Optimal Control of Nonlinear Systems with Temporal Logic Specifications

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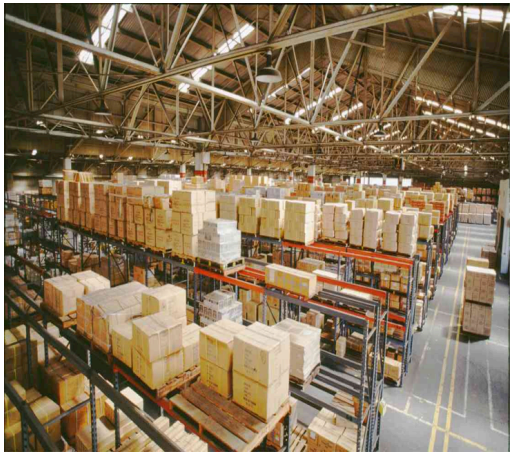
Autonomous Systems Doing Complex Tasks

Caltech



US Navy

- How to specify **complex tasks**?
- How to handle **high-dimensional**, **nonlinear** dynamics?
- How to compute time or fuel **optimal controllers**?

Temporal Logic

- Propositional logic: $\wedge$ (and), $\vee$ (or), $\Longrightarrow$ (implies), $\neg$ (not)
- Temporal operators: $\mathcal{U}$ (until), $\bigcirc$ (next), $\Box$ (always), $\Diamond$ (eventually)

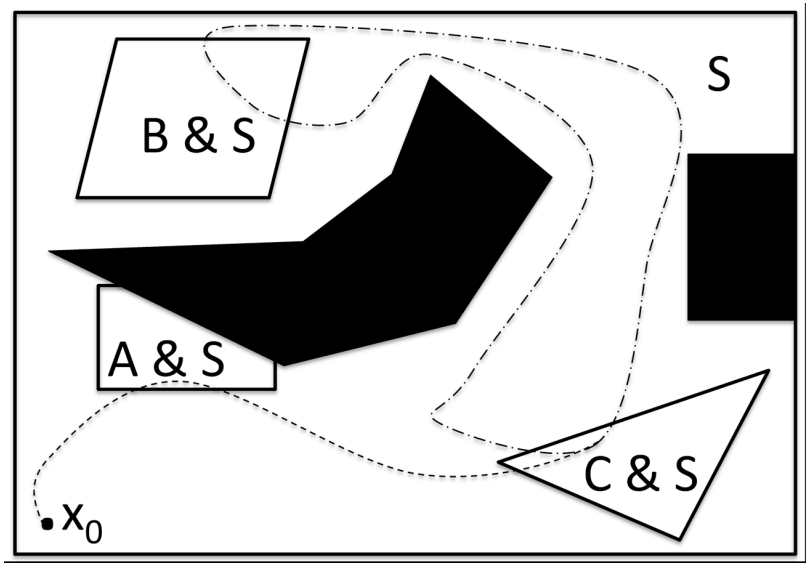
Autonomous driving

Bomb disposal

Generalizes classical motion planning

Problem Statement

- **Given**
  - nonlinear system
  - temporal logic spec **φ**
  - cost function **J**



- $\mathbf{x}(x_0,\mathbf{u})$ is the trajectory
- $L(\mathbf{x}(x_0,\mathbf{u}))$ is sequence of labels

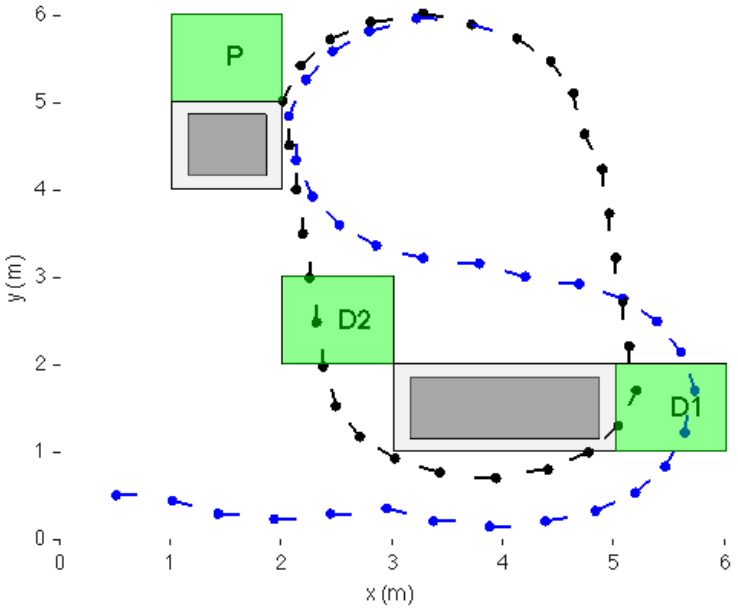
- **Goal**

$$\min_{\mathbf{u}} J(\mathbf{x}(x_0,\mathbf{u}))$$ **(minimize cost)**

$$\text{s.t. } L(\mathbf{x}(x_0,\mathbf{u})) \models \varphi$$ **(satisfy spec/task)**

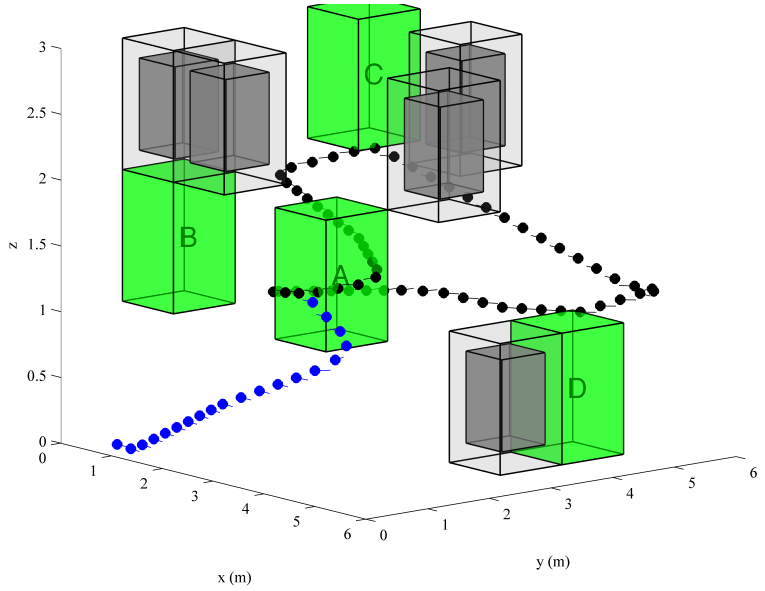
Solution Overview

1. Parameterize trajectory as a **lasso**
2. Encode **temporal logic** with mixed- integer linear constraints on lasso
3. Add **dynamic** constraints
4. Solve resulting MILP

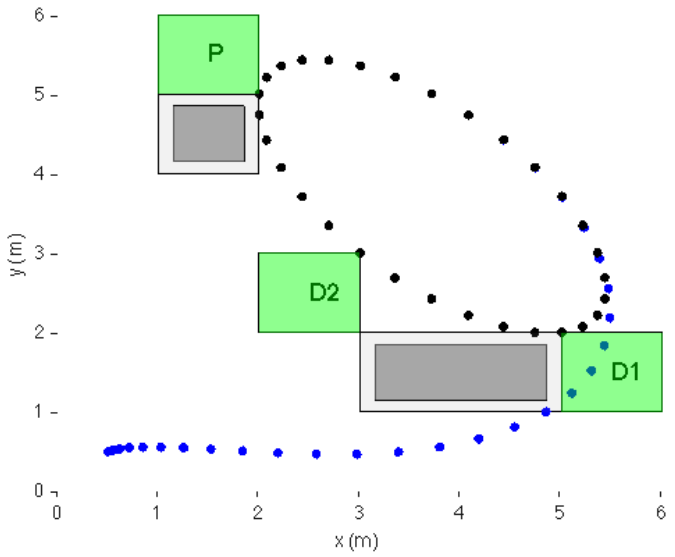


Computational Results

- Order of magnitude improvement over abstractions
- Solutions for high-dimensional systems in seconds



Aircraft (5 dim)



Quadrotor

Task: Repeatedly visit 3 regions and avoid obstacles

Thank you!

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