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Optimal Reconfiguration of Formation Flying Spacecraft —a Decentralized Approach

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***Abstract*— This paper introduces a hierarchical, decentralized, and parallelizable method for dealing with optimization problems with many agents. It is theoretically based on a hierarchical optimization theorem that establishes the equivalence of two forms of the problem, and this idea is implemented using DMOC (Discrete Mechanics and Optimal Control). The result is a method that is scalable to certain optimization problems for large numbers of agents, whereas the usual "monolithic" approach can only deal with systems with a rather small number of degrees of freedom. The method is illustrated with the example of deployment of spacecraft, motivated by the Darwin (ESA) and Terrestrial Planet Finder (NASA) missions.**

I. INTRODUCTION

For upcoming space missions like *Darwin*[1] and *Terrestrial Planet Finder (TPF)*[2] control strategies have to be devised that enable precise formation flying of a group of spacecraft. In light of the tight mass budget of these missions it is of great interest to minimize the propellant consumption in performing the associated maneuvers.

In [9], an optimal control problem relevant to this problem was formulated and solved by means of a recently developed numerical method DMOC (Discrete Mechanics and Optimal Control), which relies on a direct discretization of the variational principle [8] that underlies the dynamical model of the system. The focus was on the concrete setting of the Darwin and TPF missions: a group of six spacecraft, viewed as one large mechanical system, was placed in the vicinity of an L_2-Halo orbit and was required to adopt a certain configuration of the satellites relative to each other.

The monolithic approach in [9] does not easily scale to larger groups of vehicles. In fact, it does not exploit the structure of the given system, which is in fact composed of many identical subsystems. For each of them, a similar (sub)problem has to be solved.

In this paper we develop a decentralized approach for solving the given (large) optimal control problem. The basic idea is that if one *temporarily* neglects collision avoidance concerns, in a reconfiguration maneuver, the only coupling between the subsystems enters through a constraint on the final configuration. (As we argue later, collision avoidance can be readily reinstated, so this is not a real restriction.) We show how to derive a hierarchical formulation of the optimal control problem by exploiting this structure. The hierachical formulation is naturally suited for a solution of the associated subproblems in parallel.

Related work that was inspirational for the present paper is that of Tomlin [5], [13]: The members of the group are forced to cooperate to achieve common goals, i.e., goals concerning the entire group or subgroups, as well as independent goals, i.e., goals concerning only one agent of the group, while operating under both local and interconnection constraints.

A main challenge for formation flying is to define global goals, such as to adopt a prescribed final configuration autonomously, i.e. instead of preassigning the final positions for each spacecraft one defines a target manifold and each spacecraft is supposed to choose its final position in an autonomous way. How et al. ([4], [14]) and Mueller ([12]) solve this decision of assigning special positions to the agents by a so called "privileged method", where the vehicle with the highest minimum cost of all vehicles is assigned to the target state corresponding to its minimum cost. This procedure is repeated for all remaining members and target states. Compared to a search over all possible final configurations, this method requires less computational effort, yet it does not guarantee that a globally optimal solution is found.

In this paper, we prescribe the relative final configuration by an artificial potential. But instead of creating behavior-based feedback control laws based on this which lead to sub-optimal solutions for the formation (see Gazi ([3]) and Izzo, Pettazzi ([6])), we derive interconnecting boundary constraints for our optimal control problem.

We reformulate the resulting optimization problem as a hierarchical problem with a sum of independent cost functions and local dynamics. Our approach is similar to that of [13] in the following sense: The interconnecting constraint can be interpreted as a measure of "deviation" from the desired final configuration. Firstly, each agent optimizes its trajectory ignoring the deviations from the common final configuration. In [5], due to the linear interconnecting constraint the deviations can be interpreted as the dual variables corresponding to the dualization of the centralized problem. Therefore, an update of the deviations corresponds to the solution of the decomposed dual problem. Here, we update the initial guess for the final position iteratively within a second optimization problem including the nonlinear interconnecting constraint. This optimization problem is put above the decentralized one

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[1] http://www.esa.int/science/darwin

[2] http://planetquest.jpl.nasa.gov/TPF

such that a hierarchical optimization problem is obtained.

An outline of the paper is as follows: Based on the model for the dynamics of the spacecraft that is introduced in Section II we formalize the optimal control problem in Section III. In Section IV we introduce the decentralized approach and show equivalence of this formulation to the monolithic one. We further show how to exploit this structure in order to solve the problem in parallel. We recall the numerical method that is used for its solution in Section V and finally present our numerical results in Section VI.

II. Model

We are dealing with a group of n identical spacecraft, where we use the same model as in [9]: Each spacecraft is modeled as a rigid body with six degrees of freedom (position and orientation), i.e., its configuration manifold is $SE(3)$. We assume that each spacecraft can be controlled in this configuration space by a force-torque pair (F, τ), acting on its center of mass.

For several reasons (consistent solar illumination characteristics, lack of disturbing perturbations, relative ease of sending and retrieving spacecraft), an attractive region in space for missions like Darwin and TPF is in the vicinity of a Libration orbit around the Earth-Sun L_2 Lagrange point. Correspondingly, for each spacecraft the dynamical model for the motion of its center of mass is given by the circular restricted three body problem (cf. [9]).

In Figure 1 we plot a family of L_2-Halo orbits. This family has been computed by a predictor corrector method on an initial orbit found by a shooting technique (see [7]).

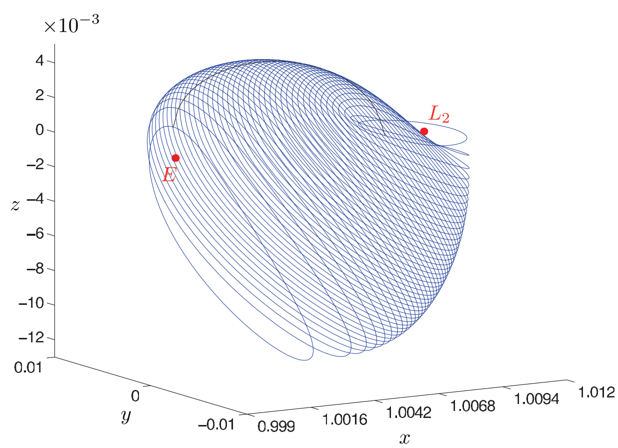

Fig. 1. Family of periodic orbits in the circular restricted three body problem in the vicinity of the L_2-Lagrange point.

In a normalized, rotating coordinate system, the potential energy of a spacecraft at $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ is

$$V(x) = -\frac{1-\mu}{|x-(1-\mu,0,0)|} - \frac{\mu}{|x-(-\mu,0,0)|}, \quad (1)$$

where $\mu = m_1/(m_1+m_2)$. Its kinetic energy is the sum of

$$K_{\text{trans}}(x,\dot{x}) = \frac{1}{2}((\dot{x_1} - \omega x_2)^2 + (\dot{x_2} + \omega x_1)^2 + \dot{x}_3^2),$$

(assuming that its mass is equal to 1 for simplicity) and $K_{\text{rot}}(\Omega) = \Omega^T J \Omega/2$, where $\Omega \in \mathbb{R}^3$ is the angular velocity and J is the spacecraft inertia tensor, which, for simplicity, we choose to be the identity; because of the presense of controls, this does not mean that the rotational dynamics is trivial.

III. The Control Problem

Our goal is to compute the force and torque $(F^{(i)}(t), \tau^{(i)}(t))$, $i = 1,\ldots,n$, for each spacecraft, such that the group moves from a given initial state $(x^{(i)}, p^{(i)}, \dot{x}^{(i)}, \dot{p}^{(i)})_{i=1}^n$ into a prescribed target manifold within a prescribed time interval $[t_0, t_f]$, where the unit quaternion $p^{(i)}$ represents the orientation of the i-th spacecraft. In our application context, the target manifold will be defined by prescribing the relative positioning of the spacecraft, their common velocity as well as a common orientation. We additionally require the resulting controlled trajectory to minimize a given cost functional—often related to the associated fuel consumption of the spacecraft.

More precisely, for their target state, we require the spacecraft to be located on a circle with center on a Halo orbit and with equidistant relative distances on the circle. Let $\nu \in \mathbb{R}^3$ be a given unit vector (the "line of sight" of the spacecraft). The target manifold $M \subset TSE(3)^n$ is the set of all states $(x^{(i)}, p^{(i)}, \dot{x}^{(i)}, \dot{p}^{(i)})_{i=1}^n$ such that:

1. all spacecraft lie in a plane with normal ν, i.e.

$$\langle x^{(i)} - x^{(j)}, \nu \rangle = 0, \quad i,j = 1,\ldots,n; \quad (2)$$

2. within that plane, the spacecraft are located equidistantly on a circle with prescribed radius and prescribed center on a Halo orbit. Let $r_0 \in \mathbb{R}$ be a given radius and $\bar{x} \in \mathbb{R}^3$ a certain point on a Halo orbit and let $\nu_1^\perp \perp \nu_2^\perp \in \mathbb{R}^3$ be two perpendicular unit vectors that are perpendicular to ν. For $i = 1,\ldots,n$ we consider the vector

$$z^{(i)} = [\nu_1^\perp \; \nu_2^\perp]^T (x^{(i)} - \bar{x}) \in \mathbb{R}^2 \quad (3)$$

and require that

$$h(z^{(i)}) = \|z^{(i)}\| - r_0 = 0, \; i = 1,\cdots,n \quad (4)$$

and

$$k(z) = 0, \quad z = (z^{(1)},\ldots,z^{(n)}), \quad (5)$$

with functions $h : \mathbb{R}^2 \to \mathbb{R}$ and $k : \mathbb{R}^{2n} \to \mathbb{R}^n$, where the constraint (4) forces each spacecraft to be a distance r_0 from the center and the constraint (5) guarantees an equidistant arrangement. We describe the constraint (5) in more detail in Section IV.

3. all spacecraft have their "line of sight" aligned with ν. For simplicity we impose a more restrictive condition, namely that each spacecraft is rotated according to a prescribed unit quaternion $p_0^{(i)}$, i.e. we require that

$$p^{(i)} = p_0, \quad i = 1,\ldots,n; \quad (6)$$

4. all spacecraft have the same prescribed linear velocity, $\dot{x}^{(i)} = \dot{x}_0, \quad i = 1,\ldots,n$, where $\dot{x}_0$ is determined on basis of the Halo orbit under consideration, and they have zero

angular velocity, i.e., $\Omega^{(i)} = 2\dot{p}^{(i)}\bar{p}^{(i)} = 0, \quad i = 1,\dots,n$, where $\bar{p}^{(i)}$ is the conjugate quaternion to $p^{(i)}$.

As mentioned, in addition to controlling to the target manifold, we would like to minimize the fuel consumption of the spacecraft. Here we consider the cost function

$$\begin{aligned} J(F,\tau) &= \sum_{i=1}^{n} J_i(F^{(i)},\tau^{(i)}) \\ &= \sum_{i=1}^{n} \int_{t_0}^{t_f} |F^{(i)}(t)|^2 + |\tau^{(i)}(t)|^2\, dt, \end{aligned} \tag{7}$$

where J_i is the cost function for spacecraft i and $F(t) = (F^{(1)}(t),\dots,F^{(n)}(t))$ and $\tau(t) = (\tau^{(1)}(t),\dots,\tau^{(n)}(t))$ denote the force and torque functions for the system.

IV. Decentralization

When one neglects collision avoidance concerns, the optimal control problem described in the previous section is "almost" decoupled in the sense that the coupling only enters through the constraints (5) on the final configuration. In this section, we show how one can exploit this fact in order to parallelize the associated computations.

A. Hierarchical Optimal Control Problem

The basic observation is that the problem can be formulated as a hierarchical optimization problem, where the *outer problem* relates to the correct arrangement of the final configuration and the n *inner problems* determine the optimal trajectory for one spacecraft with fixed initial and final configuration, respectively.

We parameterize the final positions of the spacecraft projected onto the prescribed plane by the vector $\varphi = (\varphi^{(1)},\dots,\varphi^{(n)})$ via

$$z^{(i)} = \begin{pmatrix} r_0 \cos\varphi^{(i)} \\ r_0 \sin\varphi^{(i)} \end{pmatrix}, \tag{8}$$

where $\varphi^{(i)}$ is the angle of spacecraft i, determining the final position on a prescribed circle with prescribed center (cf. Figure 2). First, we want to derive the final constraint (5) with the help of this parameterization. We define the artificial potential $G : S^n \to \mathbb{R}$ by

$$G(\varphi) = \sum_{i,j=1, i\neq j}^{n} \frac{1}{\|z^{(i)} - z^{(j)}\|^2}$$

This artificial potential acts like a gravitational potential that affects attraction or repulsion, respectively, between bodies. For an equidistant arrangement on the circle the resulting forces $dG/d\varphi$ acting on each spacecraft have to be zero. Therefore, we obtain as final constraint

$$g(\varphi) = \frac{dG}{d\varphi}(\varphi) = 0,$$

with a function $g : S^n \to \mathbb{R}^n$. With the parameterization (8) it holds by defining a function $\tilde{G} : \mathbb{R}^{2n} \to \mathbb{R}, \tilde{G}(z) := G(\varphi)$

$$0 = g(\varphi) = \frac{dG}{d\varphi}(\varphi) = \frac{d\tilde{G}}{dz}(z) \cdot \frac{dz}{d\varphi} =: k(z), \tag{9}$$

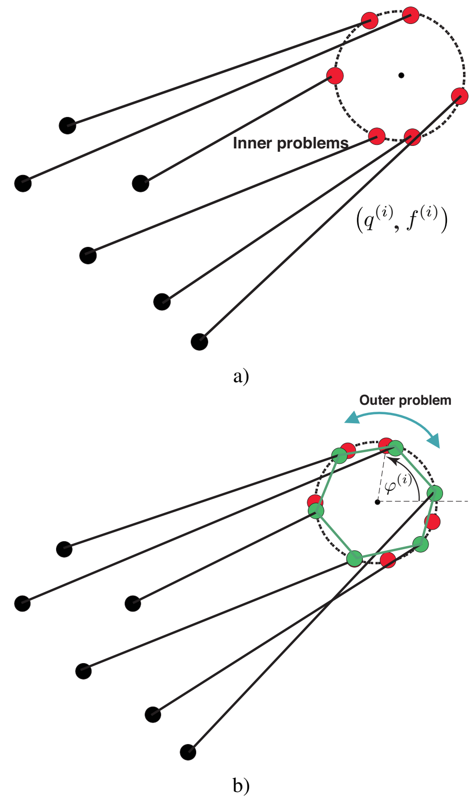

Fig. 2. Hierarchical optimal control problem. a) inner problem; b) outer problem.

which results in the final constraint (5).

With parameterization (8) the problem has the following hierarchical form: Let $q^{(i)} = \begin{pmatrix} x^{(i)} \\ p^{(i)} \end{pmatrix} \in Q = SE(3)$ denote the configuration and $f^{(i)} = \begin{pmatrix} F^{(i)} \\ \tau^{(i)} \end{pmatrix}$ the control force of spacecraft i. By optimizing within a fixed time interval $I = [0,1]$ we obtain the optimal control problem

$$\min_{\varphi} J(\varphi) = \min_{\varphi} \sum_{i=1}^{n} \min J_i(q^{(i)}, f^{(i)})$$

where the minimization on the left is subject to $g(\varphi) = 0$ and in the minimum on the right hand side, $q^{(i)} : [0,1] \to Q, f^{(i)} : [0,1] \to T^*Q, q^{(i)}(0) = q_0^{(i)},\ \dot{q}^{(i)}(0) = \dot{q}_0^{(i)}, A\, q^{(i)}(1) = b(\varphi^{(i)}),\ \dot{q}^{(i)}(1) = \dot{q}_f^{(i)}$, and $(q^{(i)}, f^{(i)})$ have to fulfill the dynamics of spacecraft i.

Here the matrix A is $A = \begin{pmatrix} [\nu_1^\perp\ \nu_2^\perp]^T & 0 \\ 0 & I_4 \end{pmatrix} \in \mathbb{R}^{6,7}$, where I_4 is the unit 4×4 matrix and the vector $b(\varphi^{(i)}) = \begin{pmatrix} z^{(i)} + [\nu_1^\perp\ \nu_2^\perp]^T \bar{x} \\ p_0 \end{pmatrix} \in \mathbb{R}^6$ is defined by equations (3) and (6). Due to the parameterization (8) we don't have to incorporate the constraint (4).

The inner problems are uncoupled since each spacecraft has to minimize its costs separately subject to fixed initial and final states and its dynamics.

The outer problem includes the constraint for the final configuration, i.e. the coupling of the system. We use an iterative method, namely sequential quadratic programming (SQP), for the solution of both, the inner and the outer problems. In each step of the solution of the outer problem all n inner problems have to be solved anew with the new boundary constraints.

In using SQP for solving the discretized system we have to provide an initial guess for the optimal trajectory of the group. This guess will typically determine to which local optimum the optimizer converges. In particular, the sequence of the spacecraft on the circle given by this initial guess will have a strong influence on their sequence in the computed solution. In particular, in this paper we do not solve the associated combinatorial optimization problem of determining the optimal sequence on the circle.

B. Equivalence of both optimal control problems

In order to show the equivalence of the "monolithic" formulation of the optimal control problem in Section III to the hierarchical one in this Section, we consider the following abstract optimization problem:

$$\min_{(x,\varphi)\in X\times\Phi} J(x,\varphi) \quad \text{s.t. } A(x,\varphi)=0,\ g(\varphi)=0, \qquad (10)$$

where $X\subset\mathbb{R}^{d_x}, \Phi\subset\mathbb{R}^{d_y}$ are compact and $J: X\times\Phi\to\mathbb{R}$, $A: X\times\Phi\to\mathbb{R}^a$ and $g:\Phi\to\mathbb{R}^g$ are continuous.

Defining $X(\varphi)=\{x\in X \mid A(x,\varphi)=0\}$, we see that

$$\bigcup_{\varphi\in g^{-1}(0)} X(\varphi)\times\{\varphi\} = \{(x,\varphi)\mid A(x,\varphi)=0, g(\varphi)=0\}.$$

Thus,

$$\begin{aligned}
&\min\{J(x,\varphi)\mid A(x,\varphi)=0, g(\varphi)=0\}\\
&=\min\left\{J(x,\varphi)\mid (x,\varphi)\in \bigcup_{\varphi\in g^{-1}(0)} X(\varphi)\times\{\varphi\}\right\}\\
&=\min \bigcup_{\varphi\in g^{-1}(0)} \{J(x,\varphi)\mid (x,\varphi)\in X(\varphi)\times\{\varphi\}\}\\
&=\min\left\{\min\{J(x,\varphi)\mid x\in X(\varphi)\}\mid \varphi\in g^{-1}(0)\right\}\\
&=\min\{\min\{J(x,\varphi)\mid A(x,\varphi)=0\}\mid g(\varphi)=0\},
\end{aligned}$$

i.e., we arrive at a hierachical formulation of the problem. The "inner problem" is given by minimizing $J(x,\varphi)$ subject to $A(x,\varphi)=0$ (for a fixed $\varphi\in\Phi$), while the "outer problem" is given by minimizing

$$\hat{J}(\varphi)=\min\{J(x,\varphi)\mid A(x,\varphi)=0\} \quad \text{s.t. } g(\varphi)=0.$$

Since in our specific application, the inner cost function, $J(x,\phi)$ is given by the sum

$$\sum_{i=1}^{n} \min J_i(q^{(i)}, f^{(i)})$$

and since all J_i are nonnegative, the inner problem decouples into n independent subproblems which can be solved independently.

C. Parallelization

The hierarchical structure of the problem enables a computational solution in parallel, since we are faced with n uncoupled inner problems in each step of the solution of the outer problem. These n subproblems are solved in n different tasks. Our implementation uses the software package PUB (**P**aderborn **U**niversity **B**SP-Library, [1]) developed within the DFG research project CRC 376 "Massively Parallel Computation" at the University of Paderborn. In the terminology of PUB, each step of the iteration scheme for the solution of the outer problem represents one *superstep*. After each superstep, the tasks have to communicate during the *synchronization*.

We use the software package PUB for several reasons: Since our implementation involves more than one superstep we rely on a frequent communication between the processes (*"coupled parallel processes"*). Moreover, the computational time of each subproblem of the inner problem depends on the initial guess for the optimal trajectory. Therefore, it is of great interest to change load on the machines for an appropriate load balancing. These coupling and migration requirements can be easily realized in PUB ([1]).

Remark 1: As noted at the beginning of §IV we neglected collision avoidance concerns in our optimization progress. The idea is to include collision avoidance maneuvers online after finding the optimal trajectories, i.e. whenever two spacecraft detect a possible collision between them, the optimal control strategy is "switched off" and both spacecraft will execute a maneuver to avoid the collision. After coming back to their pre-computed optimal trajectories, the optimal control strategy is "switched on" again. For such a scenario, a question is how much the cost of an optimal trajectory differs from the cost of the associated modified collision-free trajectory. In many applications we expect the solution to still be nearly optimal. An optimization strategy that is compatible with the methods here is that given in [2].

V. Numerical Method: DMOC

To solve the optimal control problem formulated above, we use DMOC [8], a technique that relies on a direct discretization of the variational formulation of the dynamics of the system. For convenience, we briefly summarize the basic idea.

A mechanical system with configuration space Q is to be moved on a curve $q(t)\in Q$, $t\in[0,1]$, from a state $(q^0,\dot q^0)$ to a state $(q^1,\dot q^1)$ under the influence of a force f. The curves q and f shall minimize a given cost functional

$$J(q,f)=\int_0^1 C(q(t),\dot q(t),f(t))\,dt. \qquad (11)$$

If $L: TQ\to\mathbb{R}$ denotes the Lagrangian of the system, its motion $q(t)$ satisfies the *Lagrange-d'Alembert principle*, which requires that

$$\delta\int_0^1 L(q(t),\dot q(t))\,dt+\int_0^1 f(t)\cdot\delta q(t)\,dt=0 \qquad (12)$$

for all variations δq with $\delta q(0)=\delta q(1)=0$.

Using a global discretization of the states and the controls one obtains the *discrete Lagrange-d'Alembert principle* which specifies equality constraints for the resulting finite dimensional nonlinear optimization problem: We replace the state space TQ by $Q \times Q$ and a path $q : [0,1] \to Q$ by a *discrete path* $q_d : \{0, h, 2h, \ldots, Nh = 1\} \to Q$, where N is a positive integer and where we view $q_k = q_d(kh)$ as an approximation to $q(kh)$ [11]. Analogously, we approximate the continuous force $f : [0,1] \to T^*Q$ by a discrete force $f_d : \{0, h, 2h, \ldots, Nh = 1\} \to T^*Q$ (writing $f_k = f_d(kh)$).

Based on this discretization we approximate the velocity $\dot{q}$ by forward differences and approximate the Lagrangian, the virtual force and the cost function by midpoint rule schemes.

After incorporating the boundary conditions, one obtains a discrete constrained optimization problem: Minimize

$$J_d(q_d, f_d) = \sum_{k=0}^{N-1} C_d(q_k, q_{k+1}, f_k, f_{k+1}) \tag{13}$$

subject to the constraints $q_0 = q^0$, $q_N = q^1$ and

$$\begin{aligned} D_2 L(q_0, \dot{q}_0) + D_1 L_d(q_0, q_1) + f_0^- &= 0, \\ D_2 L_d(q_{k-1}, q_k) + D_1 L_d(q_k, q_{k+1}) + f_{k-1}^+ + f_k^- &= 0, \\ -D_2 L(q_N, \dot{q}_N) + D_2 L_d(q_{N-1}, q_N) + f_{N-1}^+ &= 0, \end{aligned}$$

$k = 1, \ldots, N-1$, with the *discrete Lagrangian* $L_d(q_k, q_{k+1}) := hL\left(\frac{q_{k+1}+q_k}{2}, \frac{q_{k+1}-q_k}{h}\right)$ the *left and right discrete forces* $f_k^- = f_k^+ = \frac{h}{4}(f_{k+1} + f_k)$ and the *discrete cost function* $C_d(q_k, q_{k+1}, f_k, f_{k+1}) := hC\left(\frac{q_{k+1}+q_k}{2}, \frac{q_{k+1}-q_k}{h}, \frac{f_{k+1}+f_k}{2}\right)$.

Remark 2: Since we are interested in the relative positions of the spacecraft with respect to each other and the scales of interest differ by a factor of around 10^{11}, we performed our computations in a local coordinate system by linearizing the system around a Halo-orbit to avoid rounding errors.

VI. EXAMPLE COMPUTATIONS

As mentioned in the introduction we are particularly interested in ensembles with a large number of spacecraft. In all our computations we used $N = 10$ time intervals in the time discretization of the trajectories and solved the resulting finite-dimensional (nonlinear) optimization problem by the SQP-method as implemented in the routine `E04UEF` of the NAG-library – using numerical derivatives both for the cost and for the constraint functions.

To show the efficiency of the parallelized implementation motivated by the hierarchical problem formulation, we consider as a first example the reconfiguration of a group of 60 point masses in the plane. The group initially is located along a line, taken to be the x-axis, uniformly distributed between $x = -30$ and $x = 30$, and is required to adopt a circular, uniform formation with prescribed center at $(100, 100)$. Figure 3 shows the final positions and the final segments of the corresponding optimal trajectories.

In Figure 4 and Table I, we compare the dependence of the computation time on the number of processors. Note that the speed-up is often slightly larger than the number of different processors. We attribute this phenomenon to caching effects in the processors.

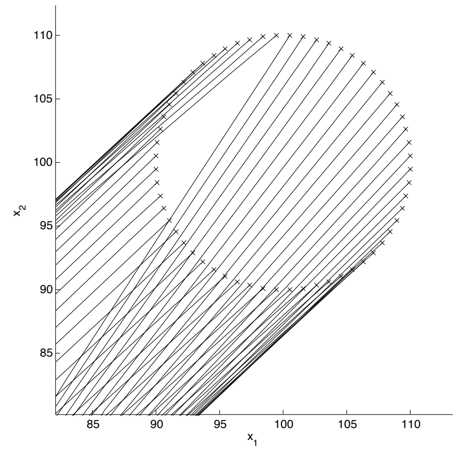

Fig. 3. Final positions for a reconfiguration of 60 point masses in the plane and the last portions of the corresponding optimal trajectories from given initial positions on the x-axis.

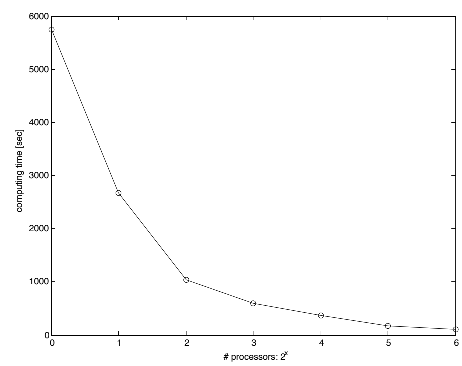

Fig. 4. Computation time in dependence on the number of processors for the reconfiguration of 60 point masses in the plane.

TABLE I

SPEED UP DIAGRAM

# of processors	2	4	8	16	32	64
speed up	2.15	5.57	9.65	15.73	34.94	53.0

As a second, more realistic example we consider a group of 30 spacecraft modeled as rigid bodies in 3-space, with the circular restricted three body problem governing their dynamics (cf. Section II).

Figure 5 shows (in normalized coordinates) the initial positions ($\circ$), the optimal trajectories as well as the final positions ($\times$). The group initially is located on a grid lying in the x_1-x_2-plane with initial orientation $p_0^{(i)} = (\cos(\frac{\pi}{2}), \sin(\frac{\pi}{2}) \cdot (1,0,0))$ for each spacecraft i (i.e. a rotation of $\theta = \pi$ around the x_1-axis) and ends in a circle formation in the plane with normal $n = (1,0,1)$ and final orientation $p_f^{(i)} = (\cos(\pi), \sin(\pi) \cdot (0,1,0))$ for each spacecraft i (i.e., a rotation of $\theta = 2\pi$ around the x_2-axis).

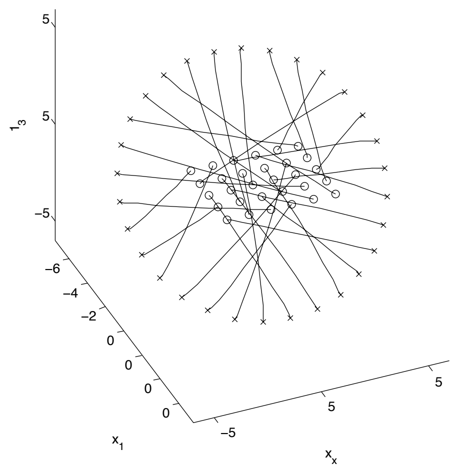

Fig. 5. Initial positions (○), optimal trajectories and final positions (×) for a reconfiguration of 30 spacecraft in the CRTBP in x_1-x_2-x_3-space.

TABLE II

SPEED UP DIAGRAM

number of processors	2	4	8	16	32
speed up	2.11	3.77	6.75	10.49	18.89

Figure 6 and Table II show the dependence of the computation time on the number of processors. The numbers are not as good as in the point masses example; however, the speed-up is still reasonably close to the optimal value, showing the effectiveness of our decentralized approach.

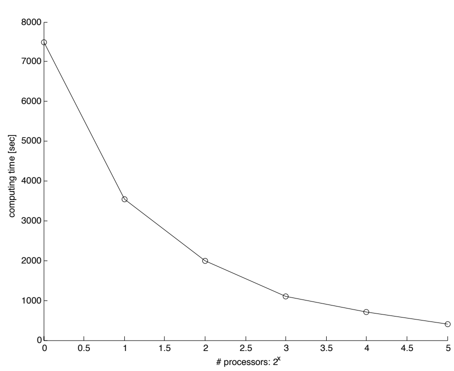

Fig. 6. Computation time in dependence on the number of processors for the reconfiguration of 30 spacecraft in the CRTBP.

VII. CONCLUSIONS AND FUTURE WORK

A. Conclusions

This paper develops a hierarchical and decentralized approach for the solution of optimal control problems with many agents which are coupled through small numbers of degrees of freedom. The method is based on a hierarchical formulation of the associated optimization problem that enables a parallelized implementation. The result was implemented using DMOC and was demonstrated on a system of 30 satellites which started in a rectangular array in a model of the three body problem appropriate to the DARWIN and TPF missions and were asked to assume a circular pattern, with the satellites equally spaced around this circle. Our numerical experiments show the computational efficiency of the proposed approach, while a "monolithic" approach would not be able to handle a problem of this size.

B. Future Work

As mentioned in Section IV all computations are done without consideration of collision avoidance concerns. In Remark 1 we gave a brief sketch about how we want to incorporate collision avoidance strategies in our future work.

Another challenge for these kinds of optimal control problems is to find the globally optimal order of the spacecraft within the target manifold. Since we use a local optimization method, this order is dependent on the initial guess fed to the method.

VIII. ACKNOWLEDGMENTS

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