Automaton-guided Controller Synthesis for Nonlinear Systems with Temporal Logic

Eric M. Wolff[1]
Ufuk Topcu[2] and Richard M. Murray[1]
[1]Caltech and [2]UPenn

IROS
6 November 2013

Modern Autonomous Systems

Caltech

NASA/JPL

http://www.andrewalliance.com/

US Navy

- How to specify **complex tasks**?
- How to handle **high-dimensional** and **nonlinear** dynamics?

Main Contributions

- Trajectory generation for **high-dimensional (10+ dim)** and **nonlinear** systems with **complex tasks**

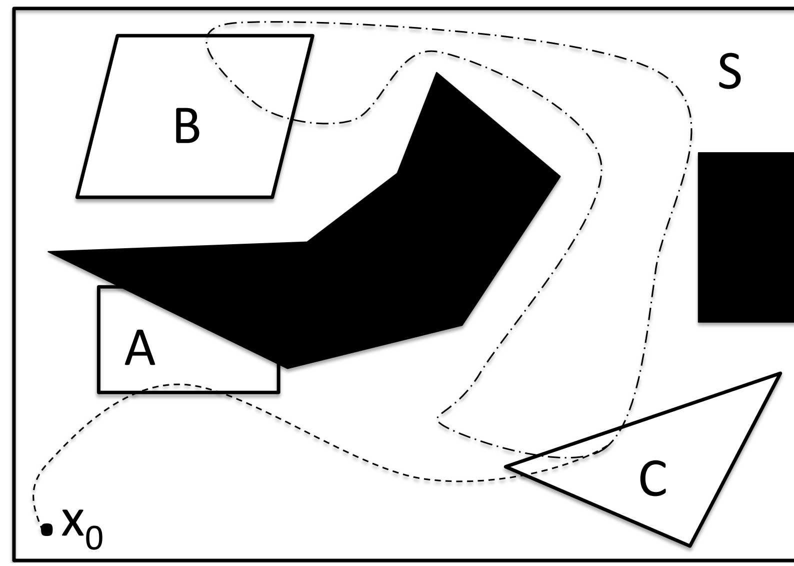
- Solves problems that discrete abstraction techniques cannot

Nonlinear System Model

- Discrete-time nonlinear system

$$x_{t+1} = f(x_t, u)$$

$$x \in X \subseteq \mathbb{R}^n, \quad u \in U \subseteq \mathbb{R}^m$$



- **Labels**: $L : X \to 2^{AP}$
- **Trajectory**:

$$\mathbf{x} = x(x_0, u) = x_0 x_1 x_2 \ldots$$

$$x_{t+1} = f(x_t, u) \text{ for some } u \in U \text{ for } t = 0,1,\ldots$$

- **Word**: $L(\mathbf{x}) = L(x_0) L(x_1) L(x_2) \ldots$

Temporal Logic

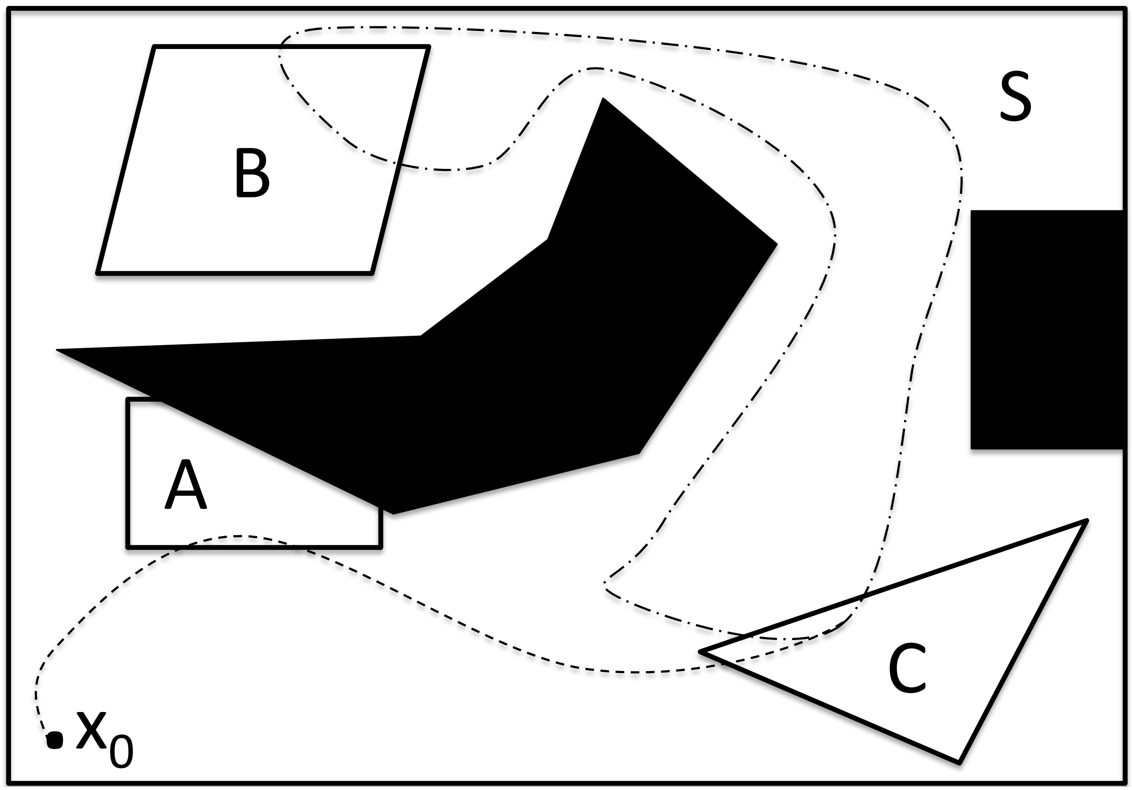
- A logic for reasoning about how properties change over time
- Reason about infinite sequences $\sigma = s_0 s_1 s_2 \ldots$ of states
- Propositional logic: $\wedge$ (and), $\vee$ (or), $\implies$ (implies), $\neg$ (not)
- Temporal operators: $\mathcal{U}$ (until), $\bigcirc$ (next), $\Box$ (always), $\Diamond$ (eventually)

Dangerous liquid handling

Bomb disposal

Generalizes point-to-point motion planning

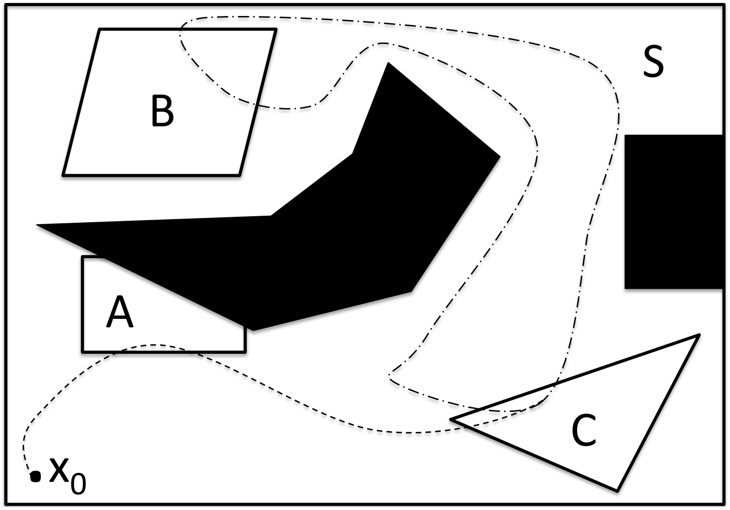
Problem Overview



Spec: Avoid obstacles, pick-up supplies at region A and then do surveillance on regions B and C.

Problem Statement

- **Given**:
  - a discrete-time nonlinear system,
  - an initial state x_0,
  - a temporal logic task φ



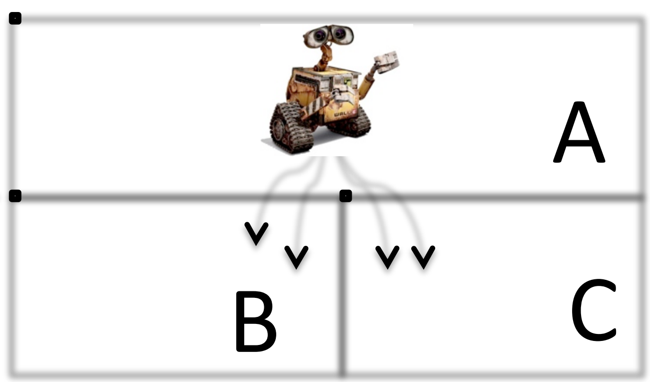
- **Goal**: Find a control input sequence $\mathbf{u}$ such that $L(\mathbf{x}(x_0,\mathbf{u})) \models \varphi$.

Related Work

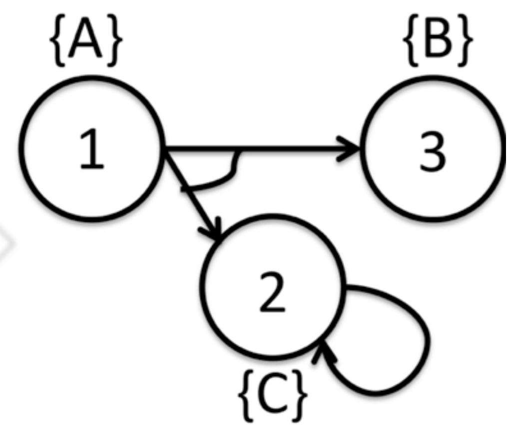
- Discrete abstractions (Alur00, Belta06, Habets06,**Gol12**, Kloetzer08, Pappas06, Tabuada06, Wongpiromsarn10,**Yordanov13**)

Low dimensional systems (<= 6)

Dynamical system



Discrete abstraction



Finite system

Related Work

- Discrete abstractions (Alur00, Belta06, Habets06,**Gol12**, Kloetzer08, Pappas06, Tabuada06, Wongpiromsarn10,**Yordanov13**)

Low dimensional systems (<= 6)

- Counter-example guided abstraction refinement (Alur03,Clarke00,Stursberg05)

Limited task search

- Hierarchical LTL motion planning (Bhatia11,Plaku10)

Limited specs & only sampling-based

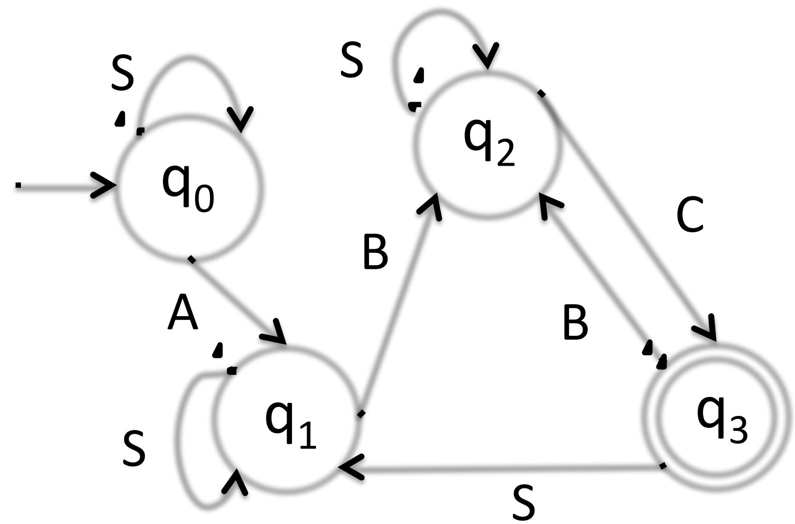
From Logic to Automaton

Informal Task

Avoid obstacles, pick-up supplies at **region A** and then do surveillance on **regions B** and **C**.

Automatic translation from **logic** to **automaton**!

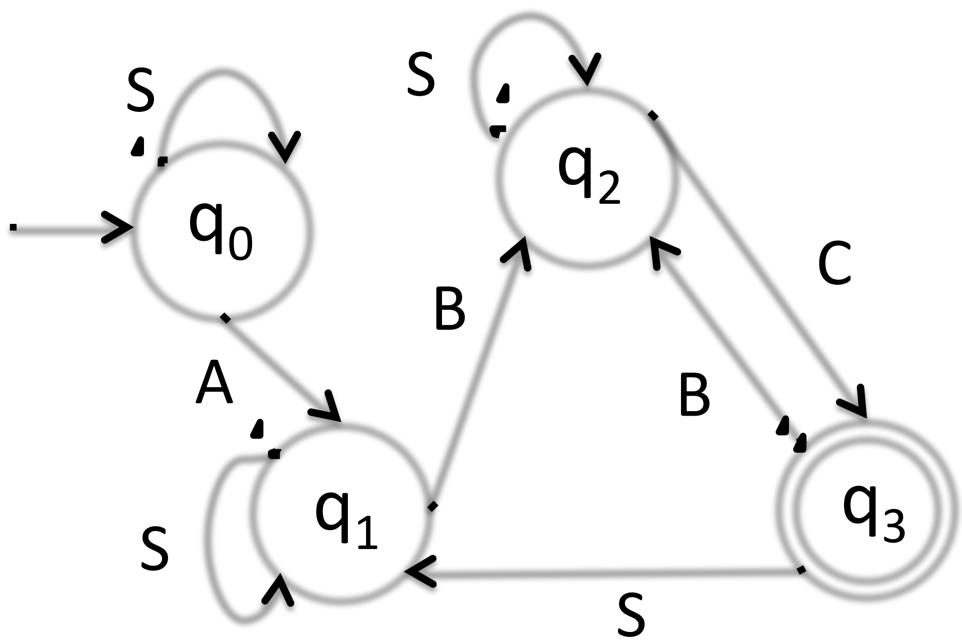
Logic Automaton



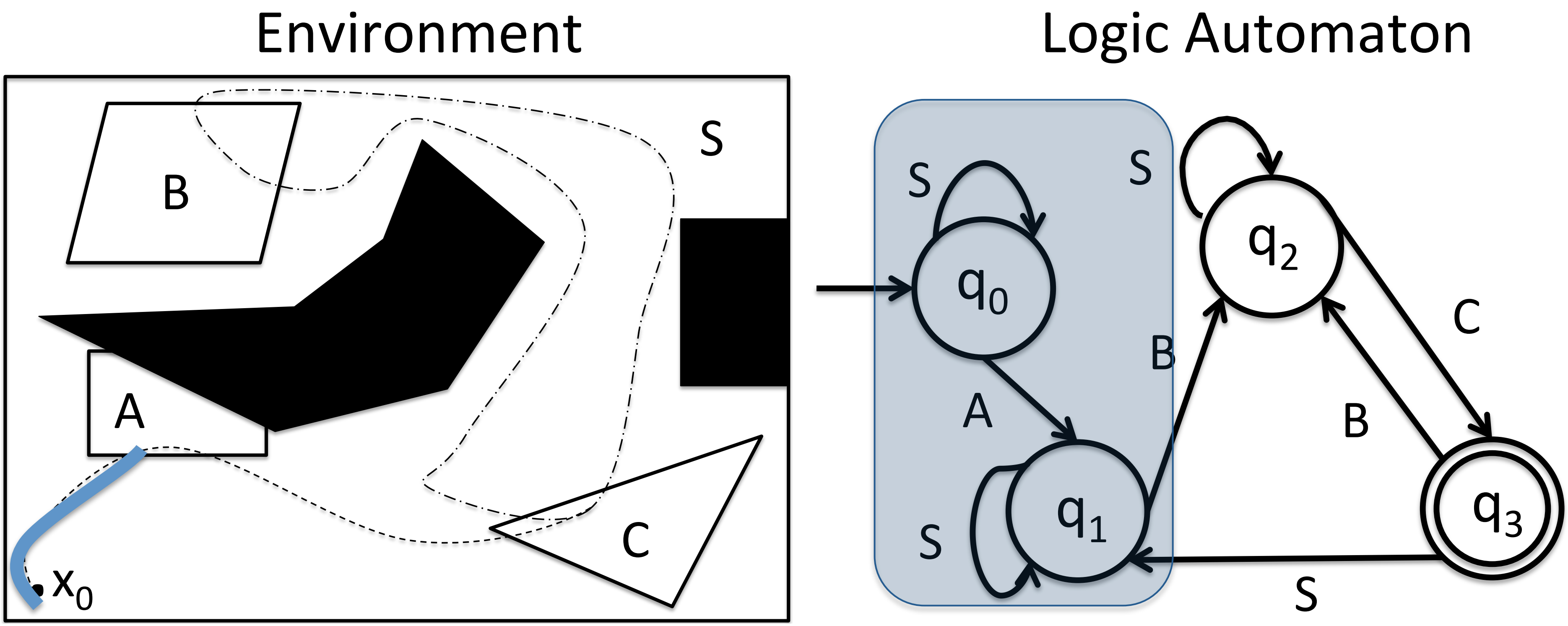
Gastin,Oddoux: http://www.lsv.ens-cachan.fr/~gastin/ltl2ba/

Automaton-guided Solution

- **Main idea**: Logic automaton guides a series of constrained reachability computations

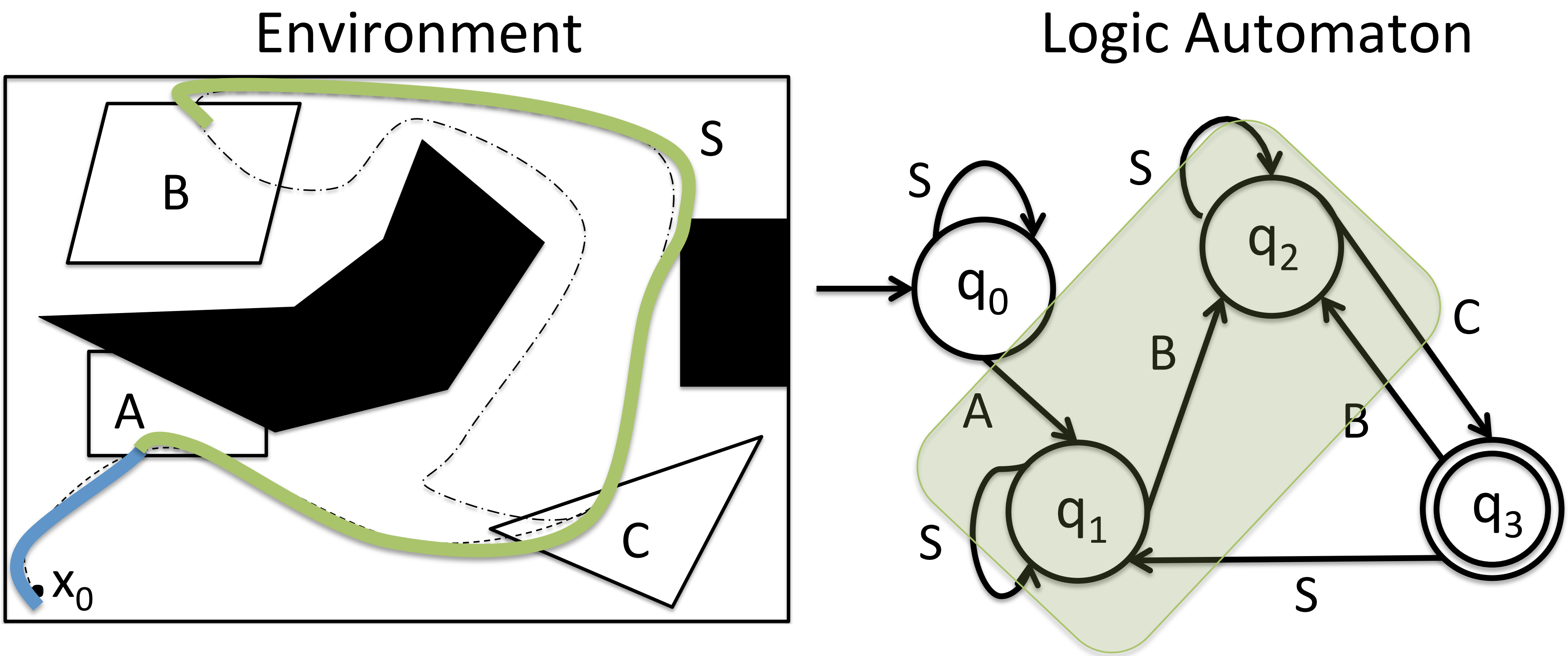


Automaton-guided Solution



- Automaton path: $q_0(q_1q_2q_3)^\omega$
- Reachability: (S,A) [(S,B)(S,C)(∅,S)]$^\omega$

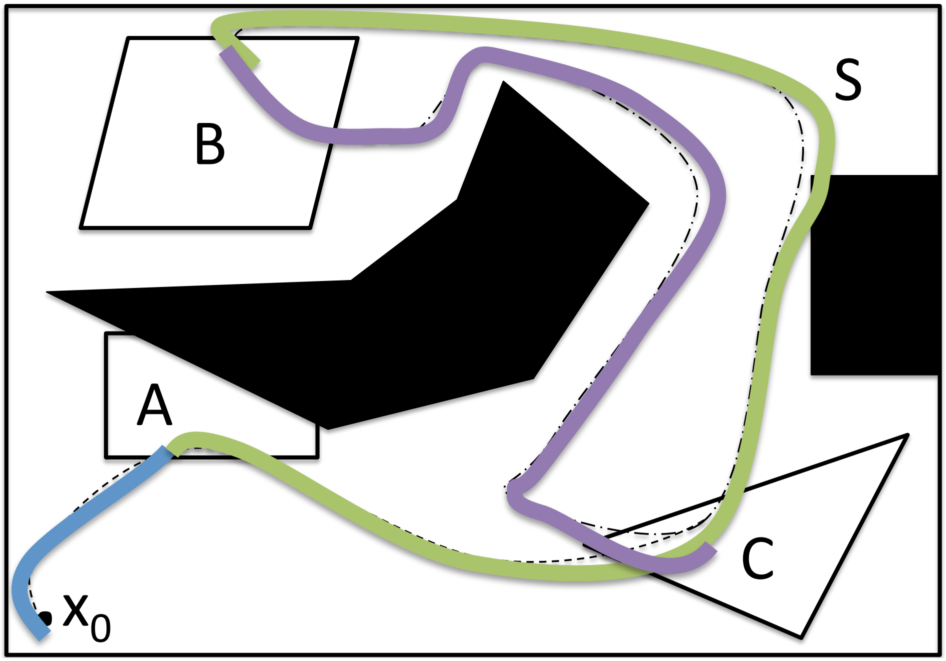
Automaton-guided Solution



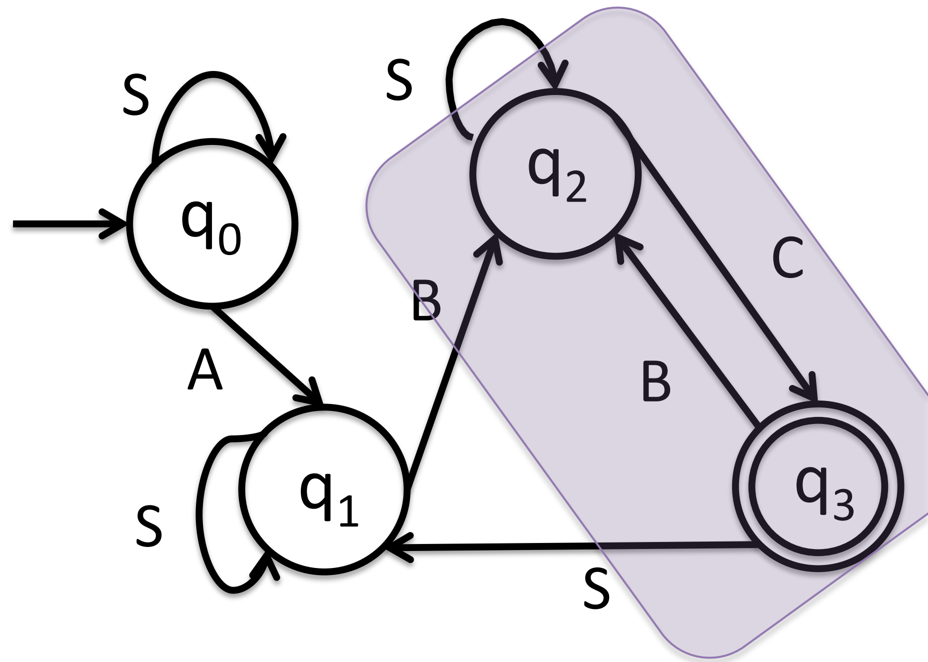
- Automaton path: $q_0(q_1q_2q_3)^\omega$
- Reachability: (S,A) [(S,B)(S,C)(∅,S)]$^\omega$

Automaton-guided Solution

Environment

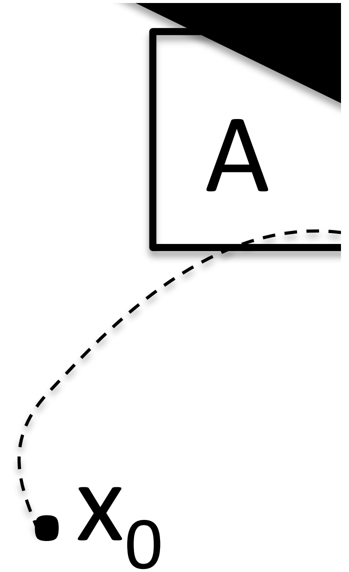


Logic Automaton



- Automaton path: $q_0(q_1q_2q_3)^\omega$
- Reachability: (S,A) [(S,B)(S,C)(∅,S)]$^\omega$

Constrained Reachability



- **Given**: safe set $X_1 \subseteq X$, reach set $X_2 \subseteq X$
- **Goal**: Find a control input u and a horizon length N such that $x_1, \ldots, x_{N-1} \in X_1, x_N \in X_2$, and $x_{t+1} = f(x_t, u)$ for $t = 1, \ldots, N-1$
- CstReach(**X_1,X_2**)
  - robotic motion planning (LaValle)
  - optimization-based methods (Betts,Milam)
  - PDE-based methods (Mitchell)

Solving Constrained Reachability

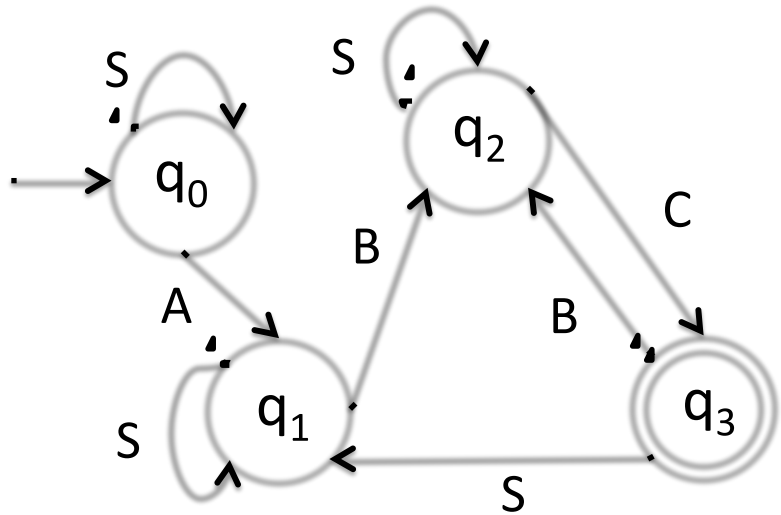
- CstReach($\mathbf{X_1}$,$\mathbf{X_2}$) can be encoded as a mixed-integer linear program
- Enforce that state is in $\mathbf{X_1}$ until $\mathbf{X_2}$ (big-M)

$$H_i x \leq K_i + M(1 - z_i), \quad z_i \in \{0,1\}, \quad \sum z_i = 1$$

- Dynamics are independent of state constraints

What if a Path is Infeasible?

- **Issue:** Lots of paths in automaton to check



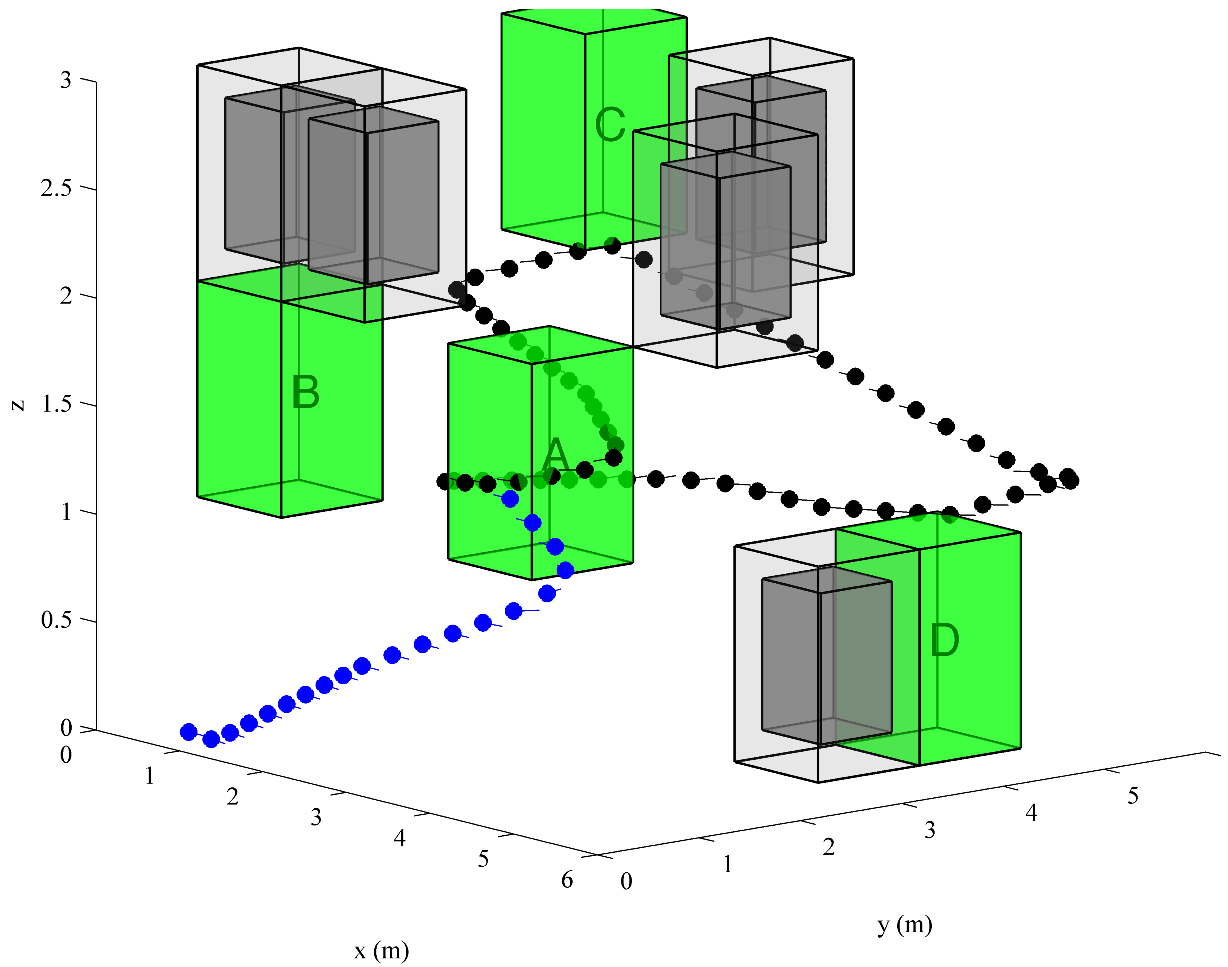
- **Fixes:**
  1) Parallel constrained reachability computations
  2) Update ordering on paths

Examples

- Systems
  - Quadrotor (10 dim)
  - Chained integrators (4, 12, 20 dim)
  - Car-like robot (nonlinear + drift)

- Specifications
  - Visit *n* goals
  - Repeatedly visit *n* goals

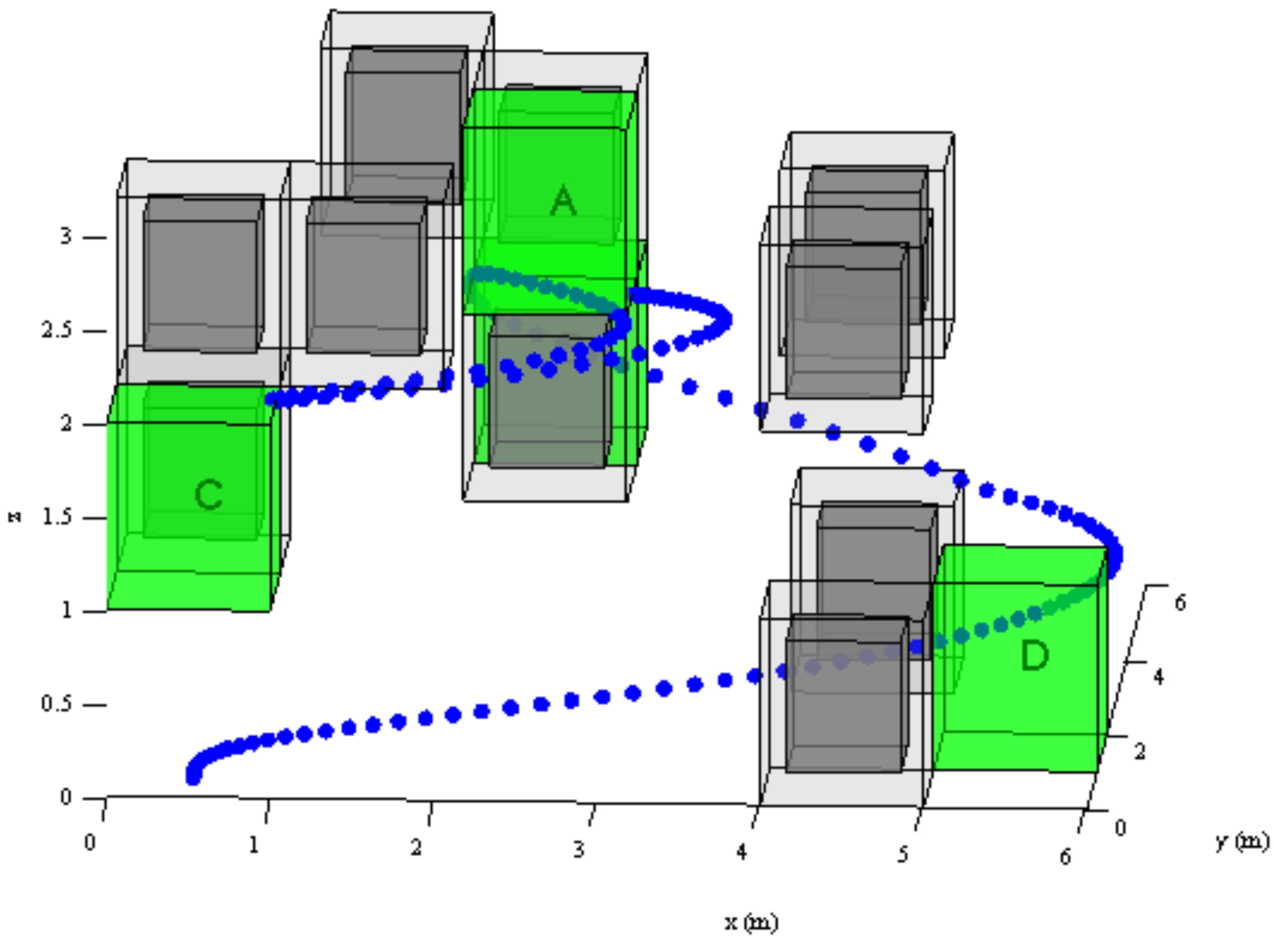
Examples



Model: car-like robot (5 dim)

Spec: Repeatedly visit 3 regions and avoid obstacles

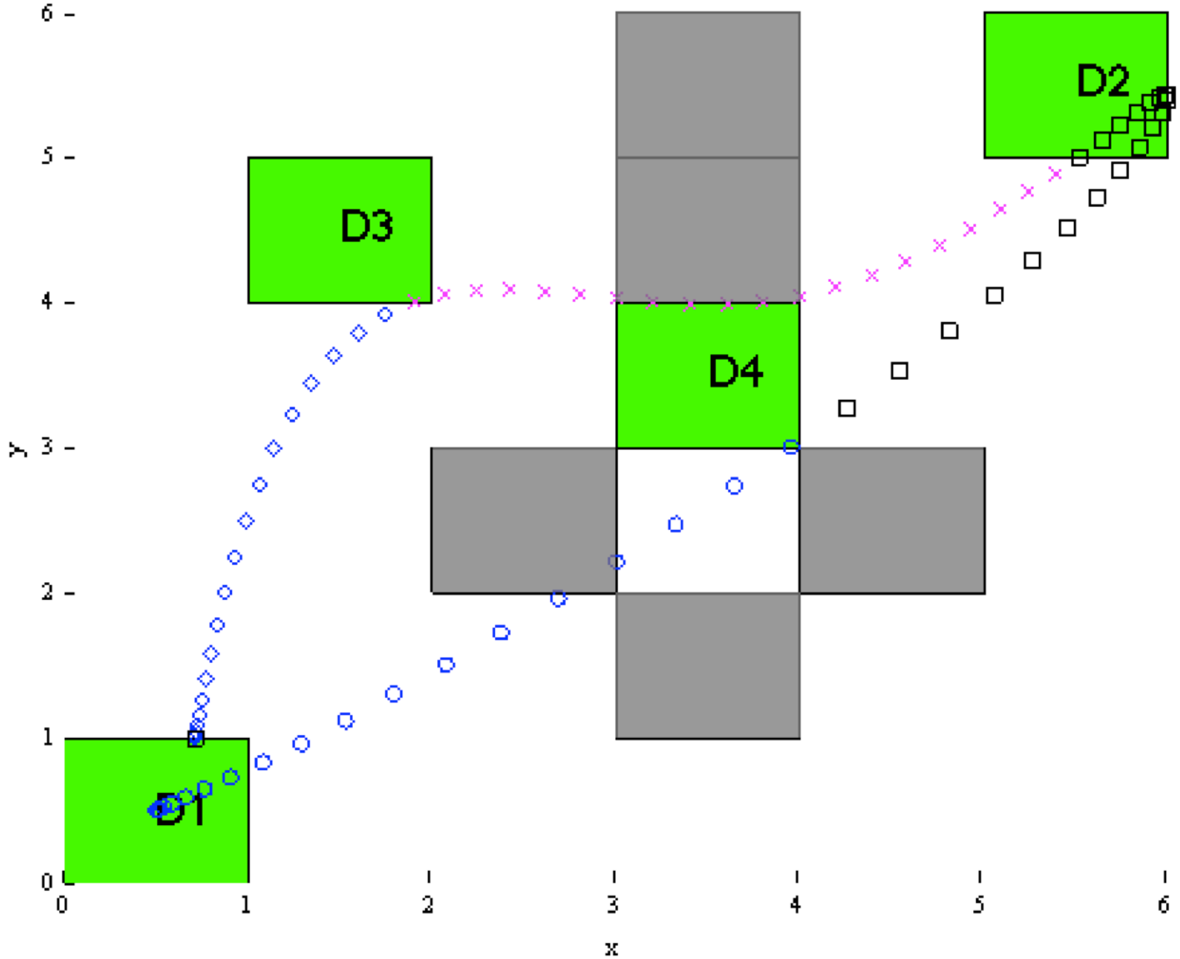
Examples



Model: quadrotor (10 dim)

Spec: Repeatedly visit 4 regions and avoid obstacles

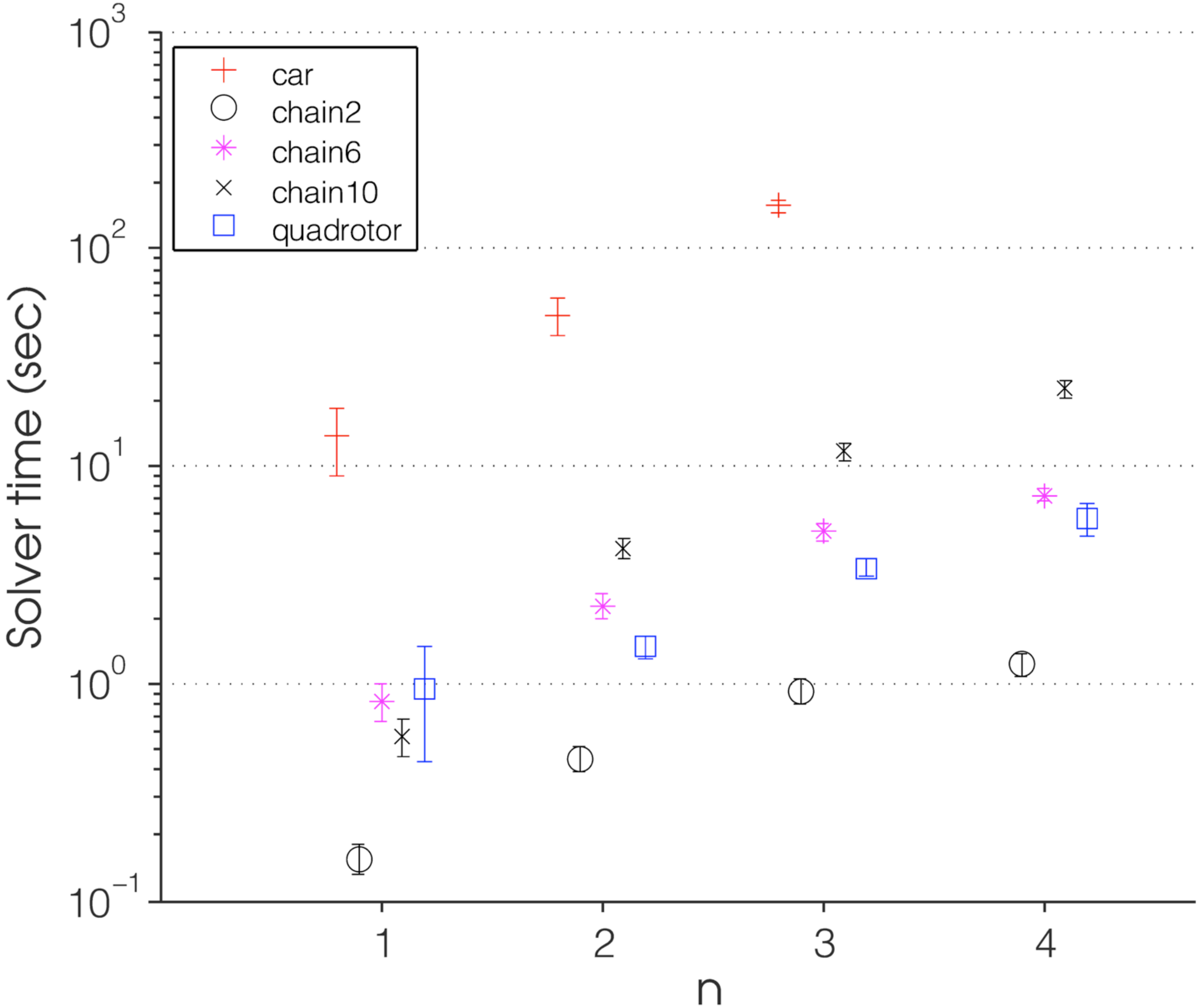
Examples



Model: quadrotor (10 dim)

Spec: Visit all 4 regions and avoid obstacles

Solver Time: Goals



Abstractions

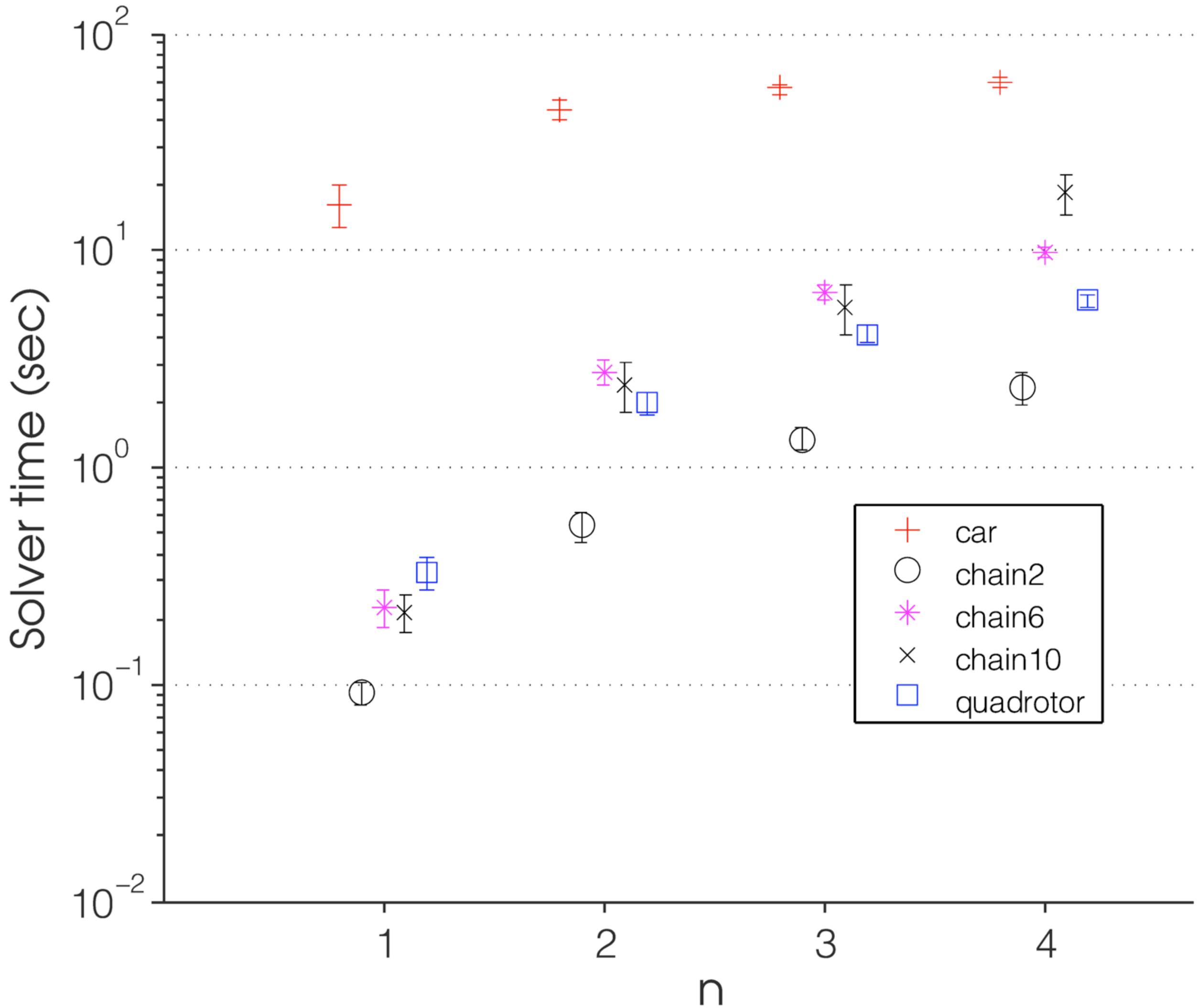
2-dim: seconds

4-dim: minutes

6-dim: hours

8-dim: ??

Solver Time: Surveillance



Abstractions

2-dim: seconds

4-dim: minutes

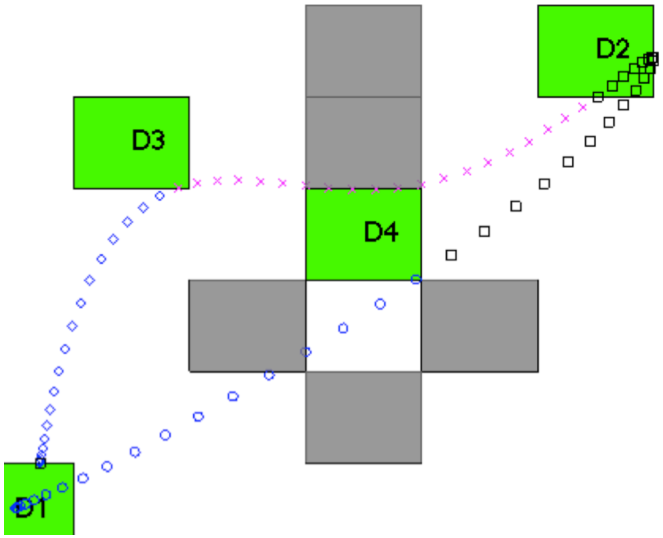
6-dim: hours

8-dim: ??

Conclusions

- Contributions
  - Temporal logic motion planning for **high dimensional** and **nonlinear** systems
  - Significant improvement on standard techniques

- Future work
  - Varying levels of abstraction
  - Improved discrete search
  - Multi-agent



Thank you!

- Contact: Eric M. Wolff
  - Email: ewolff@caltech.edu
  - Web: www.cds.caltech.edu/~ewolff/

- Funding: NDSEG fellowship, Boeing, AFOSR