Dynamical Systems and Control in Celestial Mechanics and Space Mission Design

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Introductory Remarks

- *Topics for discussion:*
  - solar system dynamics (eg, dynamics of comets)

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  - solar system dynamics (eg, dynamics of comets)
  - the role of the three and four body problems
  - space mission trajectory design
    - and the relationships between these topics

Introductory Remarks

- *Some history:*
  - ☐ ***1700-1850***: Euler, Lagrange, Gauss, began to lay the mathematical foundations
  - ☐ ***1880-1890***: Poincaré: fundamental work on the ***3-body problem***; creates the research area ***chaos***
  - ☐ ***1900-1965***: Moser, Conley, and others make fundamental contributions to the 3-body problem
  - ☐ ***1965-present***. Research in the 3 and 4 body problems and other topics in geometric mechanics and associated applications continues by many people: by no means finished!

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- *Current research importance*
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- *Acknowledgements*
  - ○ the Genesis team, the groups of Kathy Howell (Purdue), Michael Dellnitz (Paderborn), Linda Petzold (UC Santa Barbara), Gerard Gomez, Josep Masdemont, Carles Simo (the Barcelona group), etc.

Dynamical Orbits

- *Some dynamical systems concepts*
- *Simple pendulum*
  - three kinds of orbits:
    - oscillating orbits
    - running orbits
    - special separating orbits

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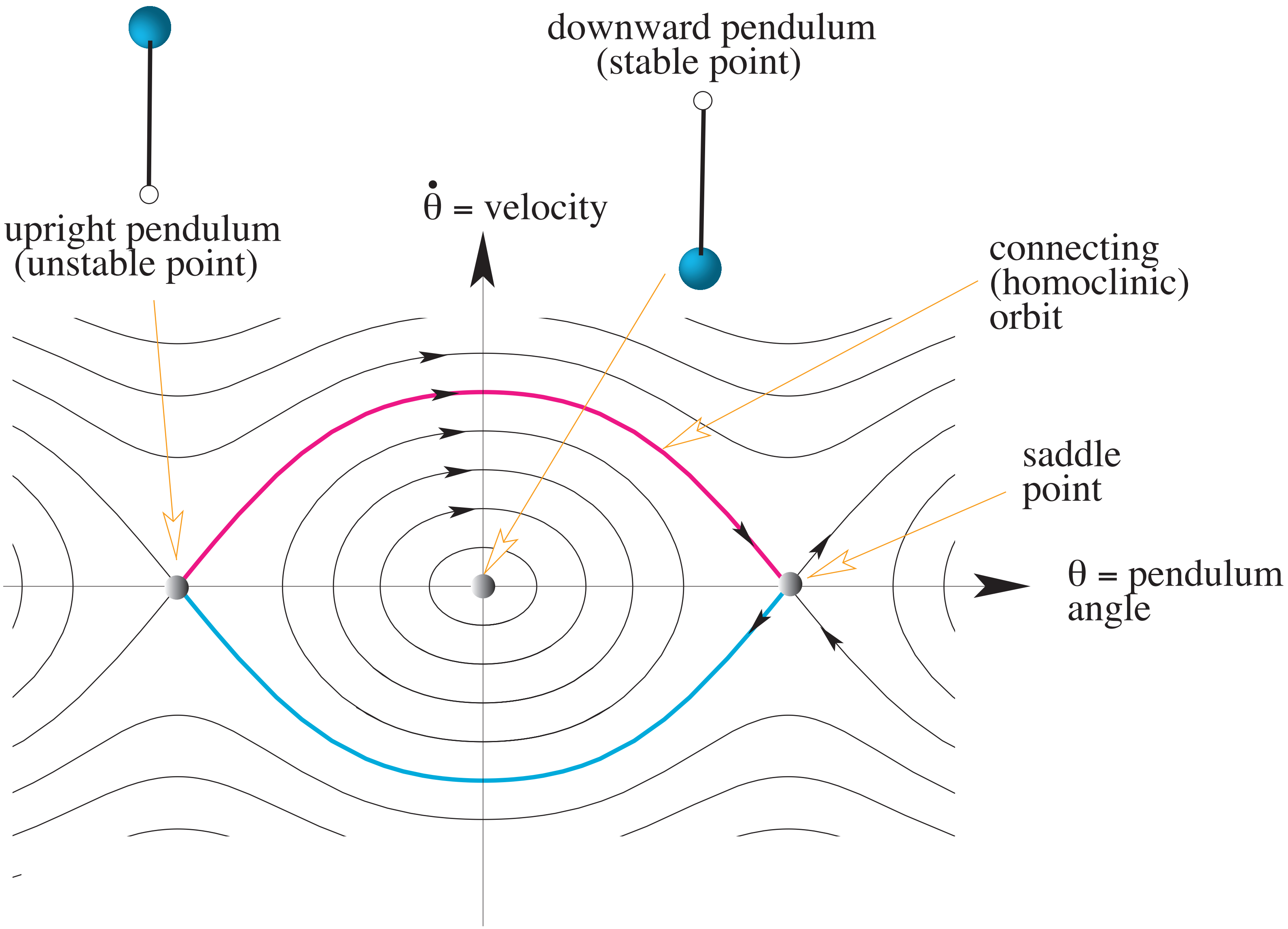
- *Some dynamical systems concepts*
- *Simple pendulum*
  - three kinds of orbits:
    - **oscillating orbits**
    - **running orbits**
    - **special separating orbits**
  - Via $F = ma$, can visualize solutions as trajectories in the $(\theta, \dot{\theta})$ plane (θ is the angle of the pendulum from the vertical downward position)
  - the resulting ***phase portrait*** allows one to put together the basic orbits in one figure:

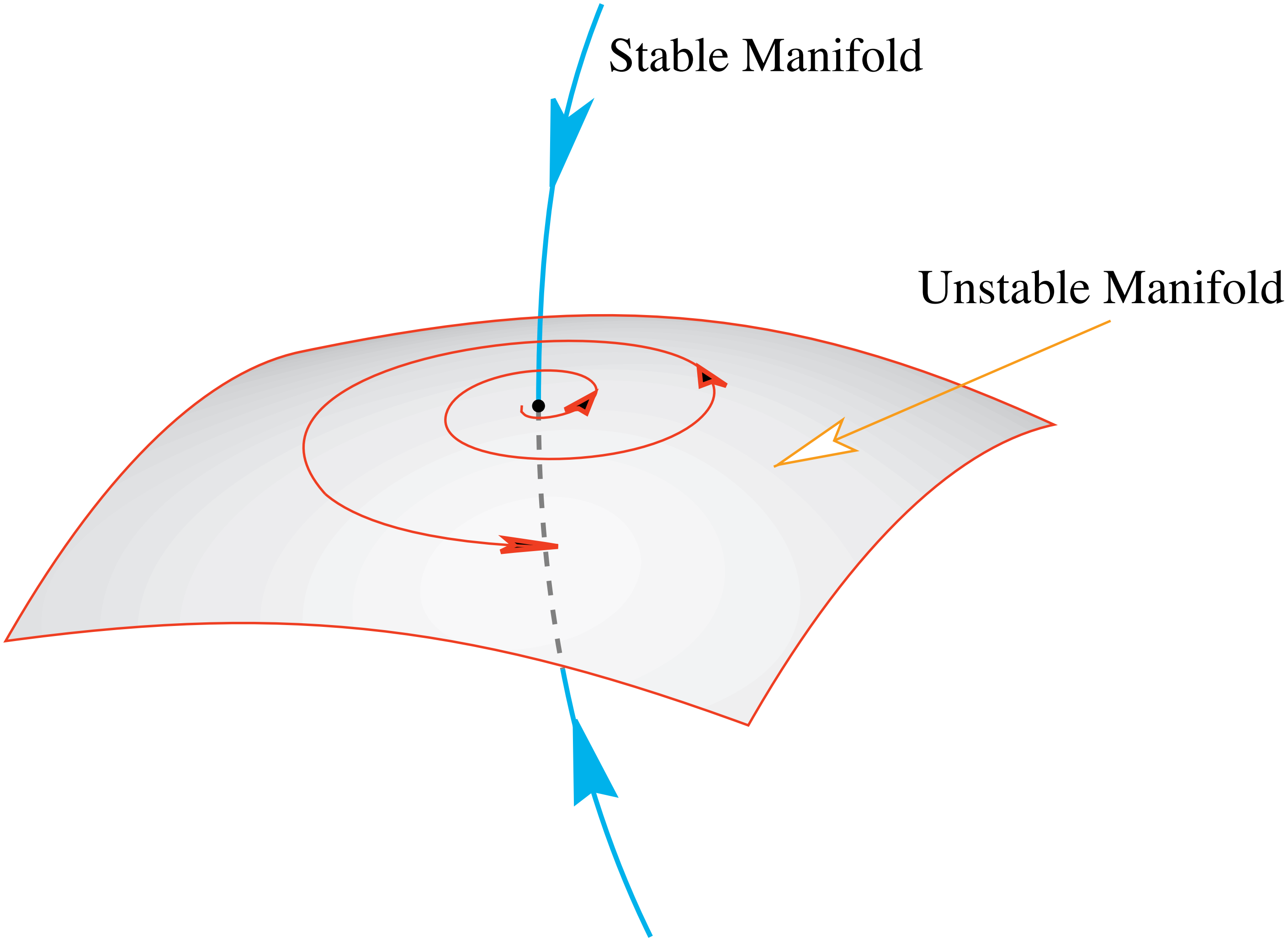
Dynamical Orbits



Phase portrait of the simple pendulum

Invariant Manifolds

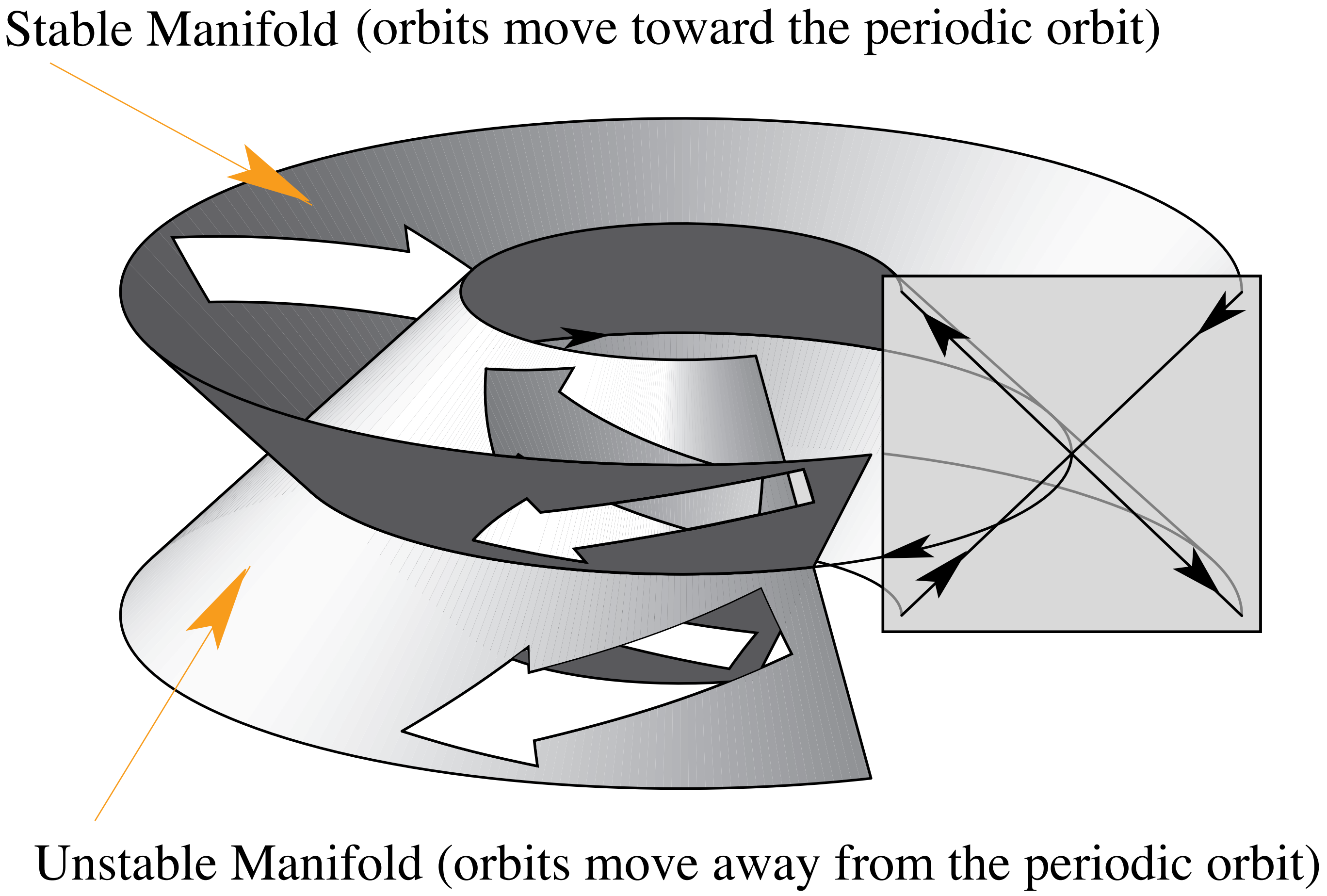
- ☐ Higher dimensional analog of the invariant curves



Invariant manifolds

Periodic Orbits

- ☐ Can replace equilibria by *periodic orbits*:



Invariant manifolds attached to a periodic orbit

Three body problem

- *General Three Body Problem*
  - Three bodies move under mutual gravitational interaction

Three body problem

General Three Body Problem

- Three bodies move under mutual gravitational interaction
- Some interesting new orbits discovered in the last few years by Richard Montgomery, Alain Chenciner, Carles Simo. Movies by Randy Paffenroth (Caltech) generated using AUTO

Three body problem

Figure 8 Orbits: 3-body-figure-8.qt

Three body problem

Fancy Orbit A: 3-body-exoticA.qt

Three body problem

Fancy Orbit B: 3-body-exoticB.qt

Three body problem

Restricted Circular Problem

- the two primaries move in circles; the smaller third body moves in the field of the primaries (without affecting them); view the motion in a ***rotating frame***
- we consider the ***planar*** and the ***spatial*** problems

Three body problem

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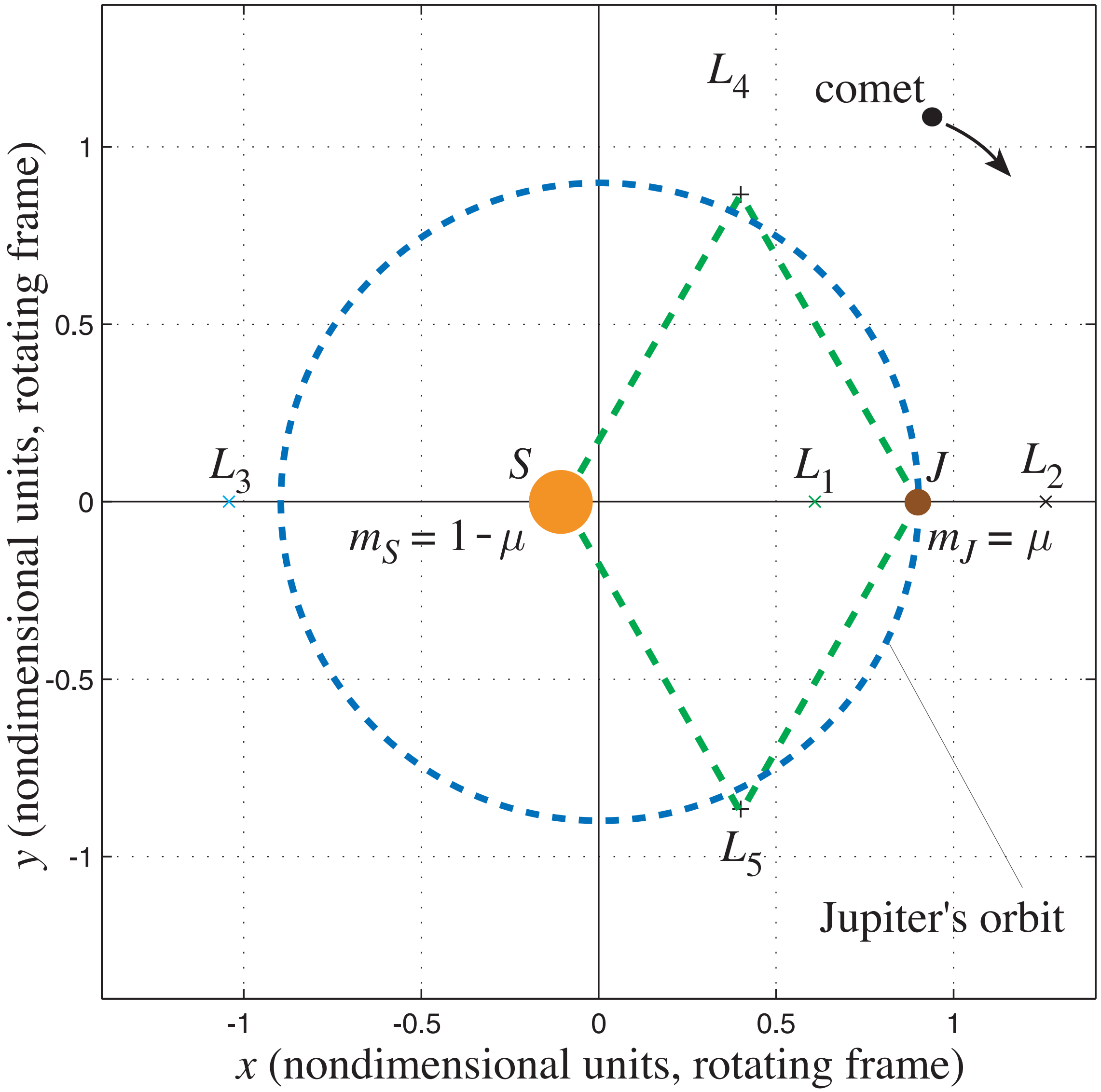
- we consider the ***planar*** and the ***spatial*** problems
- there are places of ***balance***; eg, a point between the two bodies where the attraction balances
- There are five such ***equilibrium points***:
  - Three ***collinear*** (Euler,1750) on the x-axis—L_1, L_2, L_3
  - Two ***equilateral points*** (Lagrange,1760)—L_4, L_5.

Three body problem



Equilibrium points for the three body problem

Three body problem

- if a spacecraft is at L_1 or at L_2, it will stay there
- *one can go into orbit about the* L_1 *and* L_2 *points, even though there is no material object there*!

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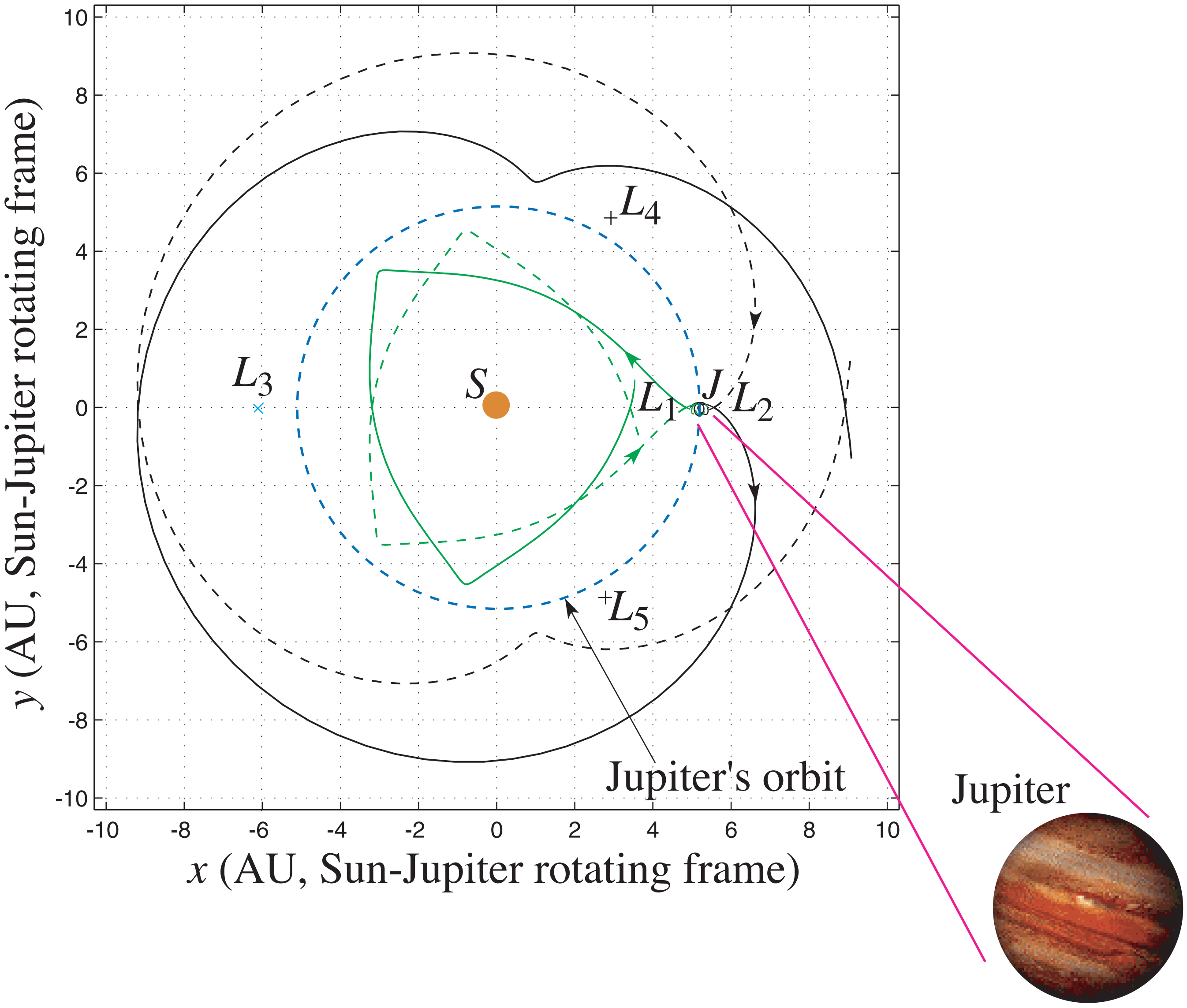
- *one can go into orbit about the L_1 and L_2 points, even though there is no material object there*!
- some of these orbits are called ***Liapunov orbits***, others are called ***halo and Lissajous orbits***.
- just as in the pendulum, one can draw the invariant manifolds associated to L_1 (and L_2) and the periodic orbits surrounding them.
- these ***invariant manifolds*** play a key role in what follows

Three body problem



Invariant manifolds for the 3-body problem

Three body problem

- consider the *planar case*; the *spatial case* is similar
- *Kinetic energy* (wrt inertial frame) in rotating coordinates:

$$K(x,y,\dot{x},\dot{y}) = \frac{1}{2}\left[(\dot{x}-\omega y)^2 + (\dot{y}+\omega x)^2\right]$$

- *Lagrangian* is K.E. − P.E., given by

$$L(x,y,\dot{x},\dot{y}) = K(x,y,\dot{x},\dot{y}) - V(x,y); \quad V(x,y) = -\frac{1-\mu}{r_1} - \frac{\mu}{r_2}.$$

- *Euler–Lagrange equations*:

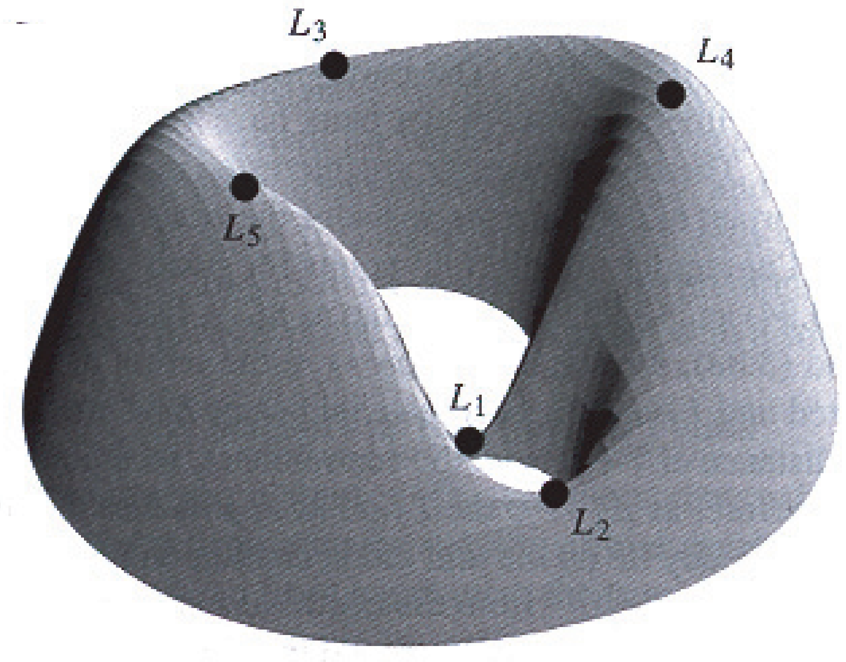
$$\boxed{\ddot{x} - 2\omega\dot{y} = -\frac{\partial V_\omega}{\partial x}, \qquad \ddot{y} + 2\omega\dot{x} = -\frac{\partial V_\omega}{\partial y}}$$

where the *effective potential* is

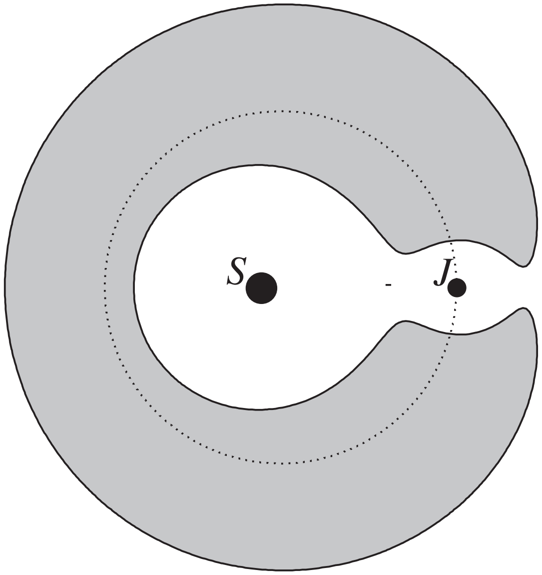
$$V_\omega = V - \frac{\omega^2(x^2+y^2)}{2}.$$

Three body problem

- *Effective potential*
  - In the circular planar restricted three body problem, and in a rotating frame, the equations for the third body are those of a particle moving in an effective potential plus a magnetic field (goes back to work of Jacobi, Hill, etc)



Effective Potential

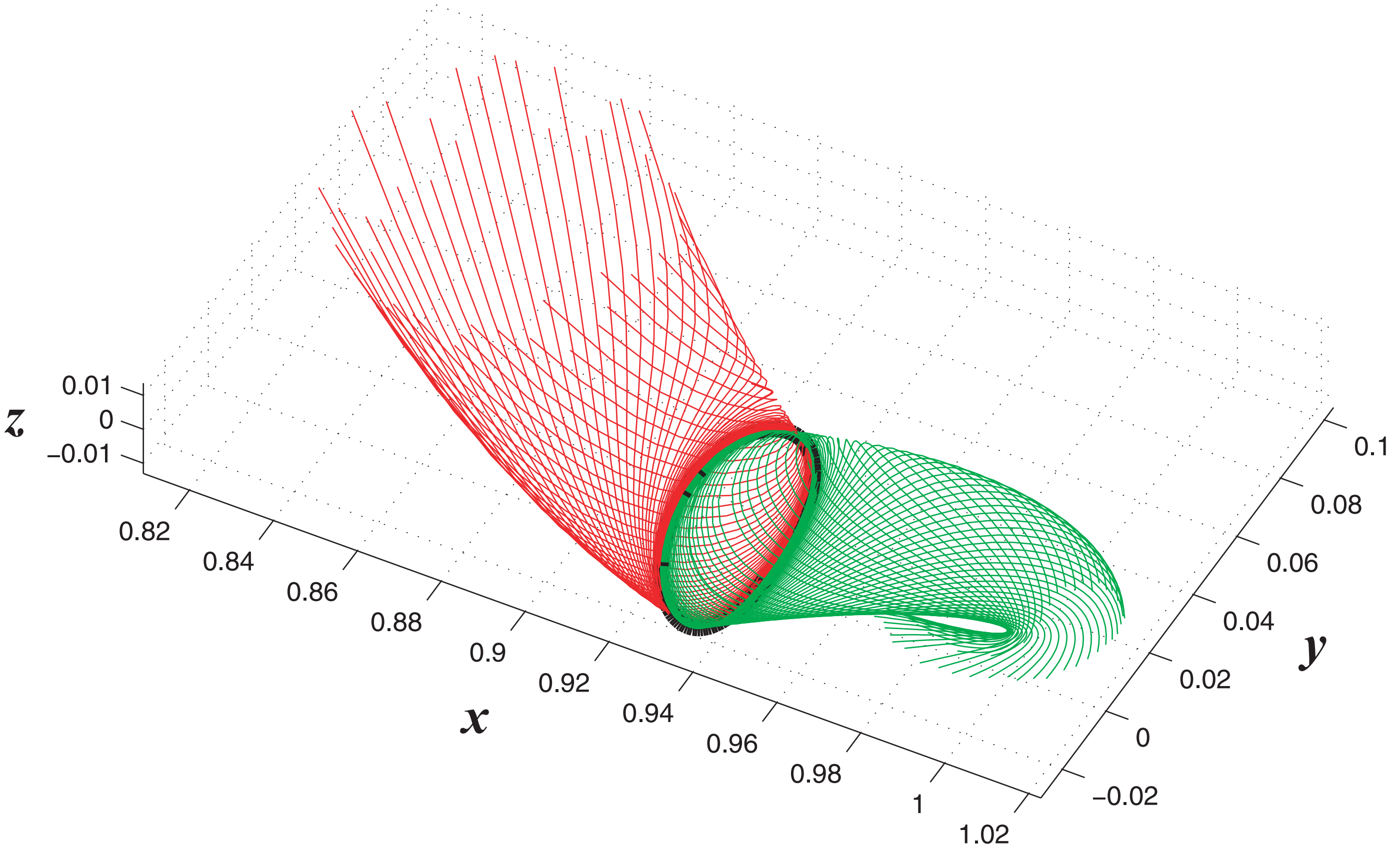


Level set shows the Hill region

Three body problem

- *Invariant Manifolds of Periodic Orbits*

☐ **red** = unstable, **green** = stable



Three body problem

- These manifold tubes play a crucual role in what ***passes through*** the resonance (transit orbits)
- and what ***bounces back*** (non-transit orbits)
- transit possible if you are “inside” the tube, otherwise nontransit—important for ***transport issues***

Comet Oterma

- we consider the historical record of the orbit of comet Oterma from 1910 to 1980
  - first in an inertial frame (fixed relative to the stars)
  - and then a rotating frame
  - very special case of *pattern evocation*
- similar pictures for many other comets

Comet Oterma

oterma-inertial.qt

Comet Oterma

oterma-rotating.qt

Genesis Discovery Mission

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- **Mission Constraints/Features:**
  - Return in Utah during the daytime
  - Descend with a parachute for a *helicopter snatch*
  - *lunar swingby contingency* in case of bad weather
  - *Energy efficient* (small thrust required): makes use of the dynamical sensitivity to design a low-cost trajectory

Genesis Discovery Mission

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Genesis Discovery Mission

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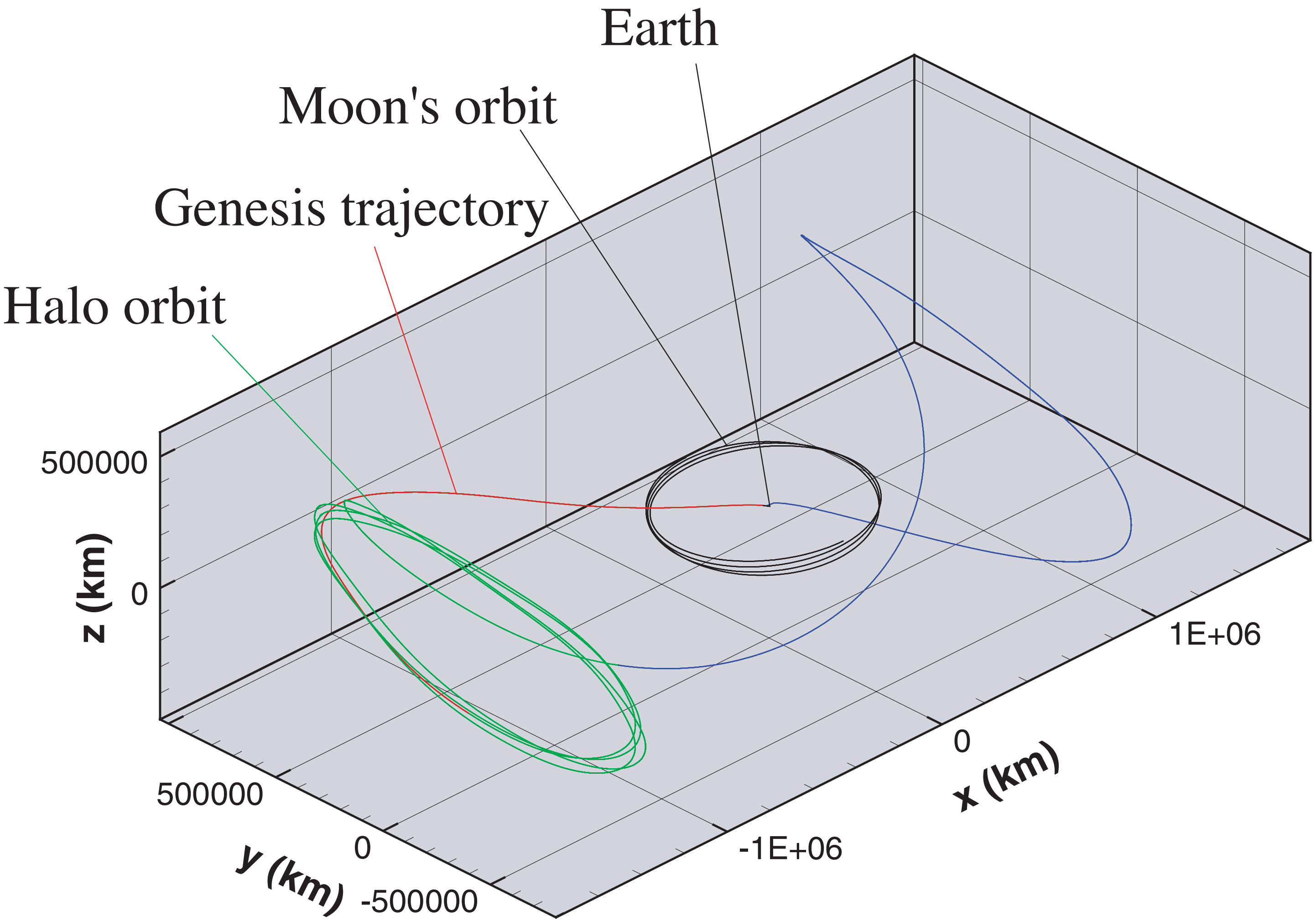
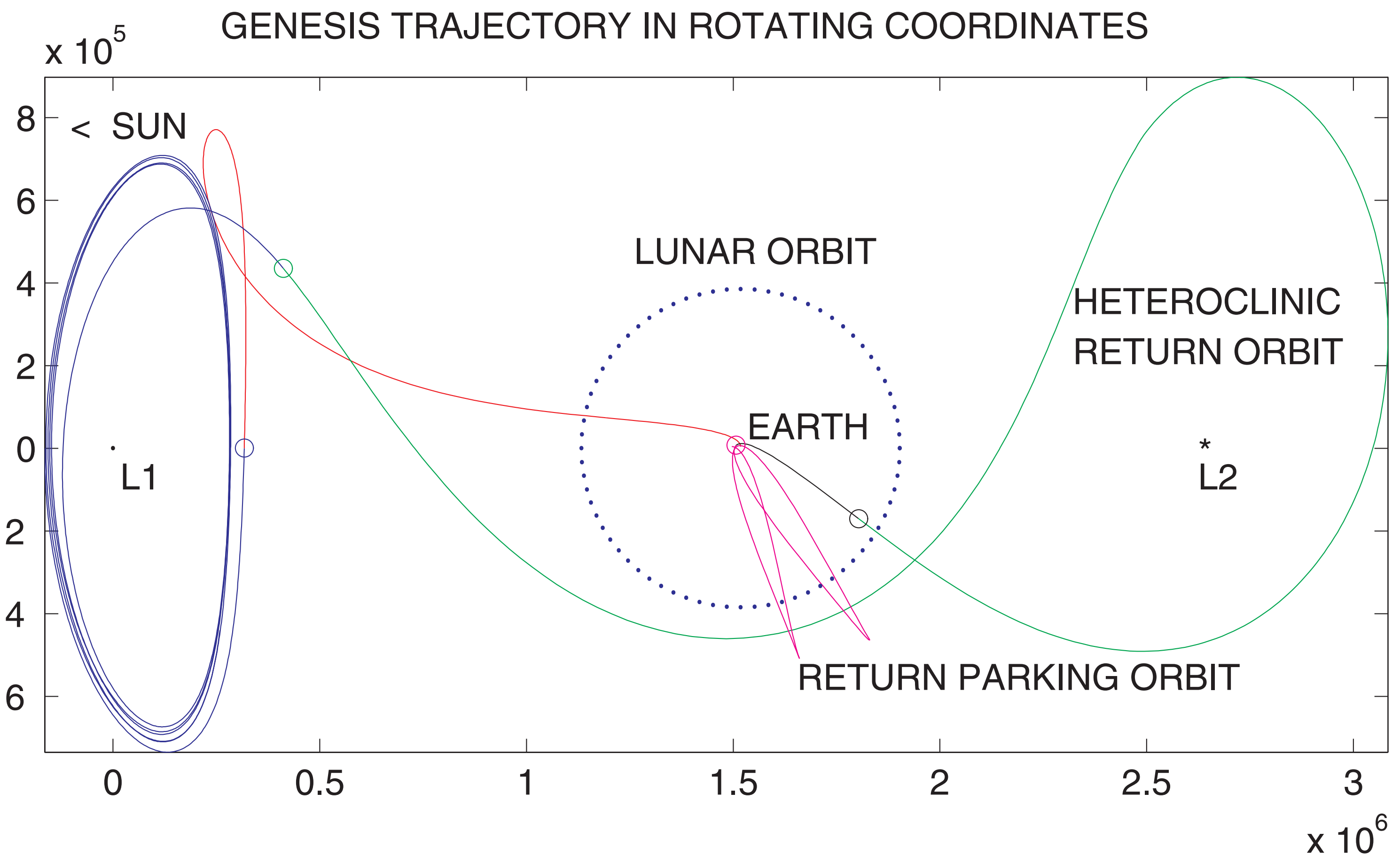
  4. return to Earth on *an impact orbit* (guided by the unstable manifold of a halo orbit around L_2).
- Final trajectory computation takes into account all the major bodies in the solar system.

Genesis Discovery Mission



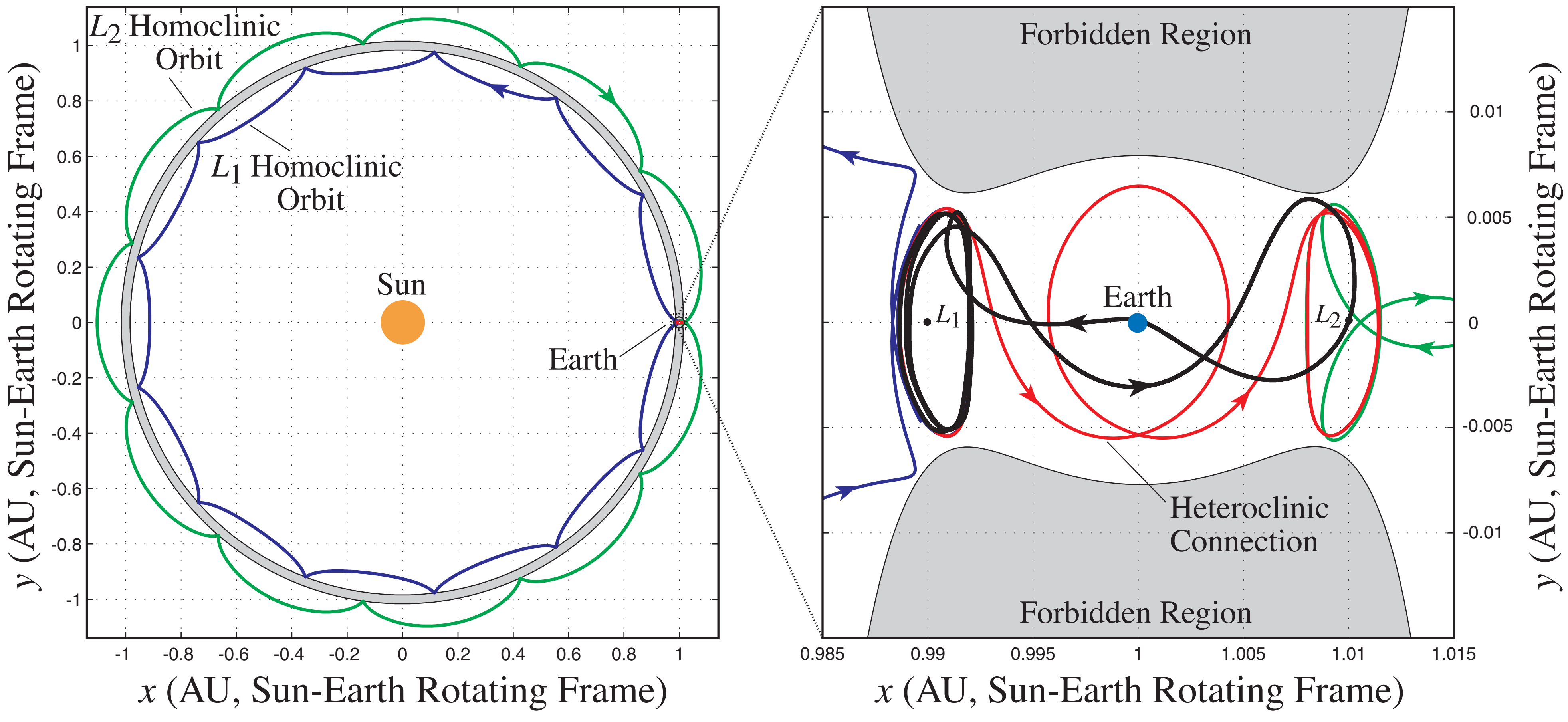
The Genesis trajectory

Genesis Discovery Mission



View of the Genesis trajectory in the plane

Genesis Discovery Mission



(a) (b)

Genesis orbit and the Sun-Earth dynamical structure

Genesis Discovery Mission

- Some *planet-impacting asteroids* use invariant manifolds as a pathway from nearby heliocentric orbits. This phenomena has been observed in the impact of comet *Shoemaker-Levy 9* with Jupiter.
- Some NEO's are subject to similar dynamics and are the most dangerous ones; perhaps the KT impact event was one of these too!
- These ideas apply to *any planet or moon system*!

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- the mission was "saved" by finding a more fuel efficient pathway to the moon (Miller and Belbruno)
- now a deeper understanding of this is emerging
- we approach this problem by
  - *systematically implementing* the view that the Sun-Earth-Moon-Spacecraft 4-body system can be modelled as *two coupled 3-body systems*
  - and using invariant manifold ideas

Lunar Missions

- Idea: *put two Genesis-type trajectories together*; we transfer from
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- *But*: takes longer (6 months as opposed to 5 days). [OK for cargo ships, but not human missions]

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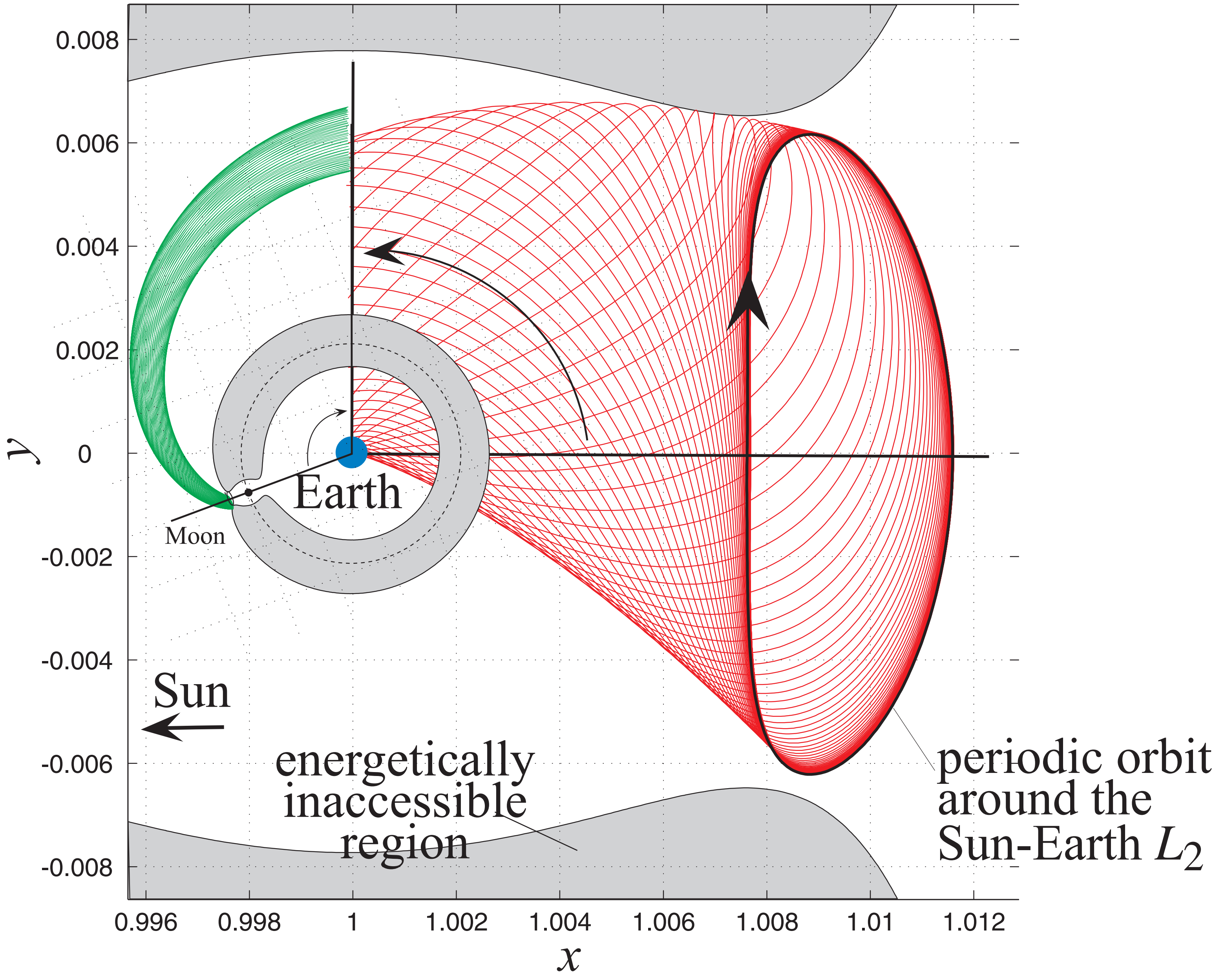
- *But*: takes longer (6 months as opposed to 5 days). [OK for cargo ships, but not human missions]
- Fuel savings *and* the time of flight in other missions (eg, to Jupiter's moon's) is more dramatic
- Schematic of the idea

Lunar Missions



Lunar Missions

shootthemoon-inertial.qt

Lunar Missions

shootthemoon-rotating.qt

Jovian Lunar Tour

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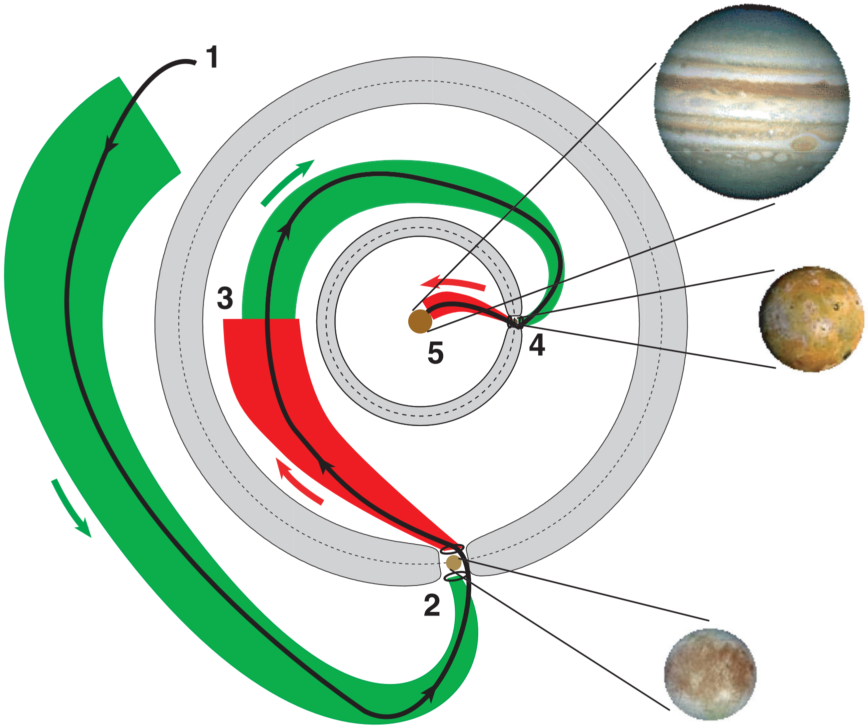
- Example 1: *Europa → Io → Jupiter collision*

1. Begin tour
2. Europa encounter
3. Jump between tubes
4. Io encounter
5. Collide with Jupiter

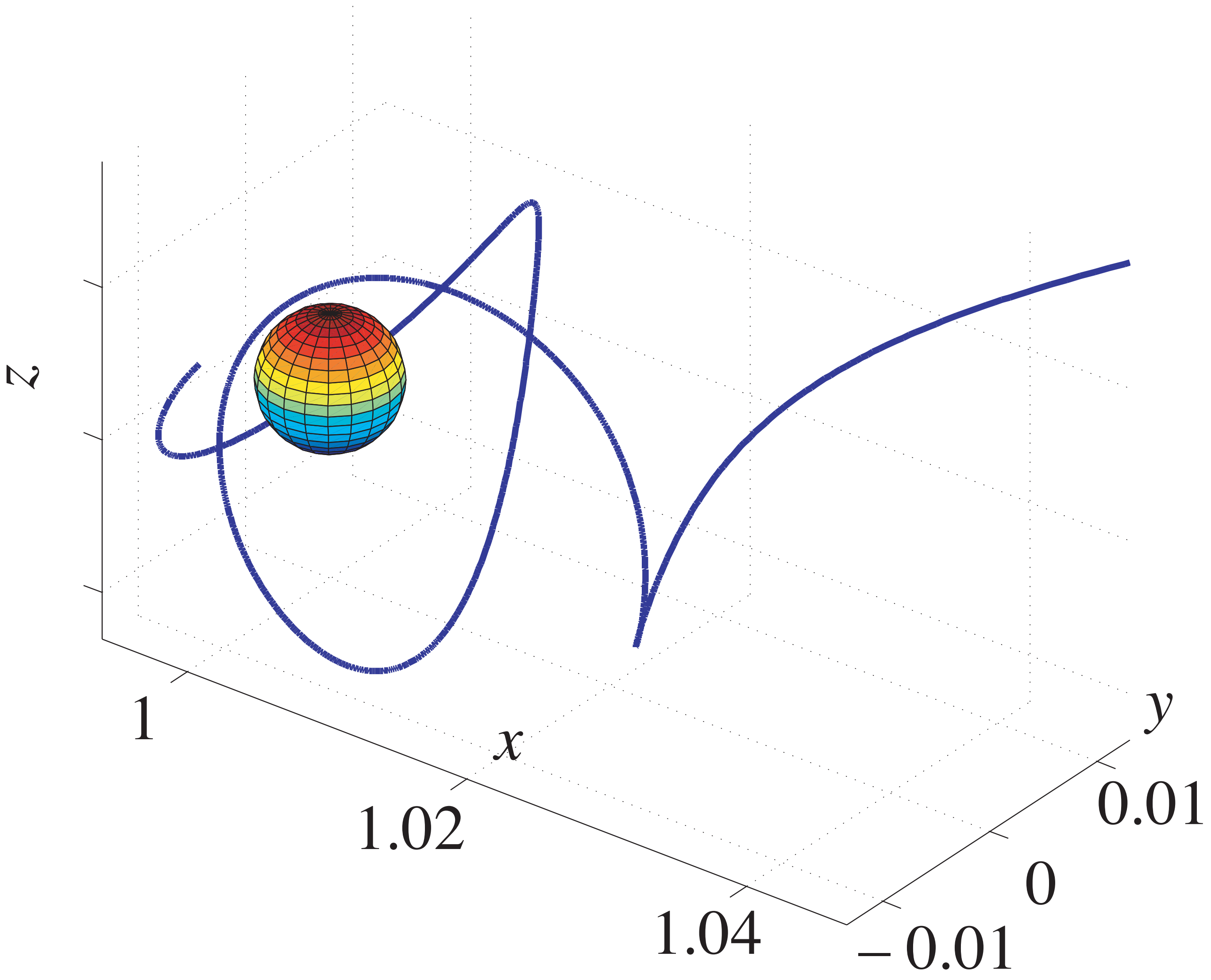


Jovian Lunar Tour

- *Strategy:*

- use burns (controls) that enable a transfer from one three body system to another
  - from the *Jupiter-Europa-spacecraft* system to
  - the *Jupiter-Io-spacecraft* system
- this strategy is similar to that used in the lunar missions together with some symbolic dynamics
- trajectories do well on fuel savings
- here is a close-up of the Io encounter

Jovian Lunar Tour



Close-up of the Io encounter

Jovian Lunar Tour

- **Example 2:** *Ganymede → Europa → orbit injection around Europa*

pgt-3d-movie-inertial.qt

Jovian Lunar Tour

pgt-3d-movie-ga.qt

Jovian Lunar Tour

pgt-3d-movie-eu.qt

Uses of Optimal Control

- *Halo Orbit Insertion*

  - After launch, the ***Genesis Discovery Mission*** will get onto the stable manifold of its eventual periodic orbit around L_1
  - Errors in, eg, launch velocity, means that there must be **corrective manouvers**
  - The software COOPT is very useful in determining the necessary corrections (burn sizes and timing) systematically for a variety of launch conditions
  - It gets one onto the orbit at the right time, while minimizing fuel (what is being **optimized**)

Uses of Optimal Control

- A number of unusual features, such as the nature of the boundary conditions
- A very nice mixture of dynamical systems (providing guidance and first guesses) and optimal control
- See the talk of **Linda Petzold** in the satellite dynamics minisymposium (work with **Radu Serban**, **Martin Lo**, **Wang Sang Koon**, **JEM** and **Shane Ross**)

Uses of Optimal Control

$\boxed{hoi}$

Uses of Optimal Control

- *Satellite reconfiguration, stationkeeping and de-configuration*
  - This application makes use of the software NTG (Non-linear Trajectory Generation) developed at Caltech
  - Details were in the minisymposium talk: Richard Murray (together with Mark Milam and Nicolas Petit)
  - This involves near Earth *spacecraft clusters*

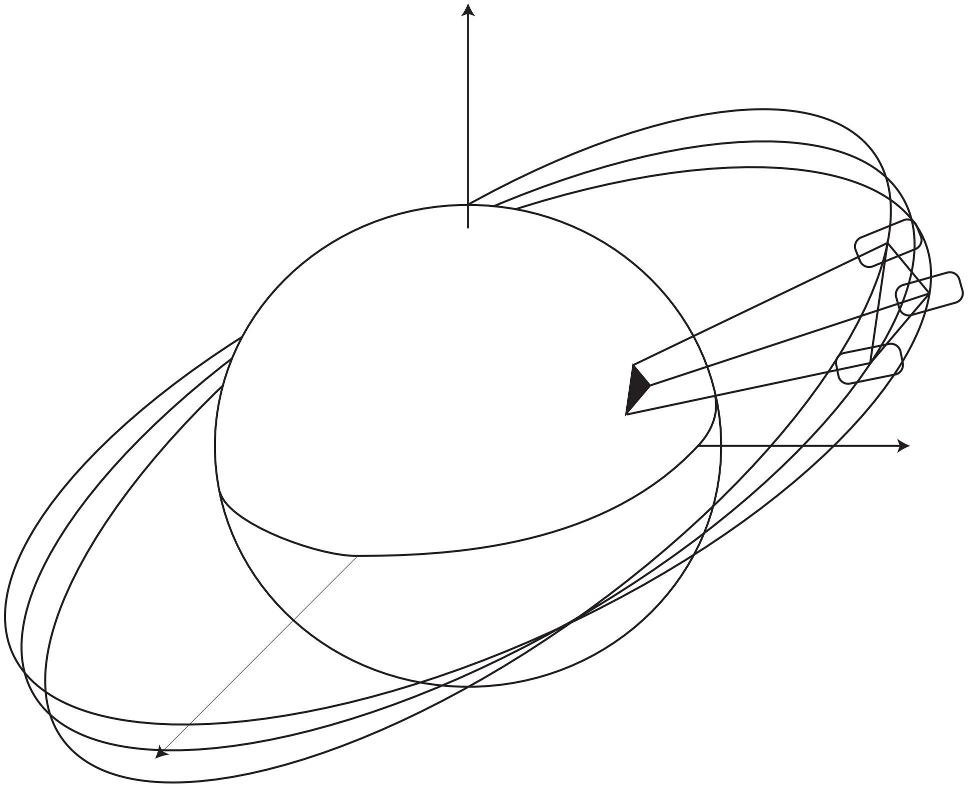
Uses of Optimal Control

- *Why clusters?*
  - Clusters can achieve the same resolution as a large telescope using vision systems coordinated in software and modern optics
  - ***coordinated clusters*** can obtain unprecedented resolution for both Earth-pointing systems and those pointing into deep space

Uses of Optimal Control

- *Two basic problems*
  - **Formation maintenance:** keep the satellites in relative position–use small controls
  - **Formation changes:** get the formation as a whole to reposition itself for the next task—use larger controls
- *Especially for reconfiguration, one wants to do this optimally (again, minimize fuel)*
  - Handles *constraints*, such as imaging and communication constraints very nicely

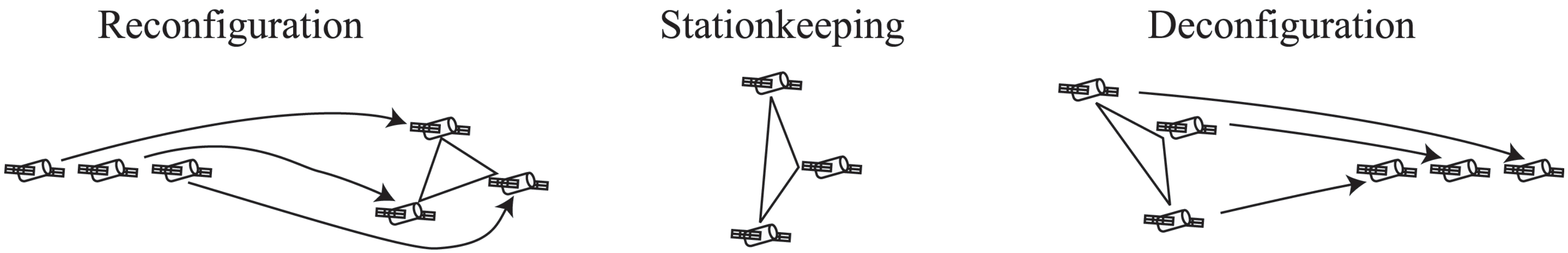
Uses of Optimal Control



Formation maintenance with guaranteed Earth coverage.

Formation Flying Methodology

Active Formation Methodology

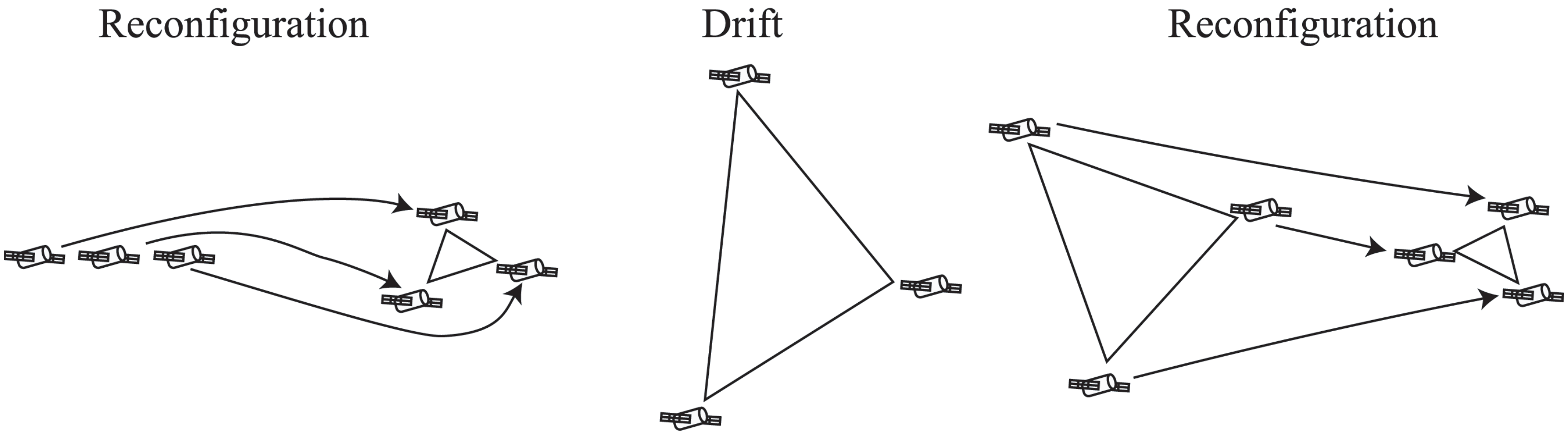


Reconfiguration

Stationkeeping

Deconfiguration

Passive Formation Methodology



Reconfiguration

Drift

Reconfiguration

Stationkeeping

Stationkeeping

Reconfiguration

Reconfiguration

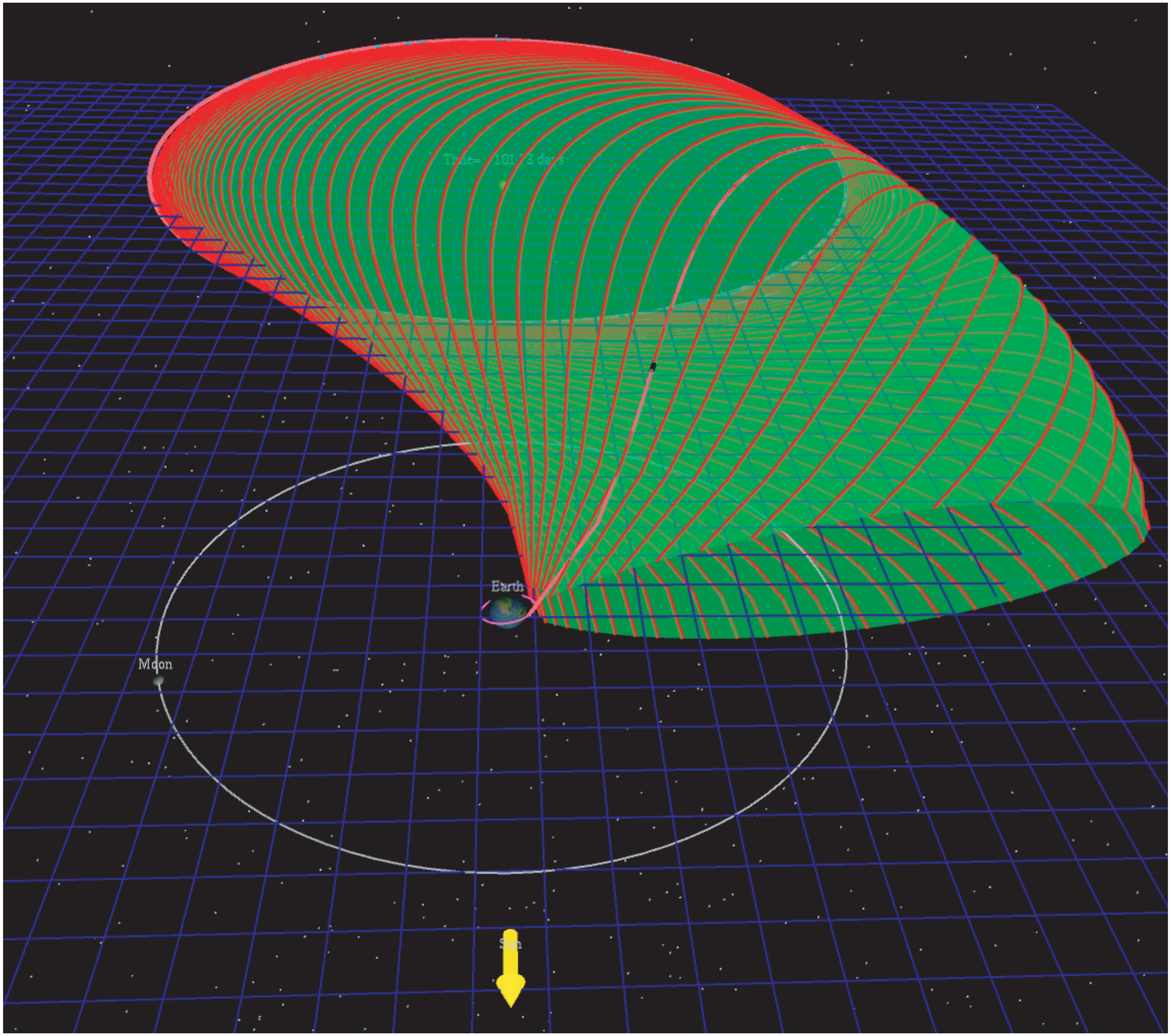
Deconfiguration

Deconfiguration

Terrestrial Planet Finder

- *Goal*: probe for Earth-like planets using a large baseline group of satellites–this time a *deep space cluster*
- Orbiting around L_2 is a candidate position: away from the Earth.
- Each halo orbit is surrounded by a torus that provides a natural dynamical formation
- Very nice visualizations of this by Ken Museth, Martin Lo and Al Barr; see Martin's talk in the satellite minisymposium

Terrestrial Planet Finder

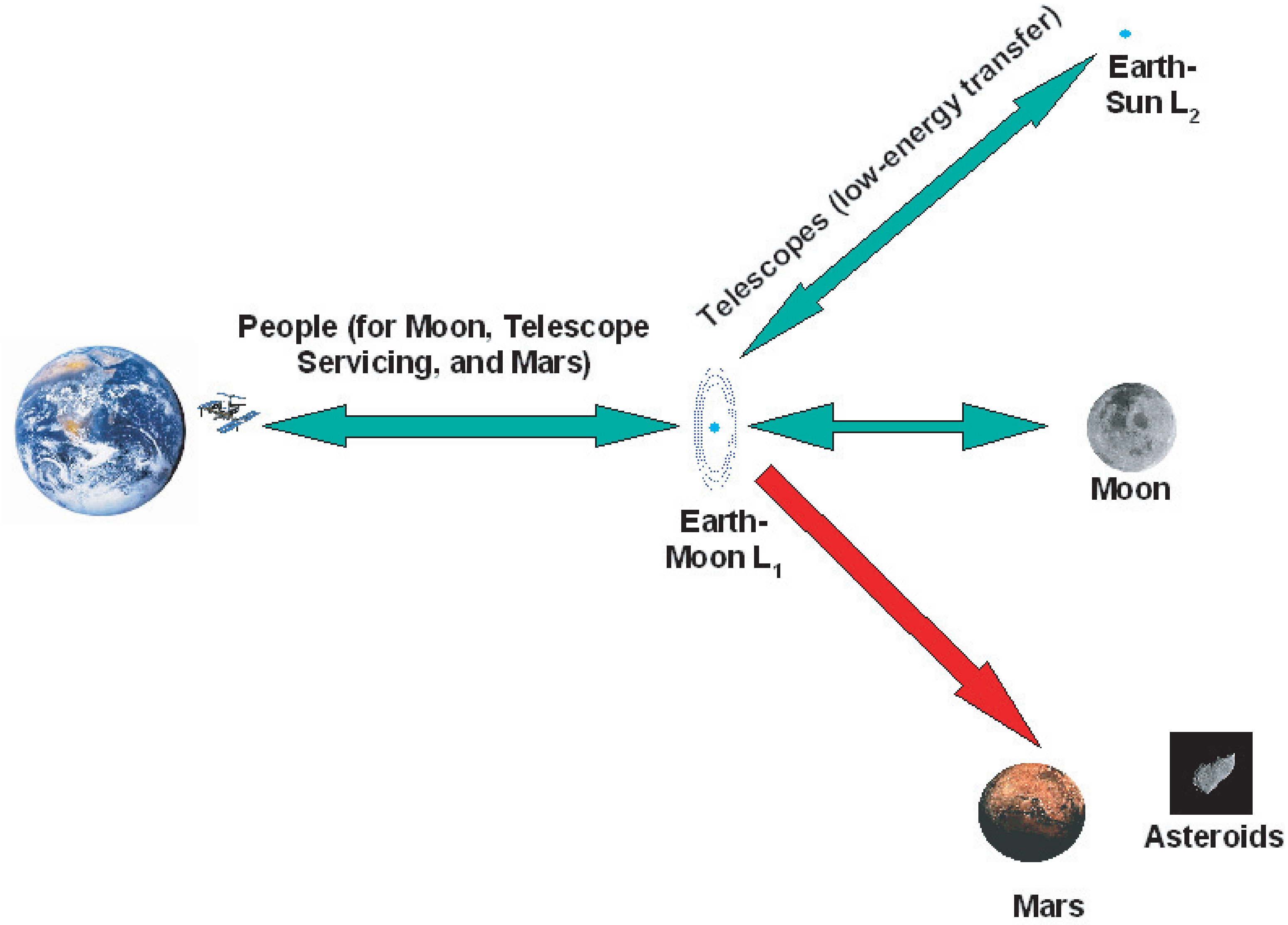


Getting to TPF and Beyond

- *The L_1 Gateway Station*

  - ☐ A gateway at the ***Earth-Moon L_1 point*** is of interest as a semi-permanent ***manned site***.
  - ☐ Can be used for going to the moon, servicing TPF and possibly for missions to other planets.
  - ☐ Efficient transfers can be created using the 3-body and invariant manifold techniques that our group has developed

Getting to TPF and Beyond



Getting to TPF and Beyond

moon-L1-to-earth-L2-mf.qt

Getting to TPF and Beyond

moon-L1-to-earth-L2-ef.qt

More Information

- *http://www.cds.caltech.edu/˜marsden/*
  http://www.cds.caltech.edu/˜koon/
- *email: marsden@cds.caltech.edu*
- *two of the main publications:*

  - Koon, W. S., M. Lo, J. E. Marsden and S. Ross [2000], ***Heteroclinic Connections between periodic orbits and resonance transitions in celestial mechanics***, *Chaos*, 10, 427–469.
  - Serban, R, W.S. Koon, M.W. Lo, J.E. Marsden, L.R. Petzold, S.D. Ross and R.S. Wilson [2001], ***Halo Orbit Mission Correction Maneuvers Using Optimal Control***, *Automatica*, to appear.

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The End

TYPESETTING SOFTWARE: TeX, *Textures*, LaTeX, `hyperref`, `texpower`, Adobe Acrobat 4.05
GRAPHICS SOFTWARE: Adobe Illustrator 9.0.2
LaTeX SLIDE MACRO PACKAGES: Wendy McKay, Ross Moore