save the *Hiten* mission using a ballistic capture by the Moon
    - *2000-present*: Koon, Lo, Marsden, and Ross develop *tube dynamics* to systematically study missions using ballistic capture and escape; patched three-body model developed to design missions such as "Shoot the Moon" and MMO—the *Multi-Moon Orbiter*

Introductory Remarks

- **Consider the following tour of Jupiter's moons**
  - Begin in an orbit about Jupiter that grazes Callisto's orbit at perijove (point of closest approach to Jupiter), which is achievable using a patched-conics trajectory from the Earth to Jupiter, just like Galileo
  - Goal: ***orbit Callisto, Ganymede, and Europa***

Example of a Resulting Orbit

- $\Delta V = 22$ m/s, but flight time is a few years



Features

- Model is a restricted bi-circular 5-body problem
- A user-assisted algorithm was necessary to produce it
- Future Goal: An automated algorithm
- The flight time is too long; should be reduced below 18 months (according to NASA)
- Evidence from other situations (such as lunar missions) suggests that *a significant decrease in flight time can be gained for a modest increase in ΔV*
- Radiation dose is not accounted for; will be included in future models—affects mission lifetime and approach strategy

Construction Procedure

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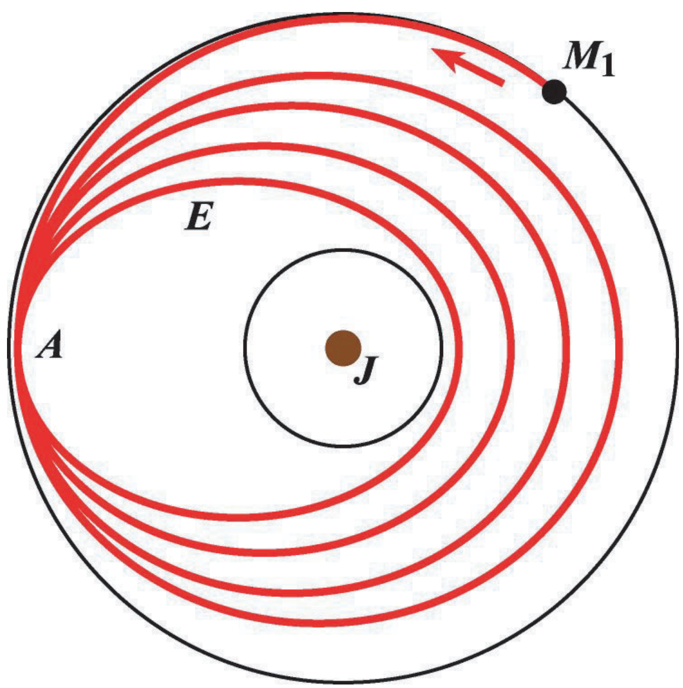
  - ***Inter-moon transfer:*** decreasing Jovian energy via resonance gravity assists
  - ***Orbiting each moon:*** ballistic capture and escape

Inter-Moon Transfer

- Spacecraft gets a gravity assist from outer moon M_1 when it passes through apoapse if ***near a resonance***
- When periapse close to inner moon M_2's orbit is reached, it takes "control"; this occurs for ellipse E

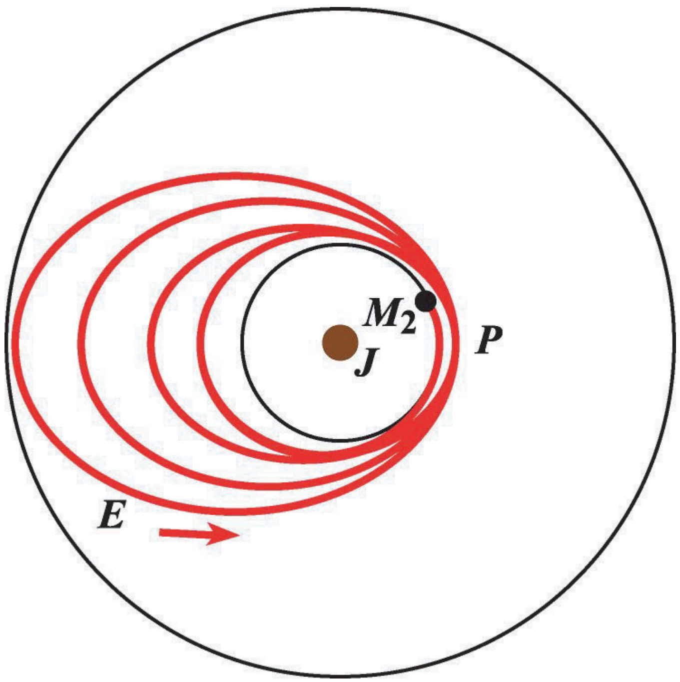
Leaving moon M_1

Apoapse A fixed



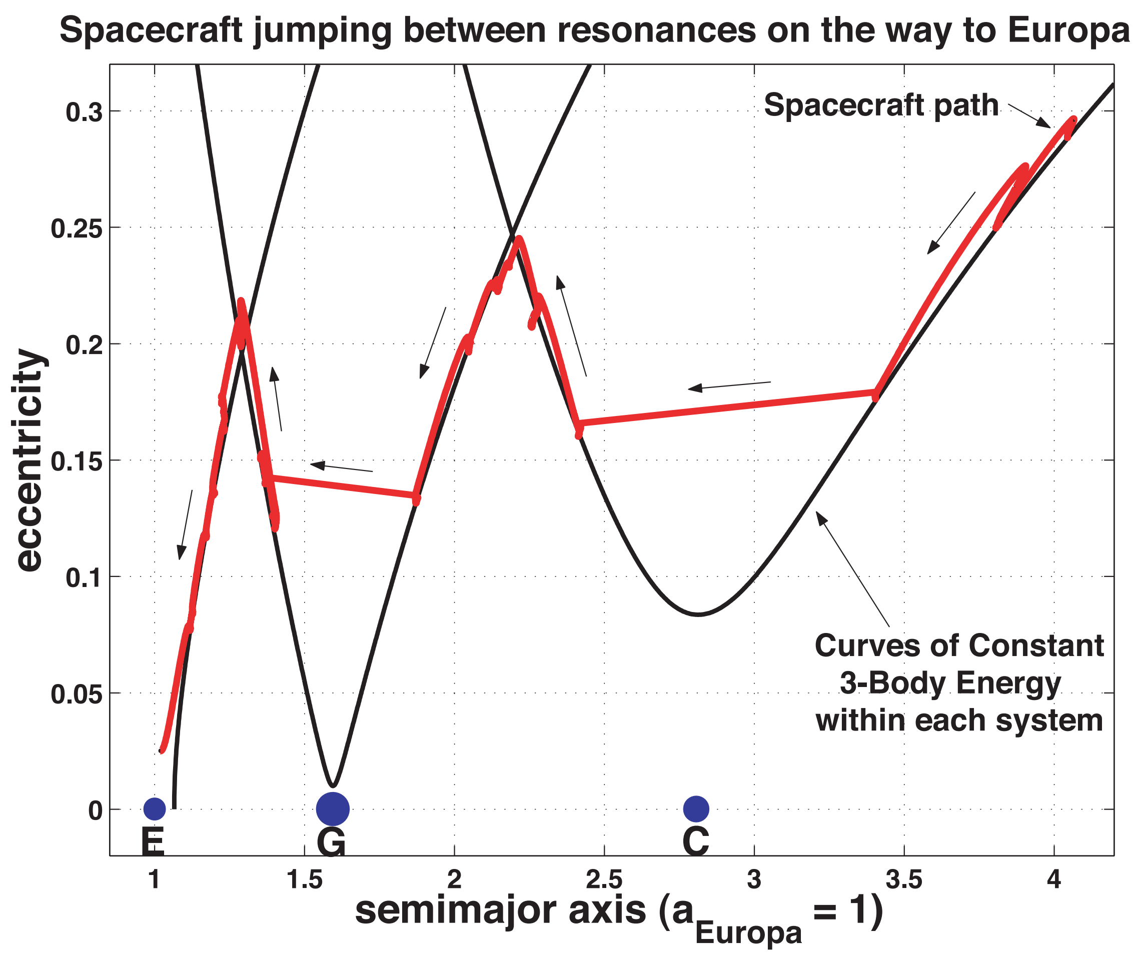
Approaching moon M_2

Periapse P fixed



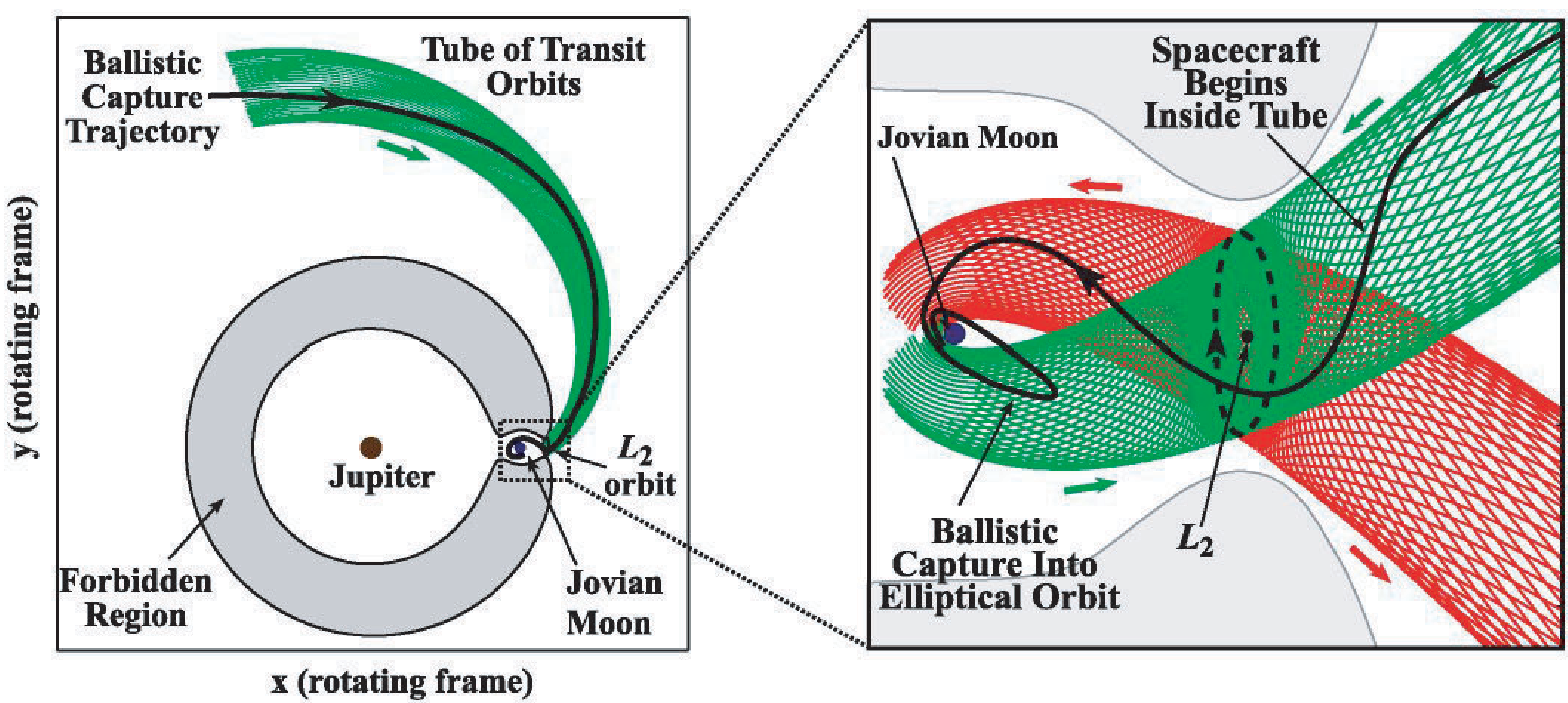
Inter–Moon Transfer

- The transfer between three-body systems occurs when energy surfaces intersect (similar to the Tisserand plots by Longuski et. al.); this can be seen on semimajor axis vs. eccentricity diagram



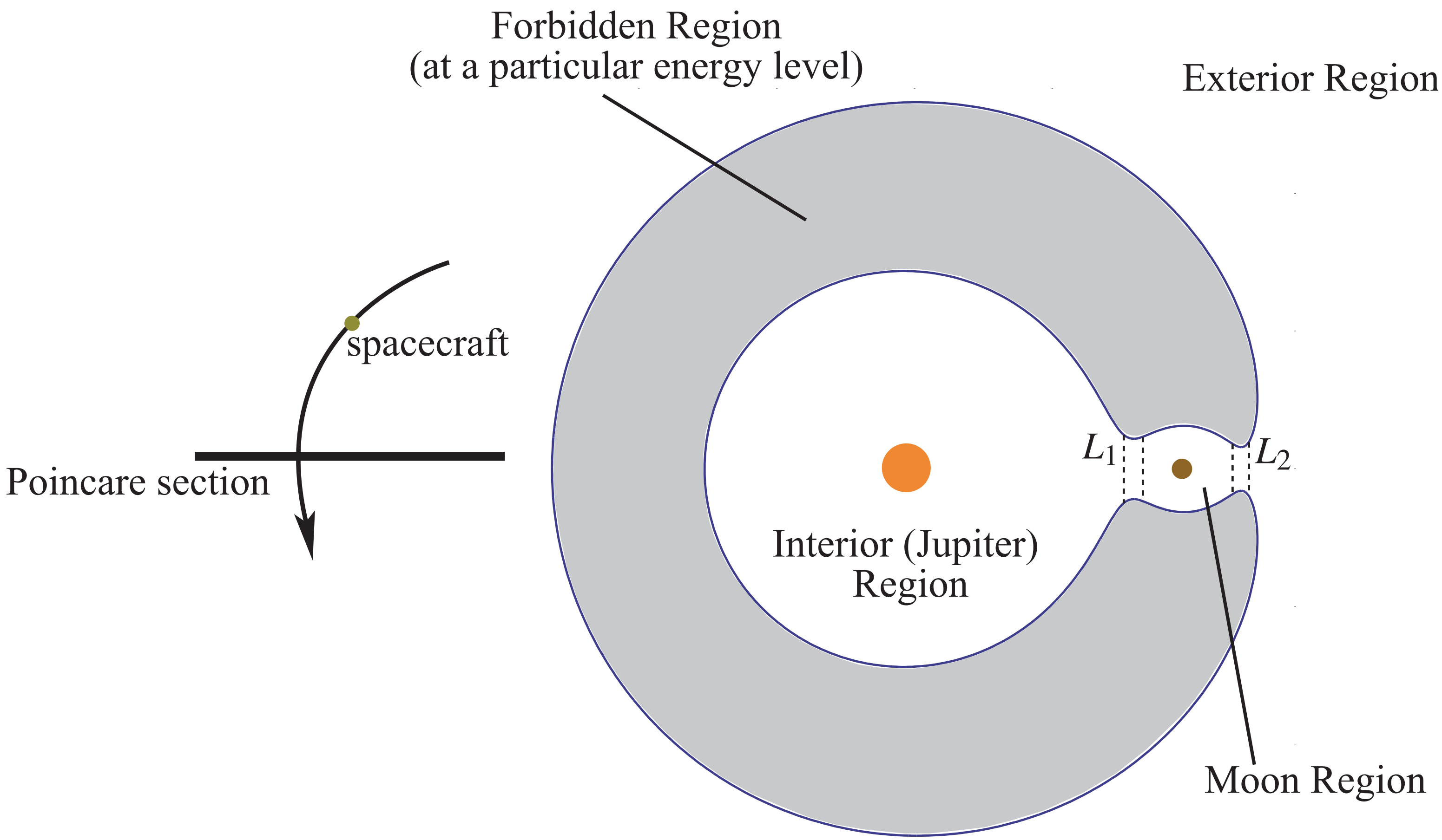
Ballistic Capture

- An L_2 orbit manifold tube leading to ballistic capture around a moon is shown schematically
- Escape is the time reverse of ballistic capture



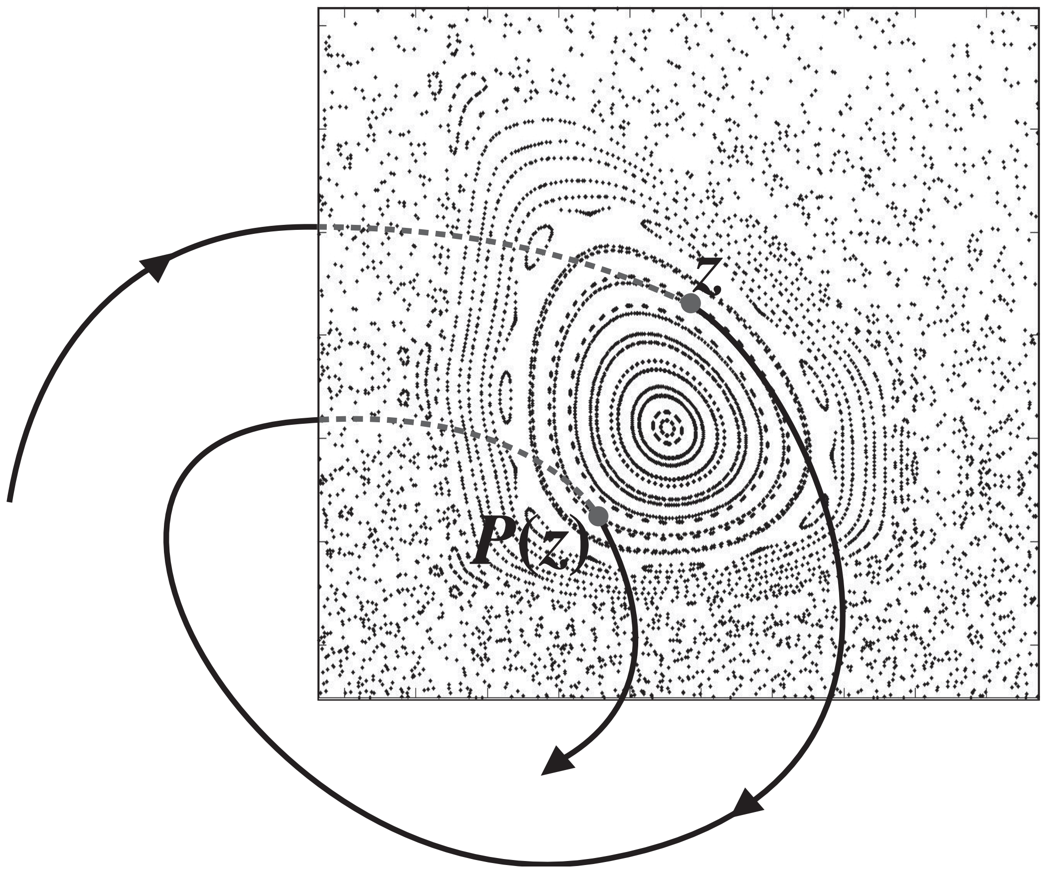
Why Does It Work?

- Recall: *planar circular restricted three-body problem*—motion of a spacecraft in the gravitational field of two larger bodies in circular motion



Poincaré Surface of Section

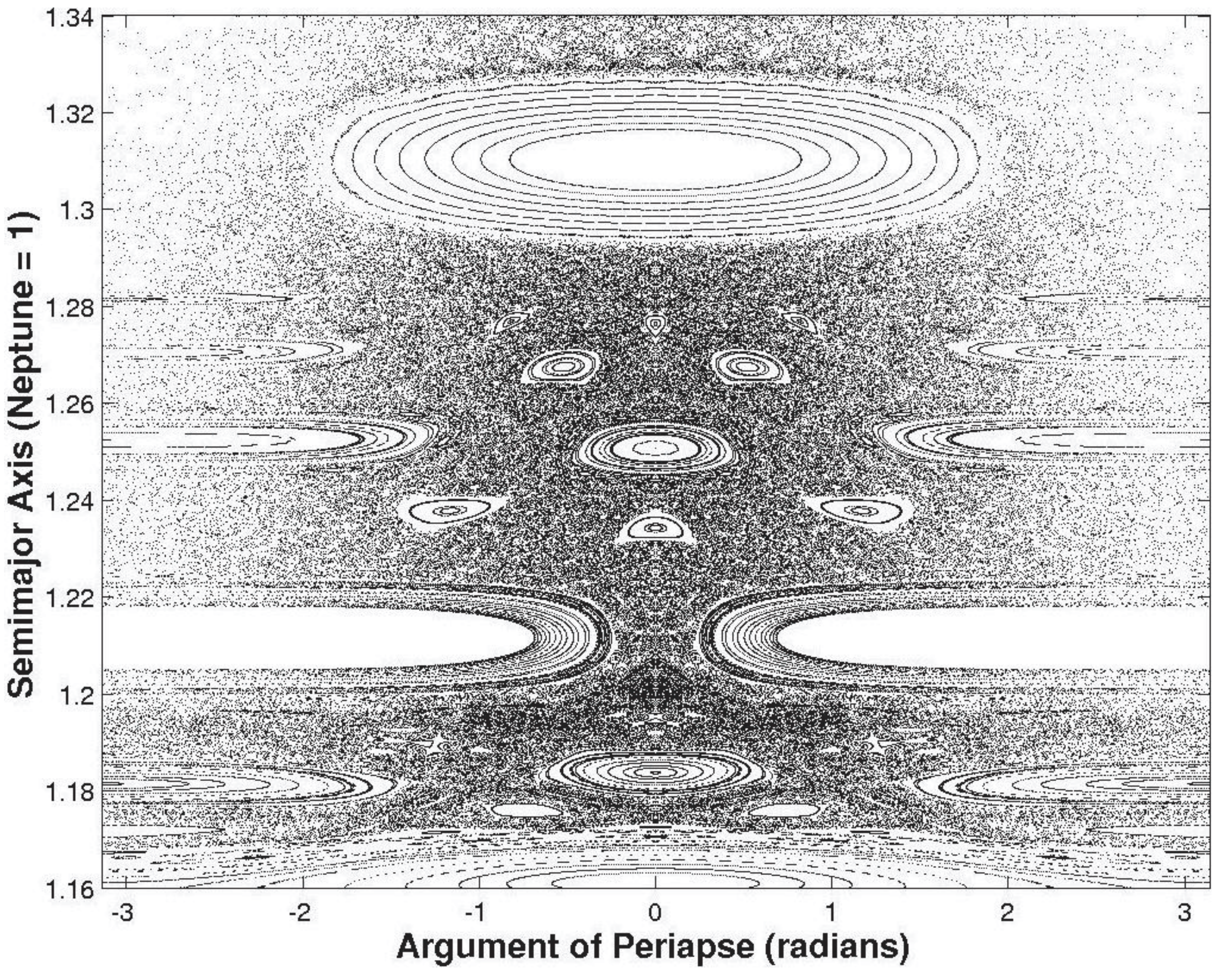
- Study Poincaré surface of section at fixed energy E, reducing system to a 2-dimensional area preserving map



Poincaré surface of section

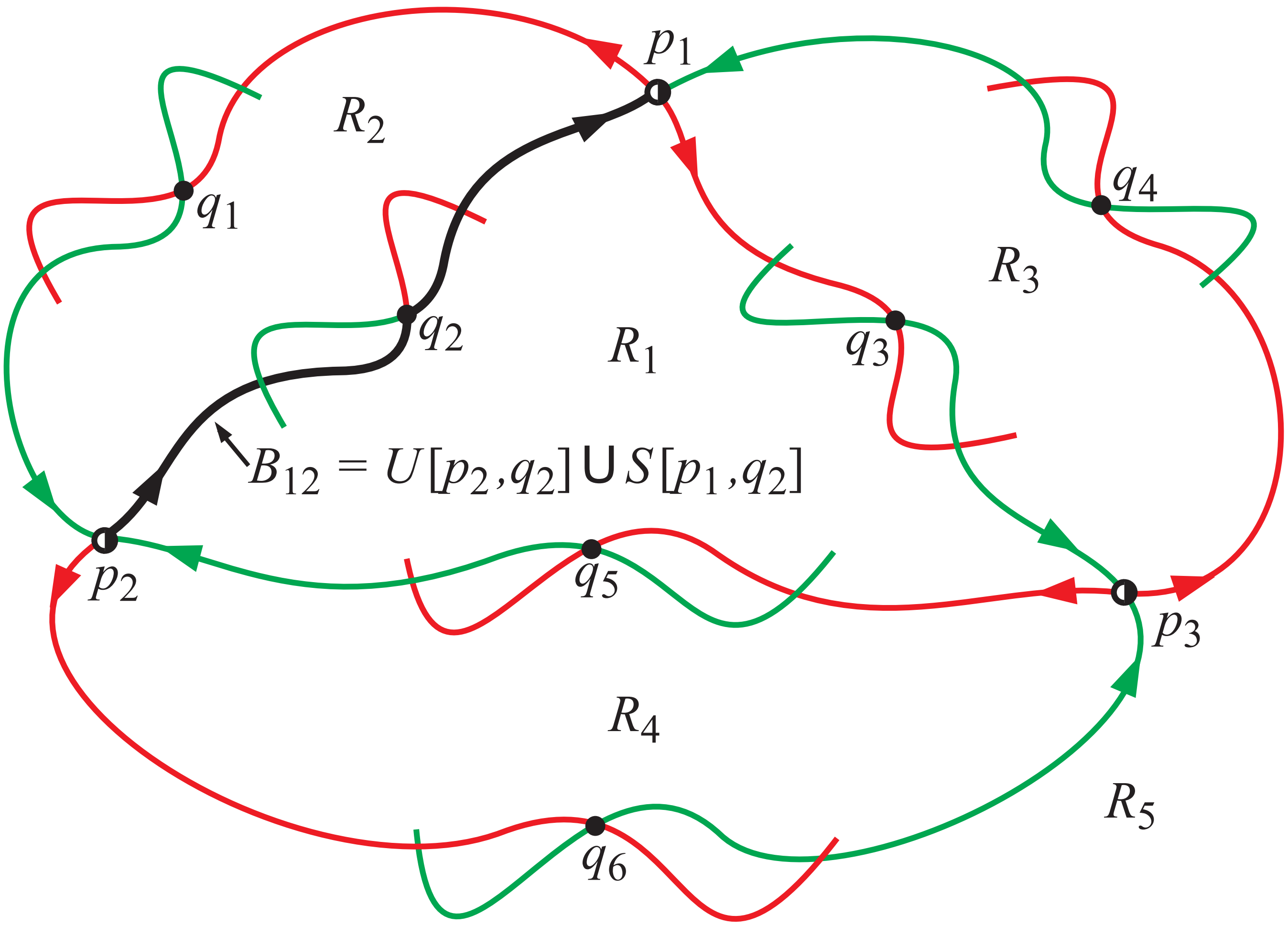
Poincaré Surface of Section

- Poincaré section reveals mixed phase space structure: KAM tori and a “chaotic sea” are visible.



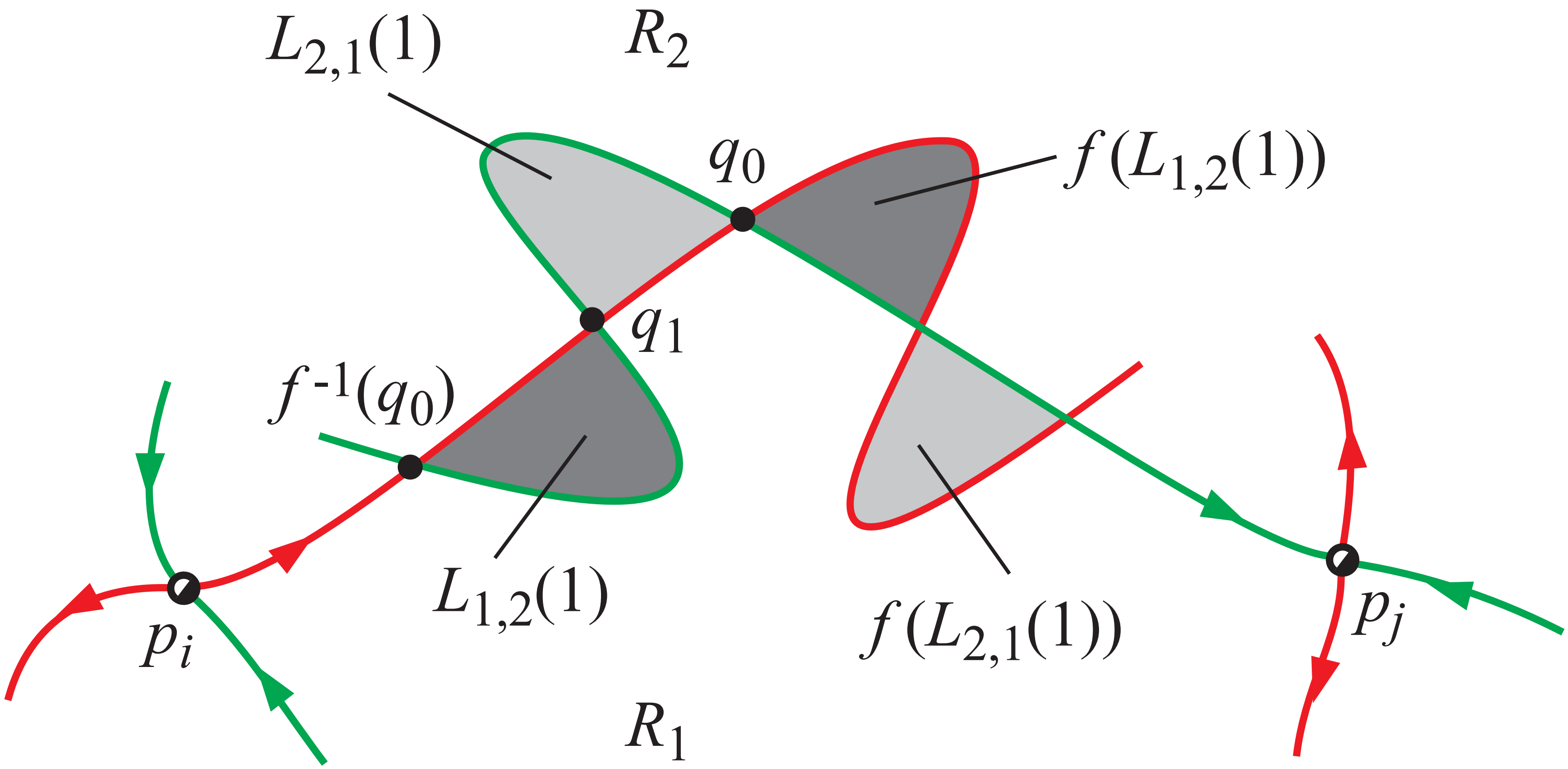
Transport in Poincaré Section

- Phase space divided into regions $R_i, i = 1, ..., N_R$ bounded by segments of stable and unstable manifolds of unstable fixed points.



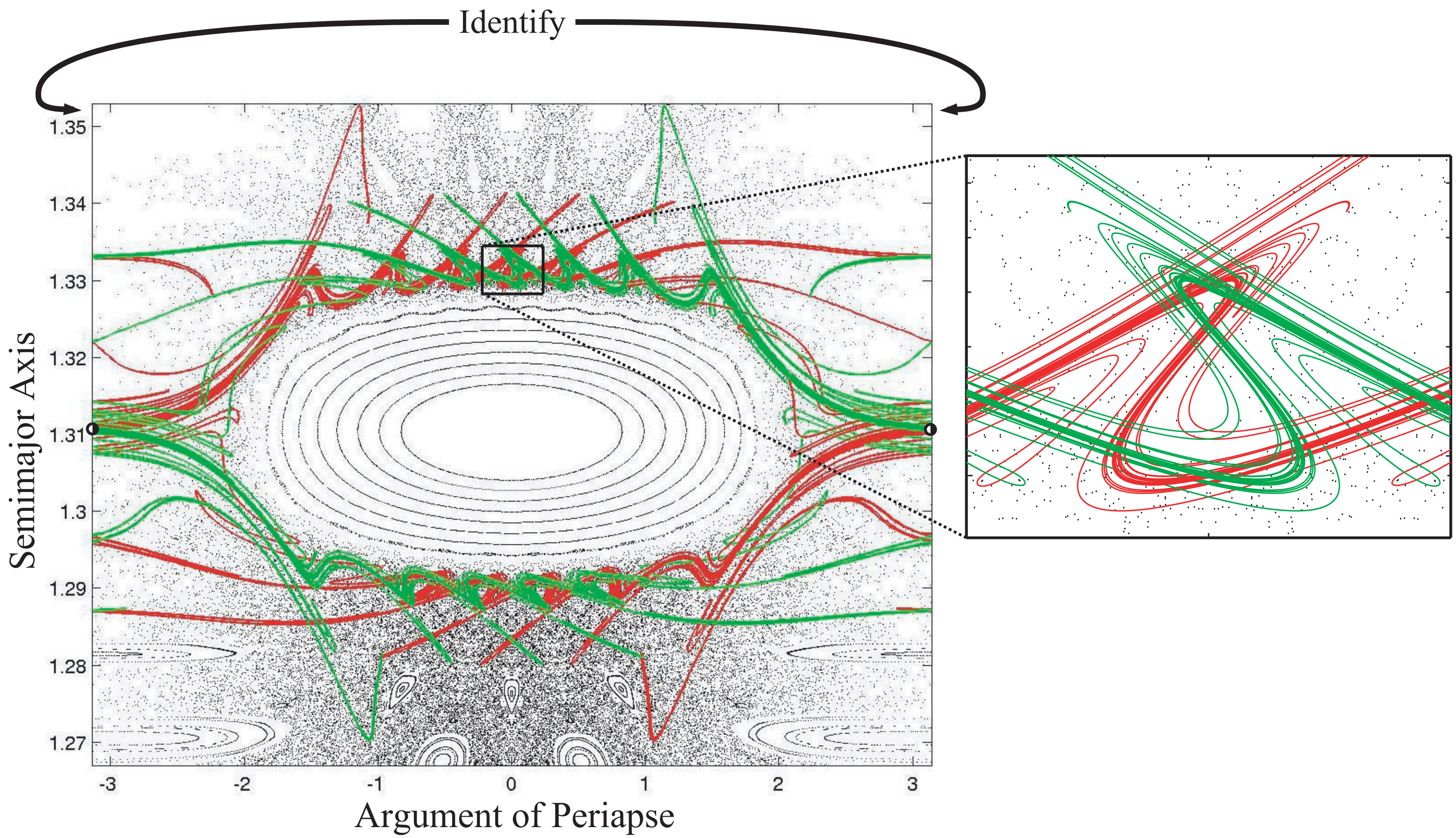
Lobe Dynamics

Transport btwn regions computed via ***lobe dynamics.***



Movement Between Resonances

We can compute manifolds which naturally divide the phase space into ***resonance regions.***

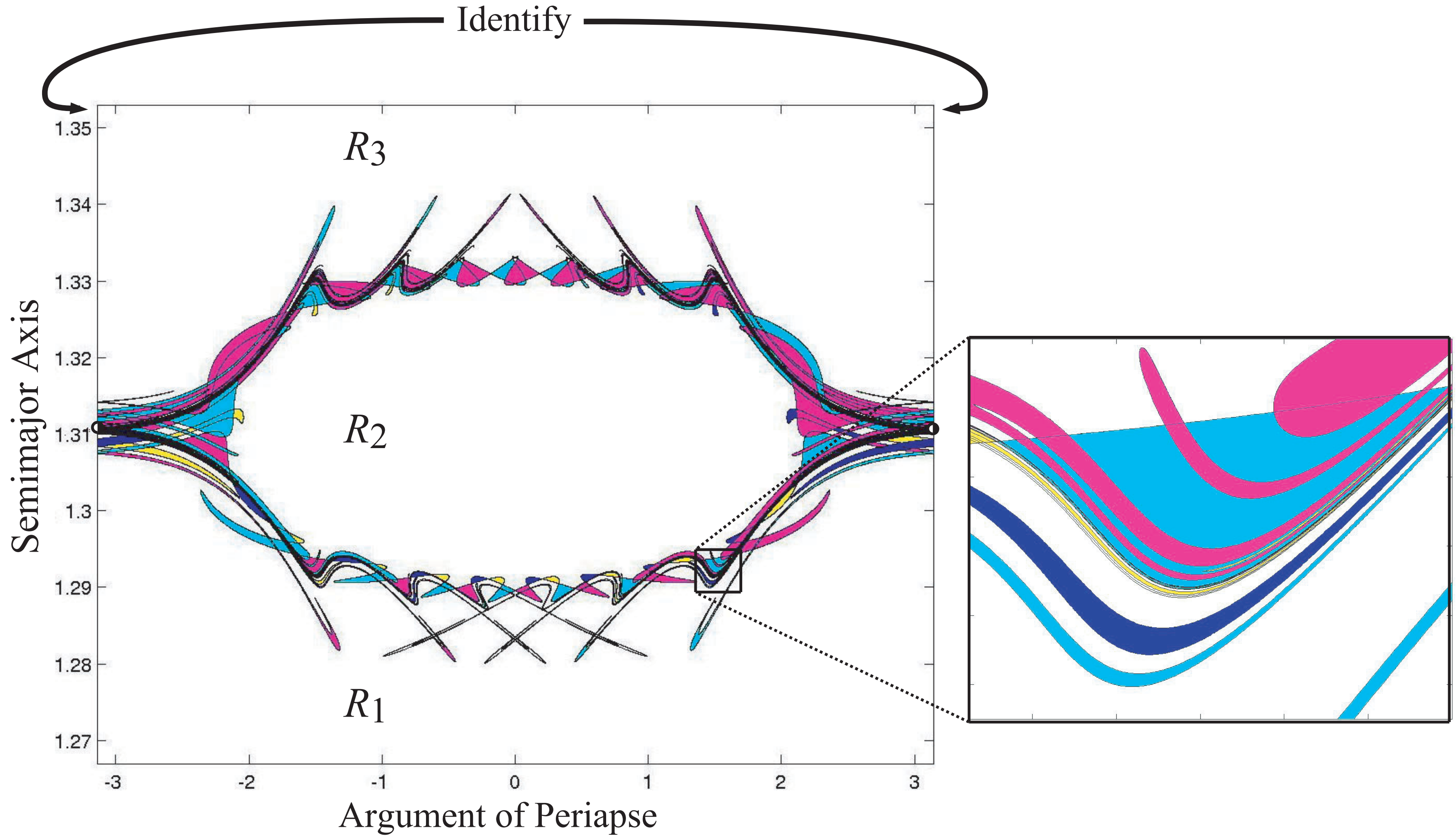


Unstable and stable manifolds in *red* and *green*, resp.

Movement Between Resonances

Transport and mixing between regions can be computed.

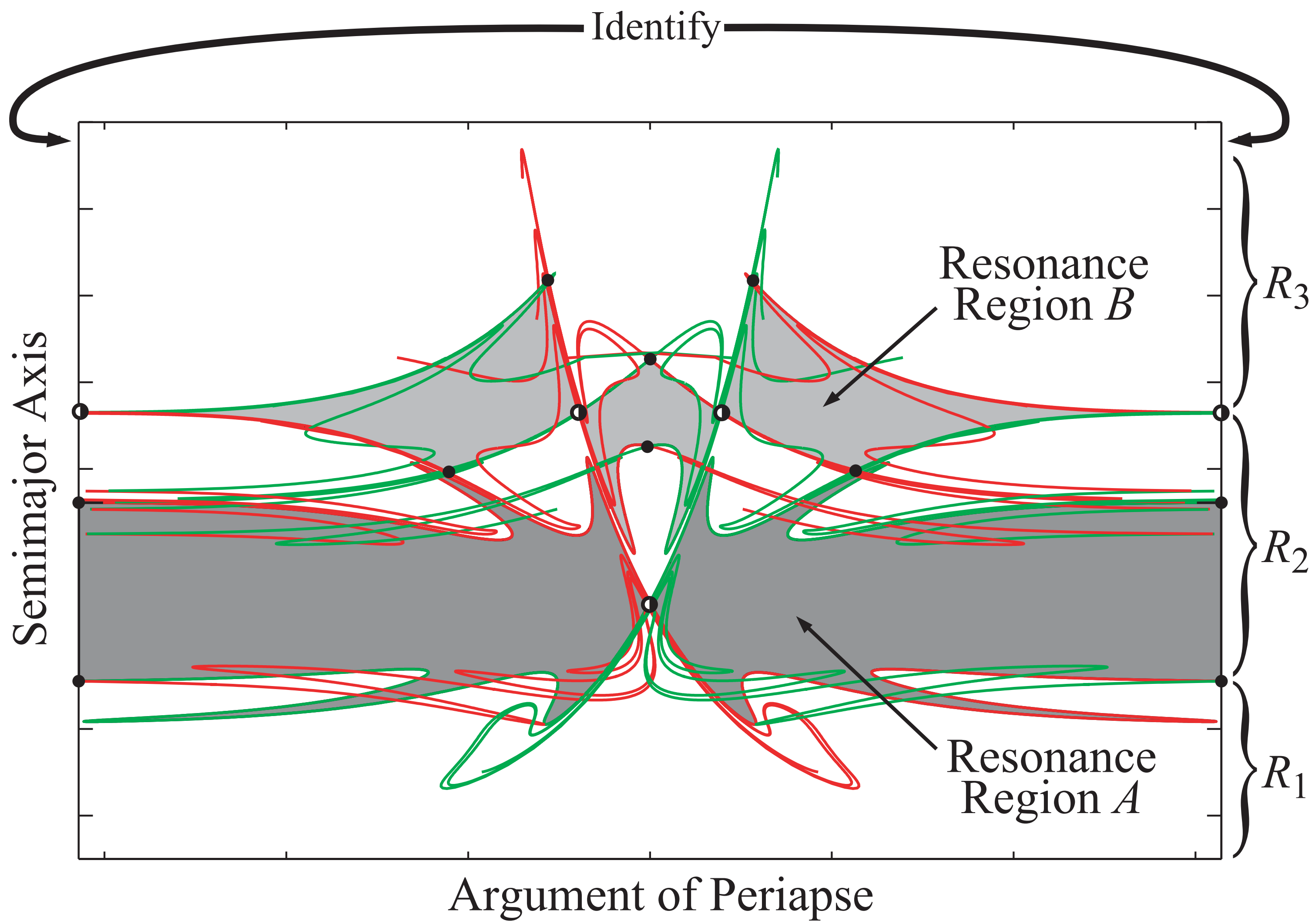
Movement Between Resonances



Four sequences of color coded lobes are shown.

Movement Between Resonances

Navigation from one resonance to another, essential for the Multi-Moon Orbiter, can be performed.



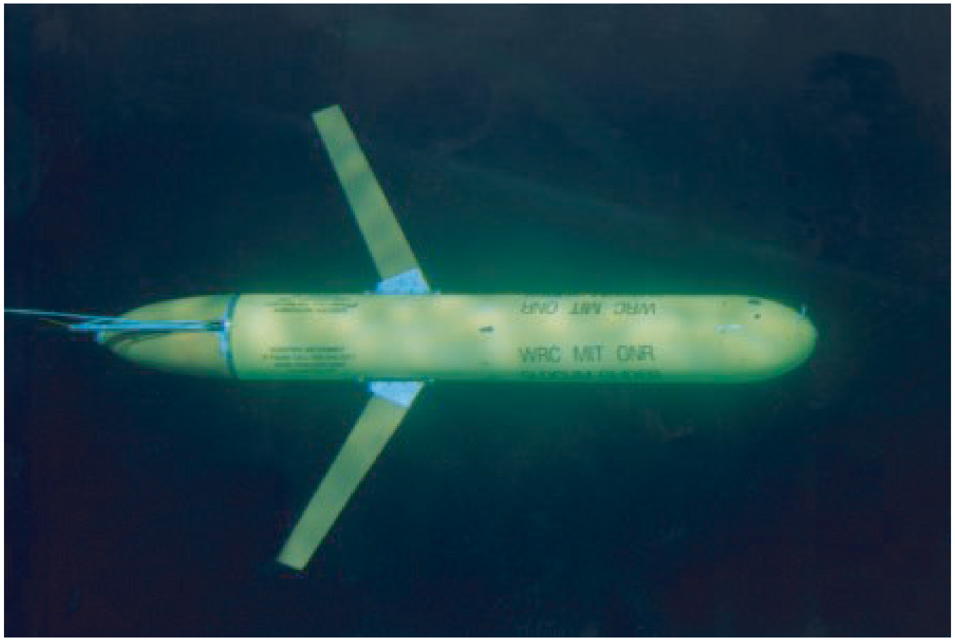
Oceanic Interlude

-

Oceanic Interlude

- The software used to compute transport by lobe dynamics, namely *MANGEN*, comes from a study of ocean dynamics.
- Interesting: there are analogs of navigating by invariant manifolds in the ocean.
- Adaptive Ocean Sampling Network (AOSN-II)
  - *Princeton:* Naomi Leonard, Clancy Rowley, Eddie Forelli, Ralf Bachmayer, ...
  - *Caltech:* Chad Couliette, Francois Lekien, JEM, Shawn Shadden
  - *MIT:* George Haller

AOSN-II Remarks



10 UWG's to be launched in summer, 2003, in Monterey Bay
Naomi Leonard, Clancy Rowley, JM $\subset$ the ONR AOSN-II team.

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  - *Management of instabilities* via invariant manifolds

Sample Maneuver: Monterey Bay

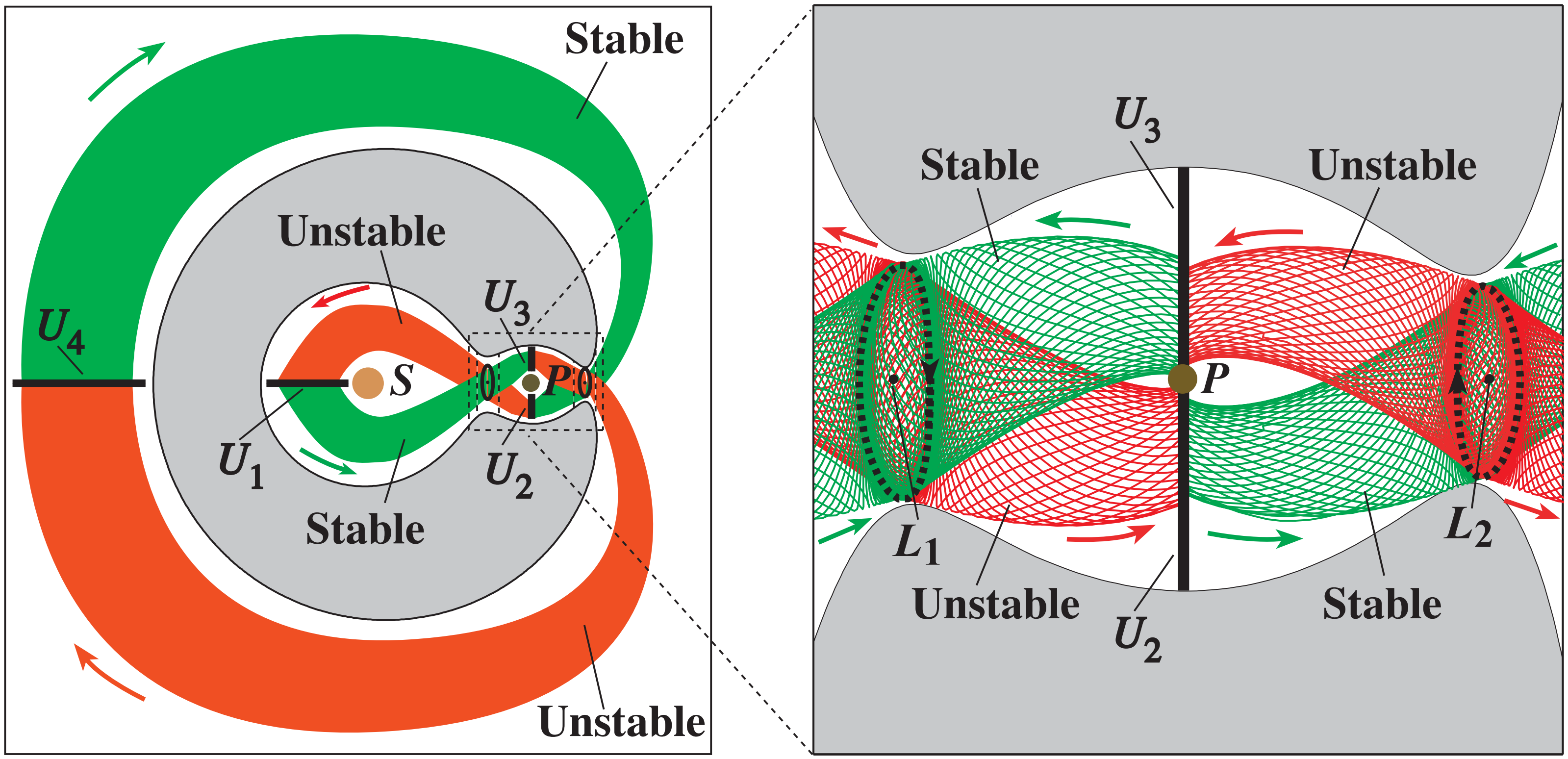
DLE-climbing-80.qt

Resonances and Tubes

- *Resonances and tubes are linked*
  - It has been observed that the tubes of capture (resp., escape) orbits are coming from (resp., going to) certain resonances.
  - Resonances are a function of energy E and the mass parameter μ
  - Koon, Lo, Marsden, Ross [2001]

Resonances and Tubes

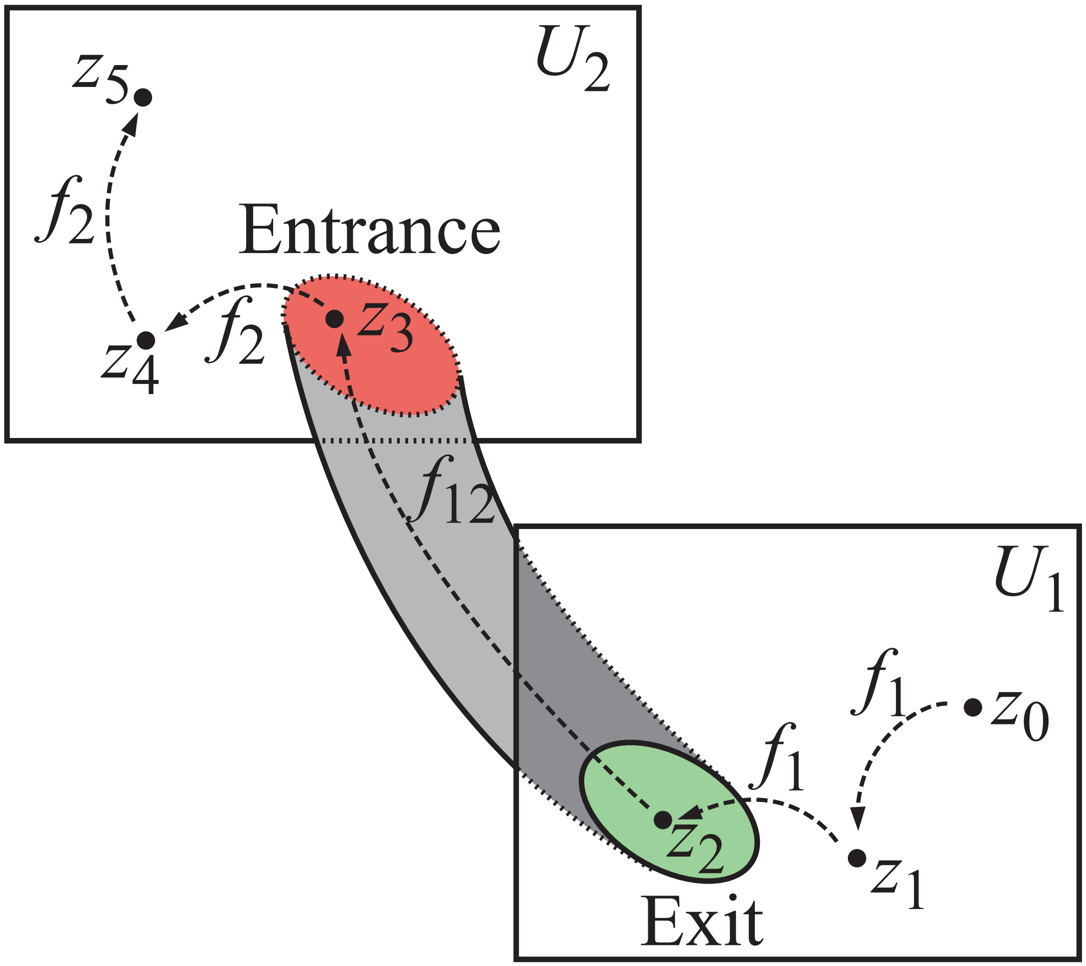
Poincaré sections in different realms (U_1 through U_4) are linked by phase space tubes. The projection of the tubes on the configuration space appear as strips.



Unstable and stable manifolds in *red* and *green*, resp.

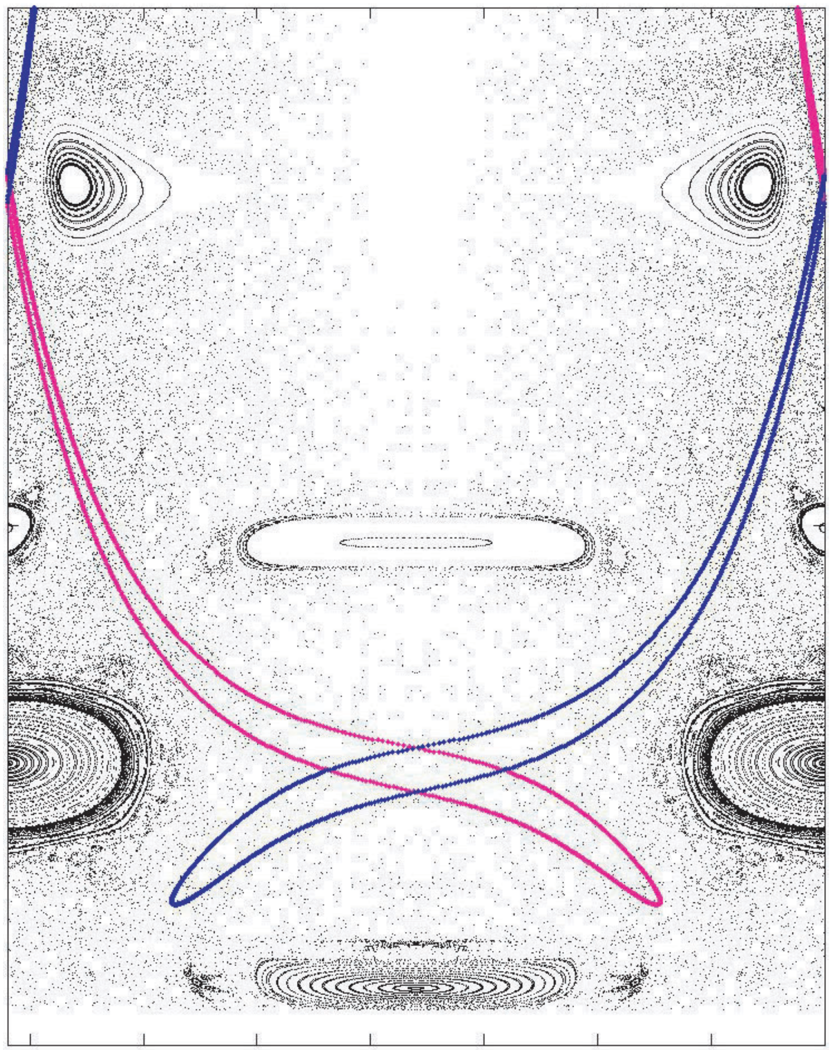
Resonances and Tubes

For example, points reach the ***exit*** in U_1 and are transported via a tube to the ***entrance*** of U_2.



Resonances and Tubes

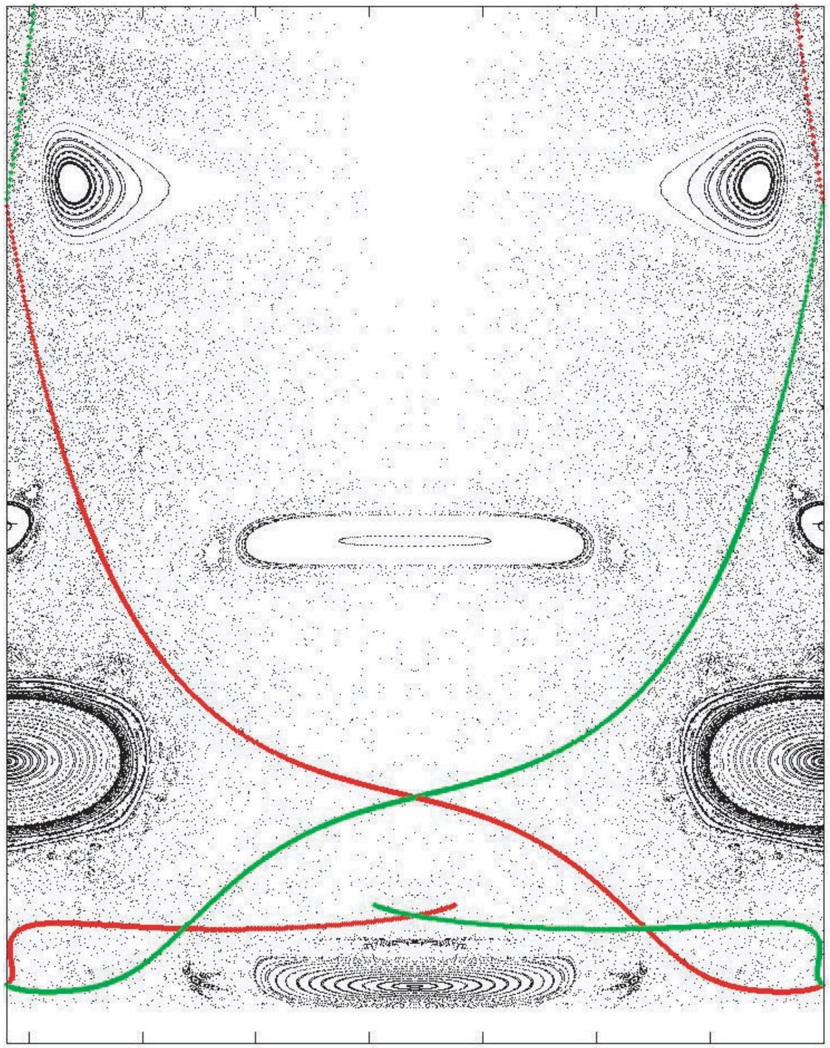
Poincaré section: tube cross-sections are closed curves



Particles inside curves move toward or away the moon

Resonances and Tubes

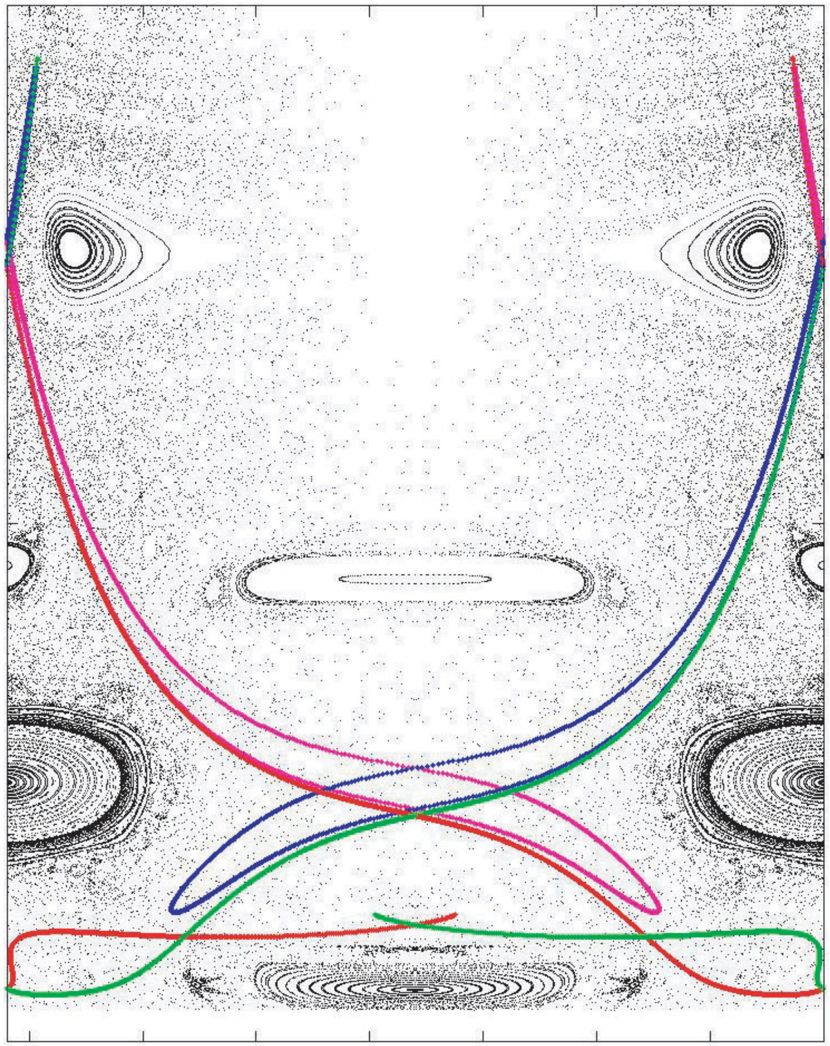
Same Poincaré section: resonance regions now plotted



2:3 exterior resonance

Resonances and Tubes

- Regions of overlap lead to ballistic capture



Regions of overlap occur

Escape Rates

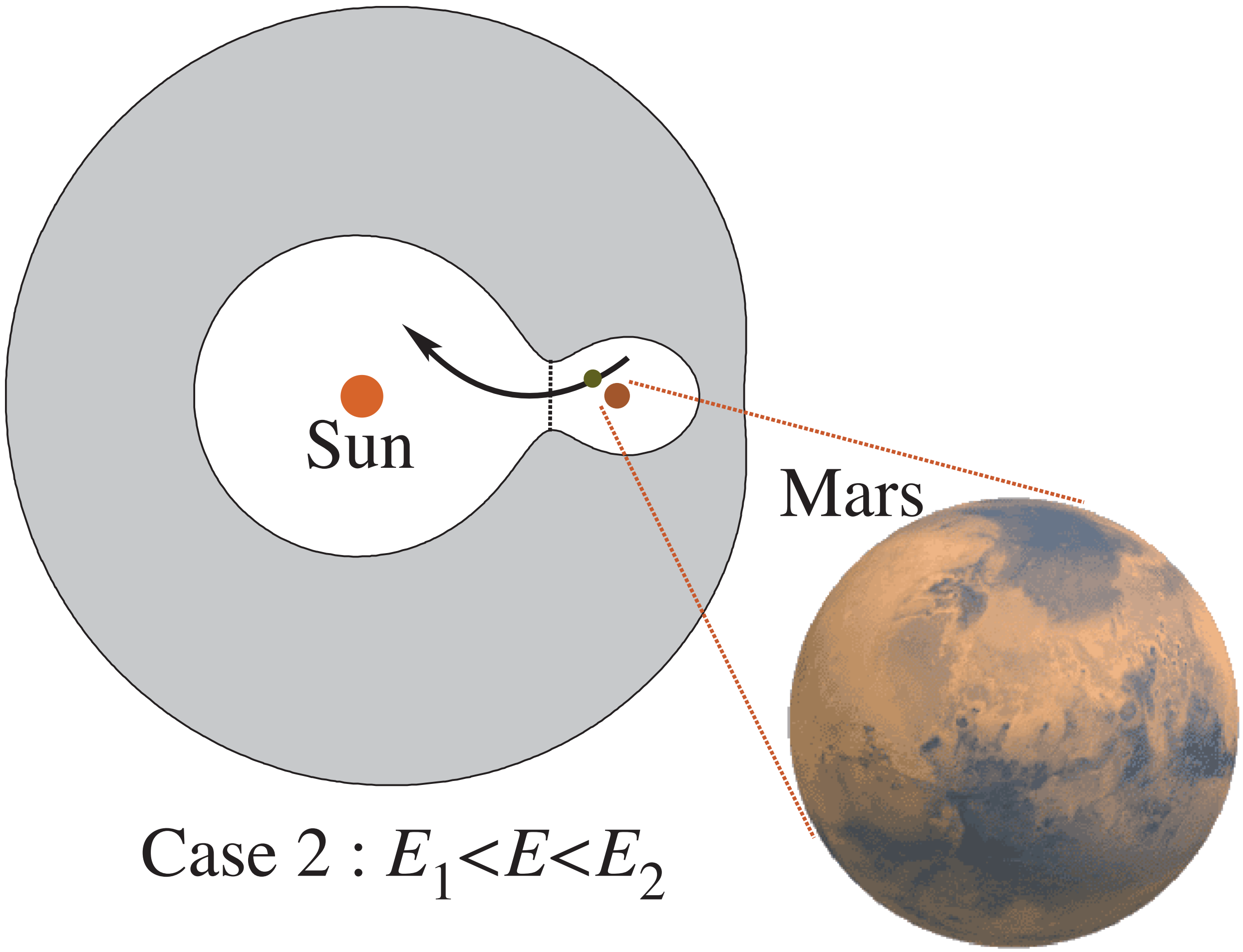
- *Applications to dynamical astronomy*
  - One can compute the rate of escape of particles temporarily captured by Mars, e.g. asteroids or impact ejecta liberated from the Martian surface.
    - Jaffé, Ross, Lo, Marsden, Farrelly, and Uzer [2002]

Mars with temporarily captured asteroids.

Escape Rates

- Consider a particle at an energy such that it can escape sunward. Using a ***statistical approach*** used in chemical dynamics, the rate of escape can be estimated.

Escape Rates



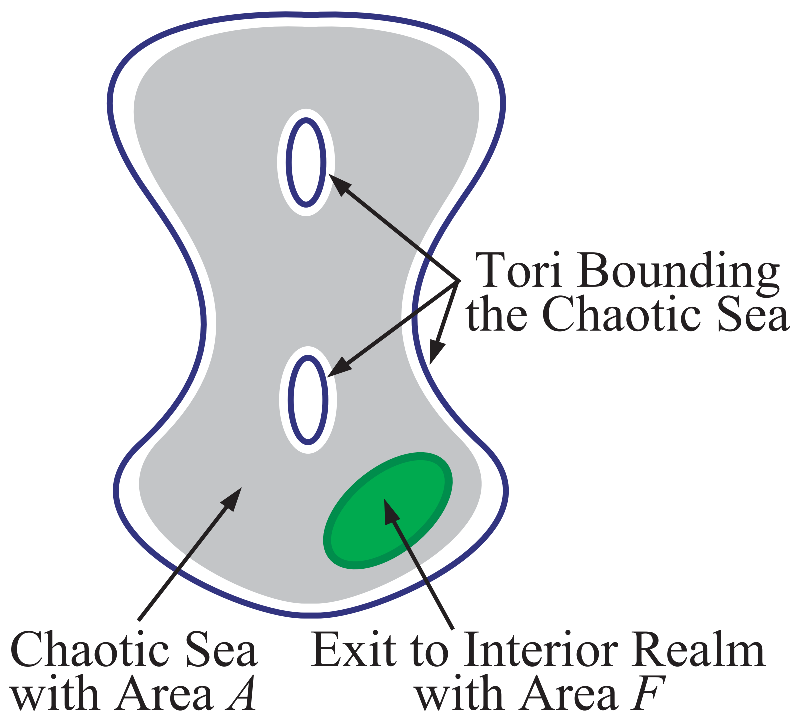
Case 2 : $E_1 < E < E_2$

Escape Rates

Mixing assumption: all asteroids in the chaotic sea surrounding Mars are *equally likely to escape*.

Escape rate $= -\log(1 - p)$, where

$$p = \frac{\text{Area of exit sunward}}{\text{Area of chaotic sea}}$$



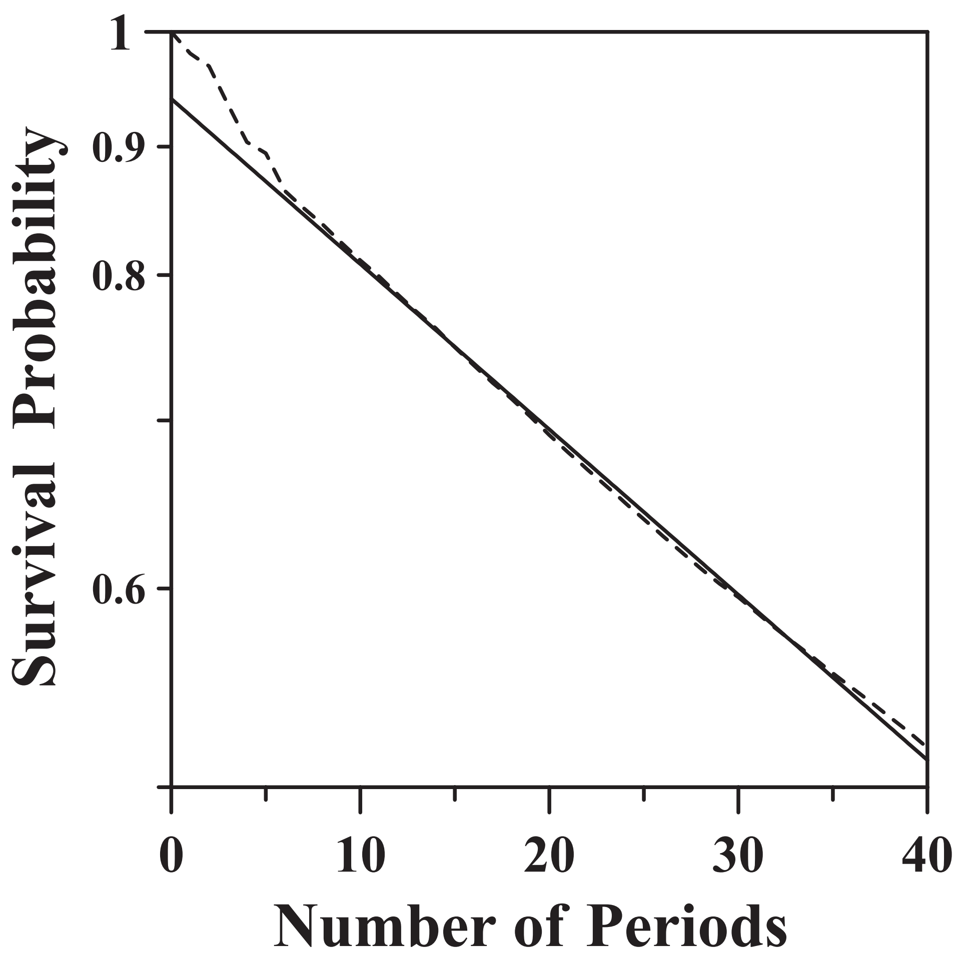
Escape Rates

Compare this rate with one obtained from a Monte Carlo simulations of 107,000 particles at randomly selected initial conditions at the same energy.

Escape Rates

Theory and numerical simulations agree well

- Monte Carlo simulation (dashed) and theory (solid)



Earth to Moon Trajectories

- *Similar methods can be applied to near-Earth space to study the ΔV verses time trade-off*

Earth to Moon Trajectories

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  - Consider a transfer from Earth orbit to lunar orbit
    - As before, use natural dynamics to lower propellant usage
    - Use planar circular restricted 3-body model
    - Bollt and Meiss [1995]: targeting through recurrence
    - Schroer and Ott [1997]: targeting passes between resonances

Earth to Moon Trajectories

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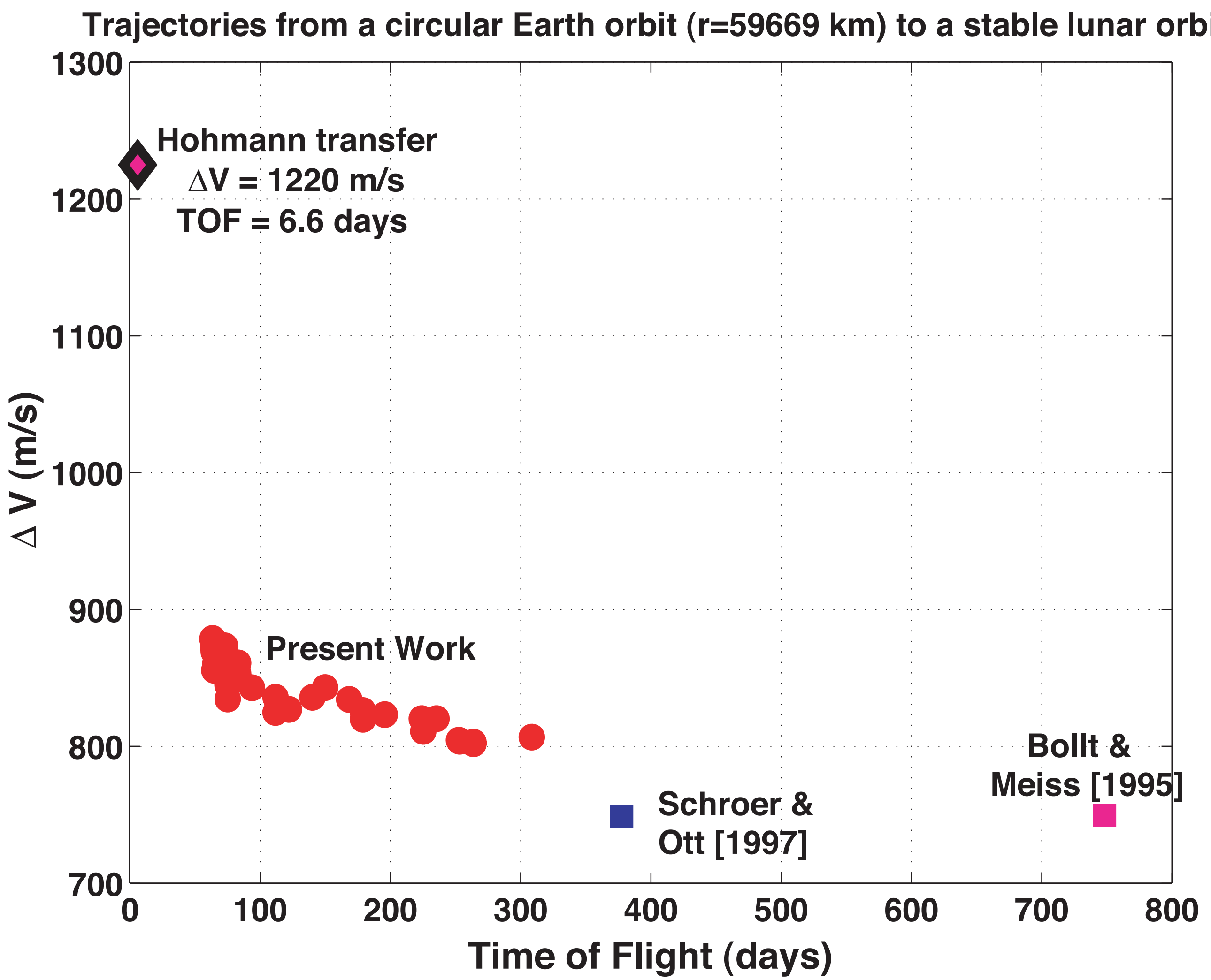
    - Bollt and Meiss [1995]: targeting through recurrence
    - Schroer and Ott [1997]: targeting passes between resonances
  - Current work: seek intersections between resonances and tubes leading to ballistic capture by the moon
    - Take full advantage of all known phase space structures

Earth to Moon Trajectories

- *Results*: much shorter transfer times than previous authors for only slightly more ΔV

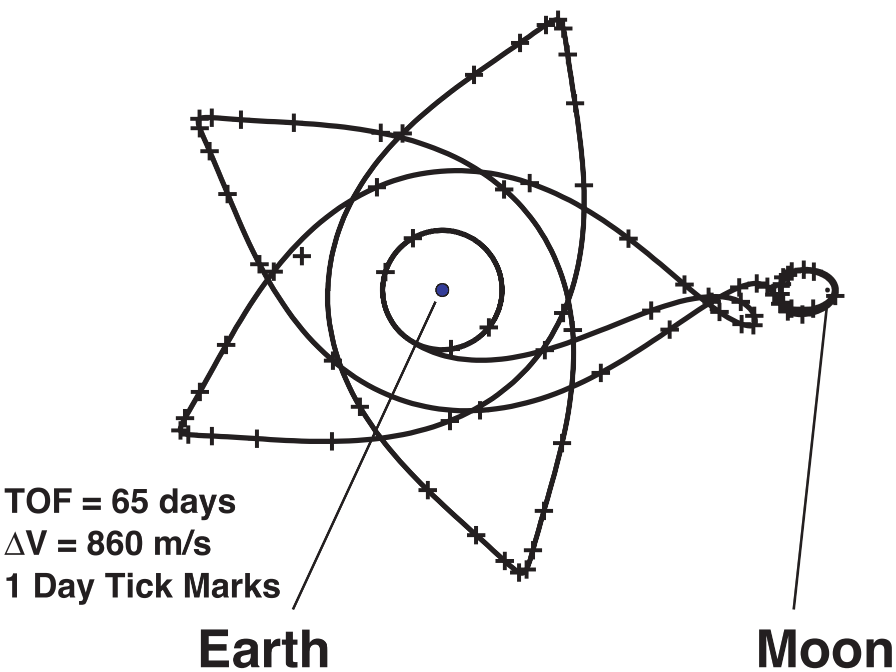


Earth to Moon Trajectories

- Compare with Bollt and Meiss [1995]
  - A tenth of the time for only 100 m/s more

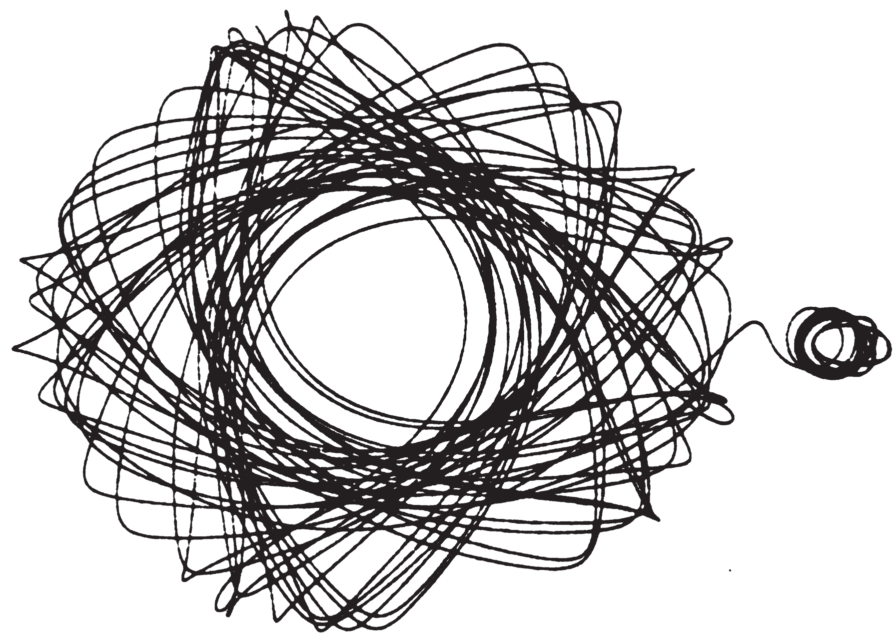
Current Result

65 days, ΔV = 860 m/s

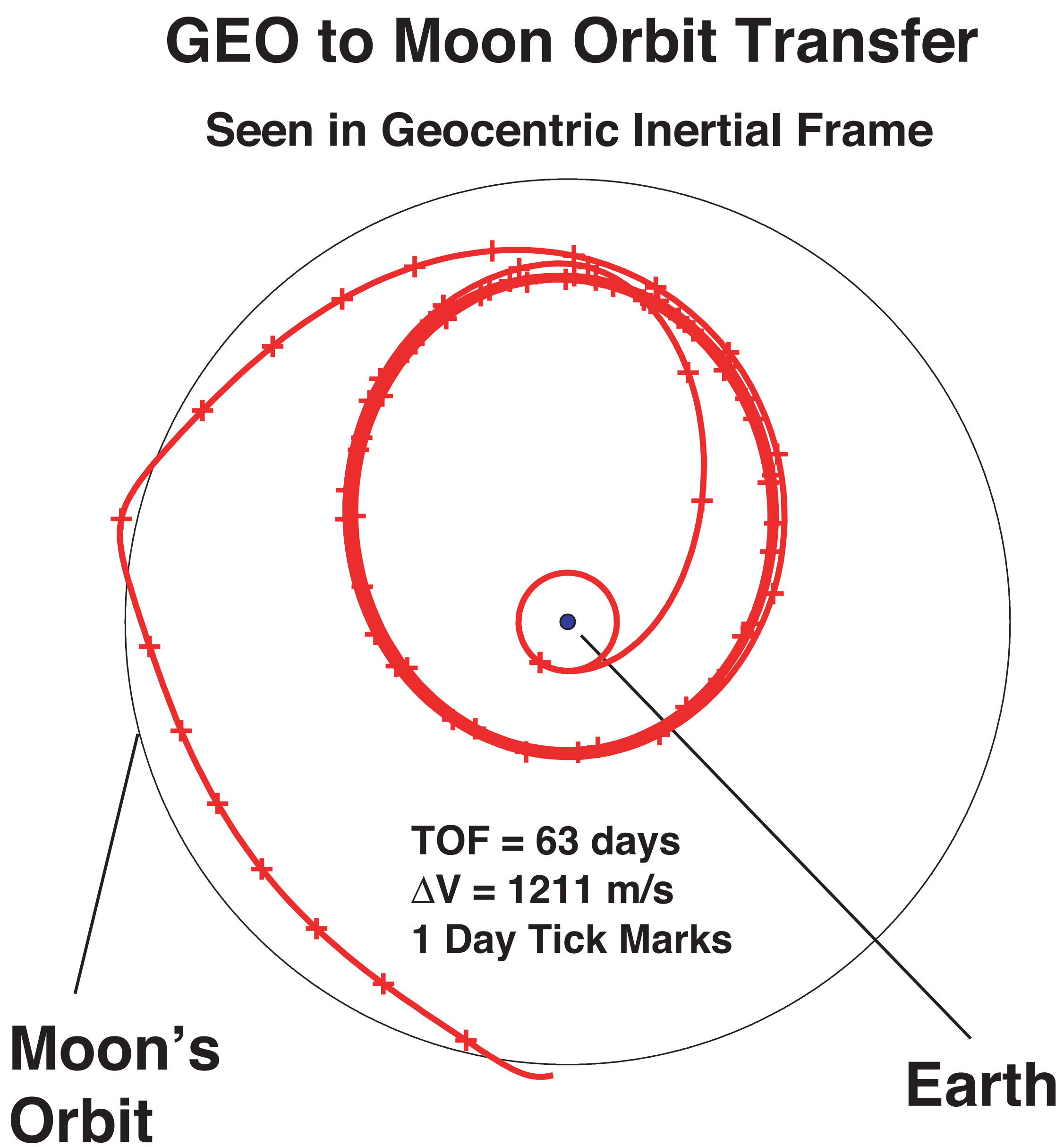


Bollt and Meiss [1995]

748 days, ΔV = 750 m/s



Example: GEO to Lunar Orbit



Example: GEO to Lunar Orbit

GEO to Moon - rotating frame

Example: GEO to Lunar Orbit

GEO to Moon - inertial frame

Selected References

- Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross [2002] *Design of a Mission to Orbit Multiple Moons of Jupiter, AAS/AIAA Space Flight Mechanics Meeting, Puerto Rico, Feb 9-12, 2003,* in preparation.
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For papers, movies, etc., visit the websites:

http://www.cds.caltech.edu/~marsden

http://www.cds.caltech.edu/~shane

The End

TYPESETTING SOFTWARE: TEX, *Textures*, LATEX, hyperref, texpower, Adobe Acrobat 4.05
GRAPHICS SOFTWARE: Adobe Illustrator 10.0.1
LATEX SLIDE MACRO PACKAGES: Wendy McKay, Ross Moore