ARNOLD DIFFUSION IN THE DYNAMICS OF A 4-MACHINE POWER SYSTEM UNDERGOING A LARGE FAULT

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Summary

We focus on the seemingly complicated dynamics of a four-machine power system which is undergoing a sudden fault. Adopting a Hamiltonian (energy) formulation, we consider the system as an interconnection of (one degree of freedom) subsystems. Under certain configuration (a star network)and parameter values we establish the presence of Arnold diffusion which entails periodic, almost periodic, and complicated nonperiodic dyanmics all simultaneously present; and erratic transfer of energies between the subsystems. In section 1 we introduce the transient stability problem in a mathematical setting and explain what our results mean in the power systems context. Section 2 provides insights into Arnold diffusion and summarizes its mathematical formulation as in [8], [1]. Section 3 gives conditions for which Arnold diffusion arises on certain energy levels of the swing equations. These conditions are verified analytically in the case when all but one subsystem (machine) undergo relatively small oscillations.

1. Introduction

Transient stability of a power system describes the dynamical phenomena caused by a sudden fault (such as short circuit) or a large impact (such as lightning). It is precisely the Lyapunov stability in a state space formulation of a simplified differential equation model (called the swing equations) which possesses multiple equilibria: $\dot{x}=f(x)$. Let x_o be a stable equilibrium point of this model which is presumably "closest" to the prefault equilibrium point (see [4],[11]). The transient stability problem is to determine whether or not a given point in the state space belongs to the region of stability of the stable point x_o. This translates the transient stability problem to one of investigation of the region of stability of a given stable equilibrium point (See [6,7,9] for simulations).

The swing equations: We write the swing equations model of an interconnected power system. We assume zero transfer conductances of the reduced network (or assume a generator connected to each node).

$$\frac{d}{dt}\delta_i = \omega_i - \omega_R \qquad (1.1.i)$$

$$M_i \frac{d}{dt}\omega_i + D_i\omega_i = P_i - \sum_{\substack{i=1\\ i\neq i}}^{n} Y_{ij} \sin(\delta_i - \delta_j) \qquad (1.2.i)$$

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where δ_i and ω_i are respectively, the angle and the velocity of the rotor of the i^{th} machine; ω_R is a constant reference velocity, usually ($2\Pi.60$ rad/sec); M_i (D_i) is the inertia (damping) constant; P_i ($: = P_{mi} - G_{ii}E_i^2$) is the exogenous power input; Y_{ij} ($: = E_iE_jY_{ij}$) is the maximum real power transferred between nodes i and j (For details of derivations and notation,refer to [3].). The damping constants D_i are known to be very small. As often done in the analysis of transient stability, we set them to zero, i.e., $D_i = 0$.

Equations (1.1) (1.2) describe an n-degree of freedom Hamiltonian system with a known energy function

$$W = \sum_{i=1}^{n} \frac{1}{2} M_i (\omega_i - \omega_R)^2 - \sum_{i=1}^{n} P_i \delta_i - \sum_{i<j} Y_{ij} \cdot \cos(\delta_i - \delta_j).$$

In the power systems literature the analysis of the transient stability had focused on utilizing this energy function. Most of these Analyses [3,6,7, 9] draw inisghts from analogies to the 1 machine - ∞ bus case (or equivalently the equal area criterion). Moreover, in [6], [9], an estimate of the stability region is produced in the δ-space (i.e. angle space) only.

The essence of our contribution, as those of [10], are to refute in certain cases the analogies to the 1 machine - ∞ bus, and prohibit the use of other than the complete state space (i.e. δ-ω space). This is so since the dynamics of (autonomous) deterministic differential equations of dimensions higher than 2, can exhibit complicated behavior (see [1], for instance). Indeed, Kopell and Washburn [10] showed that (horseshoe) chaos, i.e. unpredictable behavior of trajectories, is present is a 3-machine power system under certain configuration and parameter values. The 3-machine case describes a two-degree of freedom system.

Another form of known complicated behavior is Arnold diffusion which entails complex nonperiodic unpredictable trajectories and, moreover, erratic transfer of energies between interconnectioned degrees of freedom (subsystems). Here we show that Arnold diffusion arises in the interconnected 4-machine case (For the n-machine case, n≥4, and technical details, see [2].). This case describes a 3-degree of freedom system. The specific configuration of the power system yielding Arnold diffusion is shown in fig. 1, with the noted relative parameter ranges. In the case of 3-machines, this configuration

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produces chaos. This is analogous to that of [10] except that it focuses on energy levels of the whole system, i.e. a Hamiltonian approach, and it explicitly considers the dynamics of the reference machine (machine 4), see section 3.

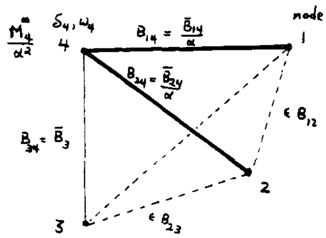

Fig. 1. A 4-machine network. (α small)

In the context of power systems, we summarize what the presence of Arnold diffusion implies: (1) analysis cannot be based on analogies to the one machine-∞ bus case which is an autonomous two-dimensional system and hence does not exhibit complicated behavior. (2) It raises questions about the adequacy of the model. Does a physical power system of the same configuration as figure 1, exhibit complicated dynamics in the form of chaos or Arnold diffusion? (3) If the model is indeed adequate, then the conclusion of our result leads to new design constraint on the parameter ranges and the configuration of the interconnected power system. (4) Transient stability analysis cannot be conducted in the angle space, i.e. δ-space, alone as in [6,9]. The complete dynamics can be understood only in the whole state space, i.e. δ-ω-space.

2. Arnold diffusion

Arnold diffusion (see [2] e.g) is a self-generated "stochastic" motion that can occur in near-integrable (i.e. weakly coupled) n-degree of freedom Hamiltonian systems where n>3. It also entails an erratic transfer of energies between these degrees of freedom. It, therefore, constitutes a new concept of instability [5] - different from the stability concept in the Lyapunov sense. Arnold [5] showed this form of instability in a specific example of a Hamiltonian system: a weakly coupled (i.e. near integrable) time-periodic two degree of freedom Hamiltonian system; one degree of freedom possesses a homoclinic orbit, i.e. a trajectory connecting a saddle point to itself; the second degree of freedom is a nonlinear oscillator; and the weakly coupling term is time periodic. There is vast experimental work on Arnold diffusion primarily in the plasma physics literature. For an account of these works, see [2]. Holmes and Marsden [8] introduced an adaptation of Arnold's result to n-degrees of freedom, where $n \geq 3$, employing a vector-Melinkov integrals version. We summarize their result (see also [8],[2]): consider the (perturbed) near-integrable Hamiltonian system

$$H^{\mu}(q,p,\underline{\Theta},\underline{I}) = F(q,p) + \sum_{i=1}^{n} G_i(I_i) + \mu H^1(q,p,\underline{\Theta},\underline{I}) \qquad (2.1)$$

The parameters $(q,p,\underline{\Theta},\underline{I})$ are canonical coordinates on a 2(n+1)-dimensional (symplectic) manifold: q,p are real and $\underline{\Theta}=(\Theta_1,\ldots,\Theta_n)$, $\underline{I}=(I_1,\ldots,I_n)$ are n-vectors. F is a Hamiltonian in the variables q and p (one degree of freedom), G_i is a one degree of freedom Hamiltonian in the action (I_i)-angle (Θ_i) coordinates, i.e., the degree of freedom describes an oscillator. H^1 is a Hamiltonian that couples the degrees of freedom (i.e. a function of all variables) and μ is the small perturbation. When $\mu = 0$, one obtains the (unperturbed) integrable Hamiltonian system. Consider the following conditions:

(C1) F possesses a homoclinic orbit $(\bar{q}(t), \bar{p}(t))$ connecting a saddle point (q_o, p_o) to itself. Let $\bar{h}$ be the energy of this orbit.

(C2) The frequencies of the oscillators $\Omega_j(I_j) = dG_j/dI_j\,(I_j) > 0$, for $I_j > 0$. (Notice that Ω_j depends on the action I_j, therefore the oscillators are nonlinear.)

(C3) The constants $G_j(I_j) = h_j$, j=1,...,n, are chosen such that the (unperturbed) frequencies $\Omega_1(I_1), \ldots, \Omega_n(I_n)$ satisfy the non-degeneracy conditions (i.e. $d\Omega_j/dI_j\,(I_j) \neq 0$, j=1,...,n) and the non-resonance condition, i.e., the equation $\sum_{j=1}^{n} k_j\Omega_j(I_j) = 0$ implies $k_j = 0\ \forall\ 1 \leq j \leq n$ (k_j are integers).

(C4) Define the Melinkov vector $M(\underline{\Theta}^\circ) = (M_1(\underline{\Theta}^\circ), \ldots M_n(\underline{\Theta}^\circ))$ by

$$M_k(\Theta_1^\circ,\ldots\Theta_n^\circ;\, h,h_1,\ldots,h_{n-1}) := \int_{-\infty}^{\infty}\{I_k,H^1\}dt,\quad k = 1,\ldots,n-1$$

$$M_n(\Theta_1^\circ,\ldots,\Theta_n^\circ;\, h,h_1,\ldots,h_{n-1}) := \int_{-\infty}^{\infty}\{F,H^1\}dt$$

where the integrals are, at least, conditionally convergent [8],[2] and {} denotes Poisson brackets. We require that the multiply 2π-periodic Melinkov vector $M(\underline{\Theta}^\circ)$ has, at least, one transversal zero, i.e., a point $\acute{\Theta}^\circ = (\acute{\Theta}_1^\circ,\ldots\acute{\Theta}_n^\circ)$ such that $M(\acute{\Theta}^\circ) = \underline{0}$ and $\det[DM(\acute{\Theta}^\circ)] \neq \underline{0}$, where DM is the nxn Jacobian matrix of the vector M with respect to the initial phases, $\underline{\Theta}^\circ$.

Theorem (Holmes and Marsden [8])

If conditions (C1)-(C4) hold for the perturbed system (2.1), then, for μ sufficiently small, Arnold diffusion arises in this system.

Remarks: (1) An extension of this Theorem to include non-Hamiltonian systems of differential equations is given in [2]. Specifically, this extension allows the degree of freedom described by the energy function F above to be non-Hamiltonian.

(2) Intuitively, the phenomenon of Arnold diffusion can be thought of as a weak coupling ($\mu H'$) between 2-dimensional subsystems (degrees of freedom); where one subsystem (F) has a homoclinic orbit, the other subsystems (G_i) are nonlinear oscillators. Under the conditions (C2) and (C3) the oscillators survive the perturbation μ, and remain nonlinear oscillators with the same frequencies. These oscillators then act collectively as a periodic forcing to the subsystem (F).

§3. The application to the swing equations of a 4-machine power system.

Here we consider an interconnected power system with the configuration of a star network (fig. 1 with-out the dotted lines). We require that machine (or area) 4 to be relatively large, machine (or area) 3 to be relatively small; and machines 1 and 2 to be intermediate. We also choose appropriate parameters P_i and Y_{ij} and define μ to be a measure of the ratio between machine k (M_k, k = 1,2,3) and the large (reference) machine (M_1) [2]. The latter machine serves to produce the coupling between the first three machines. One obtains, after appropriate scaling of constants, and expansions in μ (see [2]), the following Hamiltonian which describes the dynamics of the interconnection of the star network.

$$H^{\mu} = \sum_{i=1}^{2} \frac{1}{2}\omega_i^2 - \rho_i\sigma_i - \beta_i[\cos(\sigma_i+\Delta_i^s) - \cos(\Delta_i^s)]$$
$$+ \alpha[\tfrac{1}{2}\omega_3^2 - \rho_3\sigma_3 - \beta_3[\cos(\sigma_3+\Delta_3^s) - \cos(\Delta_3^s)]]$$
$$+ \alpha\mu\tfrac{1}{2}[\omega_1 + \omega_2 + \alpha\omega_3]^2 + \alpha\mu[\rho_n(\sigma_1 + \sigma_2 + \alpha\sigma_3)]$$
$$+ \alpha\mu \sum_{i=1}^{2} \beta_i \sin(\sigma_i+\Delta_i^s)[\sigma_1+\sigma_2+\sigma_3]$$
$$+ \alpha^2\mu\beta_3\sin(\sigma_3 + \Delta_3^s)[\sigma_1+\sigma_2+\alpha\sigma_3] + O(\alpha^2\mu^2) \qquad (3.1)$$

where $\sigma_i(= \delta_i - \delta_i^s$, δ_i^s a constant) is the angle and ω_i is the velocity; $\rho_k, \beta_k, \Delta_k^s$ are constants; α is a small constant; and μ is the perturbation parameter. The first three terms, each, describes the dynamics of a pendulum with constant forcing; the other terms, which are functions of μ, represent the coupling function. (The phase portrait of a forced pendulum is shown in figure 2.) The first two terms are pertaining to the

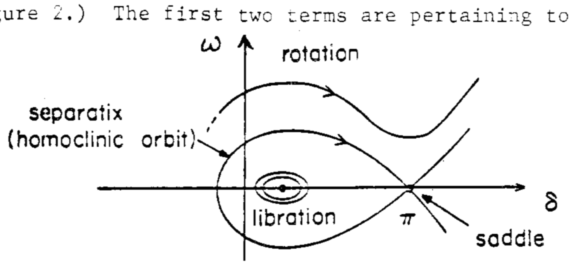

Fig.2. The phase portrait of a pendulum with constant forcing.

two degrees of freedom associated with the intermediate machines (1) and (2). The third is associated with the small machine, machine 3. It is scaled by the (fixed) small nonzero parameter α, which serves to guarantee that, for certain energy levels of the unperturbed system (μ = 0), subsystem 3 possesses a homoclinic orbit, while subsystems 1 and 2 act only as nonlinear oscillators.

Our Hamiltonian (3.1) describes a coupling of subsystems each of which is a Hamiltonian of a forced pendulum. From figure 2, and by the known properties of forced pendulums, conditions (C1)-(C3) of the Arnold diffusion theorem (section 2) can easily be satisfied. It only remains to satisfy condition (C4) on the Melnikov vector. After simplifications of the expressions utilizing equation (1.2), one obtains [2],

$$\tilde{M}_i(t_i) = \int_{\infty}^{\infty}\{-\frac{d}{dt}\ \dot{\omega}_i(\Omega_i(t-t_i)) \cdot \bar{\sigma}_3(t)$$
$$+ \dot{\omega}_i(\Omega_i(t-t_i)) \cdot \bar{\omega}_3(t) - \dot{\omega}_3(t) \cdot \bar{\sigma}_i(\Omega_i(t-t_i))\}dt, \quad i = 1,2 \qquad (3.2.a)$$

$$\tilde{M}_3(t_1.t_2) = \sum_{k=1}^{2} \int_{-\infty}^{\infty} -\frac{d}{dt}\ \dot{\omega}_3(t) \cdot \bar{\sigma}_k(\Omega_k(t-t_k))$$
$$+ \dot{\omega}_3(t) \cdot \bar{\omega}_k(\Omega_k(t-t_k))$$
$$- \dot{\bar{\omega}}_k(t-t_k) \cdot \bar{\omega}_3(t) \quad dt \qquad (3.2.b)$$

where the overbar denotes the variables along a homoclinic orbit before perturbation. We stress that these improper integrals must be <u>shown to exist</u> and condition (C4) must be <u>verified analytically</u>, in order to prove the existence of Arnold diffusion. When $\bar{\omega}_i$ and $\bar{\sigma}_i$ are merely t-periodic, such an analytic verification is not apparent. (We note though that Fourier expansions may be utilized to perform the evaluation computationally, see [2].)

In the case when machines 1 and 2 undergo small oscillations; analytic proofs of the Melinkov integrals, so as to possess transversal zeros can be established. In this case the variables of machines 1 and 2, i.e. $\bar{\omega}_i$ and $\bar{\sigma}_i$ become sinusoidal (plus small error terms). The Melinkov integrals are then evaluated to (we relegate the technical details to [2]).

$$\tilde{M}_i(t_1,t_2) = a_{i1} \cos \Omega_i t_i + b_{i1} \sin \Omega_i t_i \quad i=1,2 \quad (3.3.a)$$

$$\tilde{M}_3(t_1,t_2) = \sum_{k=1}^{2} a_{3k} \cos(\Omega_k t_k) + b_{3k} \sin(\Omega_k t_k) \quad (3.3.b)$$

where a_{i1}, b_{i1}, a_{3k} and b_{3k} are nonzero constants; and Ω_1 and Ω_2 are commensurate frequencies. Let $\tilde{\tilde{M}}(t_1,t_2) = [\tilde{M}_1, \tilde{M}_3]$ (note that we dropped $\tilde{M}_2$ since we consider machine 2 to be acting as the forcing). Thus,

$$\det[D\tilde{\tilde{M}}] = \frac{\partial\tilde{M}_1}{\partial t_1} \cdot \frac{\partial\tilde{M}_3}{\partial t_2} \qquad (3.4)$$

Equation (3.3.a) has transversal zeros t_1^*, i.e. $\tilde{M}_1(t_1^*, t_2) = 0$ and $\partial\tilde{M}_1/\partial t_1\ (t_1^*) \neq 0$, such that $\sin(\Omega_1 t_1^*) = \frac{-a_{11}}{[a_{11}^2 + b_{11}^2]^{1/2}}$ and $\cos(\Omega_1 t_1^*) = \frac{b_{11}}{[a_{11}^2 + b_{11}^2]^{1/2}}$. Plug one such zeros, t_1^*, into (3.3.b) and obtain a t_2^* such that: $\tilde{M}_3(t_1^*,t_2^*) = 0$ and $\partial\tilde{M}_3/\partial t_2 \neq 0$. Then the pair (t_1^*,t_2^*) constitute a transversal zero for the vector $\tilde{\tilde{M}}(t_1,t_2)$ as seen from (3.4). Carrying out these steps, we set $M_3(t_1^*,t_2^*)$ to zero, i.e.,

$$a_{32} \cos \Omega_2 t_2^* + b_{32} \sin \Omega_2 t_2^* + \frac{a_{31}\ b_{11} - b_{31}\ a_{11}}{[a_{11}^2+b_{11}^2]^{1/2}} = 0$$

or

$$[a_{32}\ b_{32}] \begin{pmatrix} \cos \Omega_2 t_2^* \\ \sin \Omega_2 t_2^* \end{pmatrix} = +\frac{a_{11}b_{31} - a_{31}b_{11}}{[a_{11}^2 + b_{11}^2]^{1/2}} \triangleq C \qquad (3.5)$$

and require that

$$\frac{\partial \tilde{\tilde{M}}_3}{\partial t_2} := \Omega_2 \,[b_{32} \;\; - \;\; a_{32}] \begin{bmatrix} \cos \Omega_2 t_2^* \\ \sin \Omega_2 t_2^* \end{bmatrix} \neq 0 \qquad (3.6)$$

Conditions (3.5) and (3.6) have simple geometric interpretation as can be seen in figure 3. Let

$$A(t) = \begin{bmatrix} a(t) \\ b(t) \end{bmatrix} := \begin{bmatrix} \cos \Omega_2 t \\ \sin \Omega_2 t \end{bmatrix}.$$ Then condition (3.6)

means that A(t) does not coincide with the point z on the circle in figure 3. Equation (3.5) requires that the projection of the vector $(a_{32}, b_{32})^t$ on the vector A(t), i.e., the point p, to have a length equal to the constant $|C|$. It is easy to see, geometrically, that there exist two values of t_2^* (in each period) that satisfy equation (3.5) provided $|C| \leq |(a_{3,2}, b_{3,1})^t|$. This latter condition is readily satisfied, and therefore Arnold diffusion arises on certain energy levels. (These levels are given explicitly in [2] as $H^{M} = h$, for all $h > \bar{h}$, but 'near' $\bar{h}$, see [8].)

As a closing remark we note that our result on the presence of Arnold diffusion extends to the configuration of figure 1, which adds weak interconnections between the subsystems, as shown by the dotted lines.

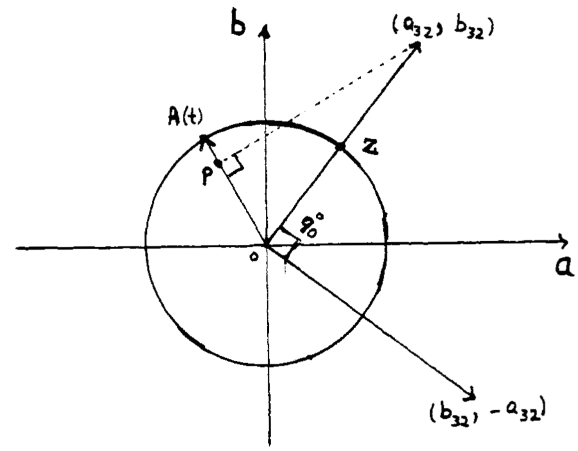

Fig.3. A Geometric View of conditions (3.5) + (3.6)

References

[1] F.M.A. Salam, J.E. Marsden, and P.P. Varaiya, "Chaos and Arnold Diffusion in Dynamical Systems", IEEE Trans. on Circuit and Systems, September 1983.

[2] ______, "Arnold Diffuions in the Swing Equations of a Power System", Memo. UCB/ERL 83/13, Electronics Research Lab., Univ. of California, Berkeley, CA.

[3] P.M. Anderson and A.A. Fouad, Power System Control and Stability, Vol. 1, Ames 10, 10: The Iowa State University Press, 1977.

[4] A. Arapostathis, S. Sastry, P. Varaiya, "Global Analysis of Swing Dynamics", IEEE Trans. on Circuits and Systems, Vol. CAS-29, No. 10, October 1982, pp.673-679.

[5] V.I. Arnold, "Instability of Dynamical Systems with Several Degrees of Freedom", Dokl. Akad. Nauk SSSR 156:9-12, 1964.

[6] T. Athay, R. Podmore and S. Virmani, "A Practical Method for the Direct Analysis of Transient Stability", IEEE Trans. on Power Apparatus and Systems, vol. PAS-98 (1979), pp. 573-584.

[7] A. A. Fouad and S.E. Stanton, "Transient Stability of a Multimachine Power System, Part I: Investigation of System Trajectories: Part II: Critical Transient Energy", IEEE Trans. on Power Apparatus and Systems, vol. PAS-100 (1981) pp. 3408-3424.

[8] P.J. Holmes and J.E. Marsden, "Melnikov's Method and Arnold Diffusion for Perturbations of Integrable Hamiltonian Systems", Journal of Mathematical Physics, Vol. 23 no. 4, 1982, pp. 669-679.

[9] N. Kakimoto, Y. Ohsawa, M. Hayashi, "Transient Stability Analysis of Electric Power Systems via Lure Type Lyapunov Function" "Part I. New Critical Value for Transient Stability", pp. 62-71. "Part II. Modification of Lure Type Lyapunov Function with Effect of Transfer Conductances", pp. 72-79. Trans. IEE of Japan, Vol. 98, No. 516, May/June 1978.

[10] N. Kopell and R.B. Washburn, Jr., "Chaotic Motions in the Two Degree-of-Freedom Swing Equations", IEEE Transactions on Circuits and Systems, November 1982, pp. 738-746.

[11] M.A. Pai, "Some Mathematical Aspects of Power System Stability by Lyapunov's Method", International Conference on "Electric Power Problems: The Mathematical Challenge" sponsored by SIAM, Seattle, Washington, March 18-20, 1980.