Optimal Control of Non-deterministic Systems for a Fragment of Temporal Logic

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CDC

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Modern Autonomous Systems

Caltech

NASA/JPL

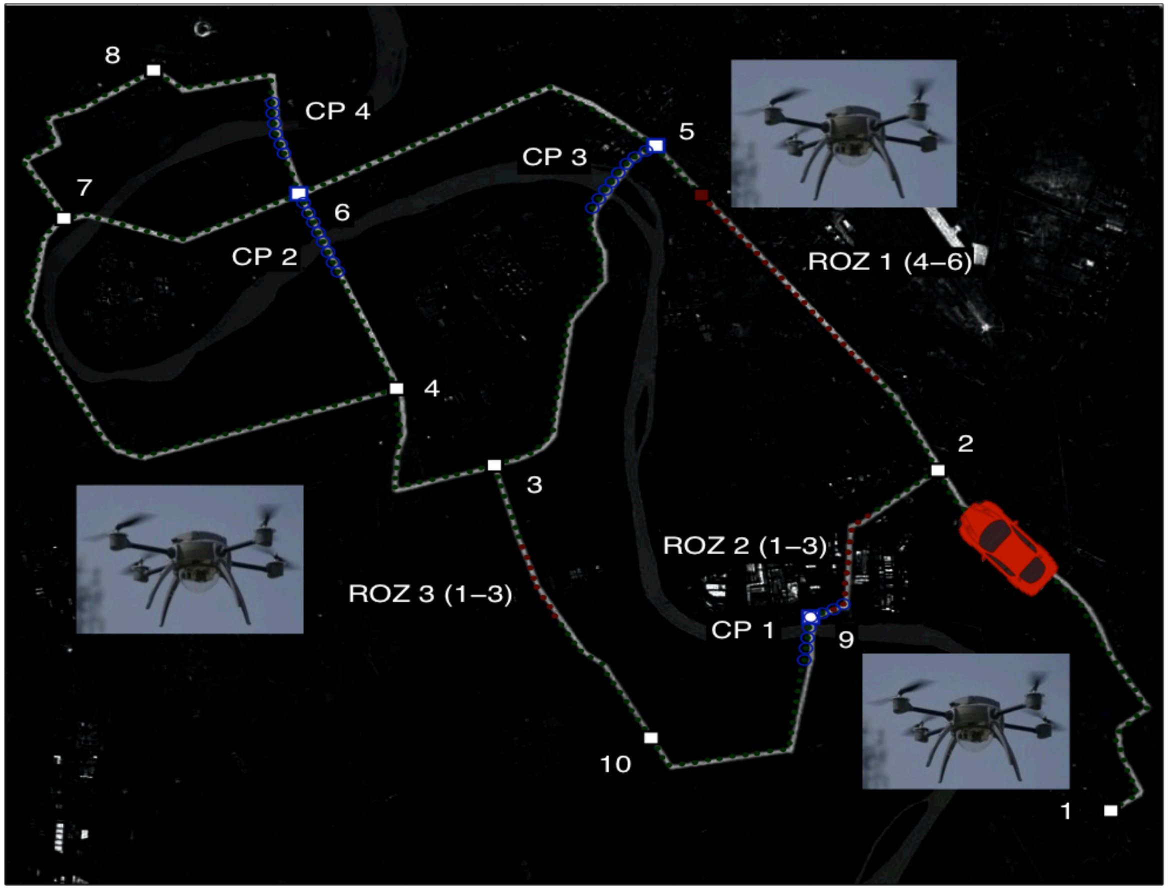
http://www.andrewalliance.com/

US Navy

- How to specify **complex tasks**?
- How to create **optimal** controllers?
- React to **adversarial** environment?

UAV Surveillance Tasks

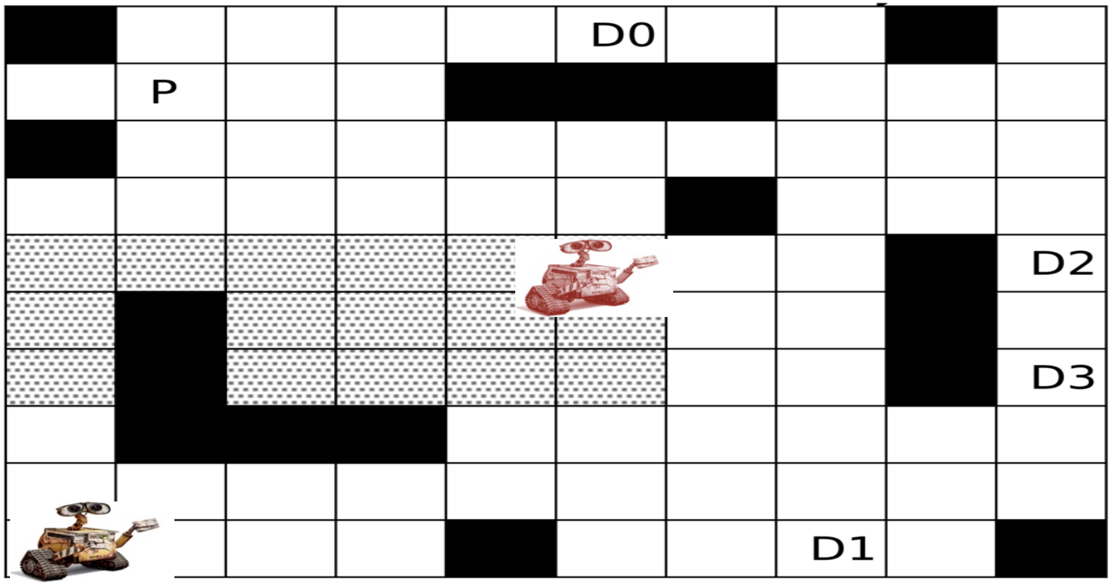
- Tracking a vehicle with a team of UAVs

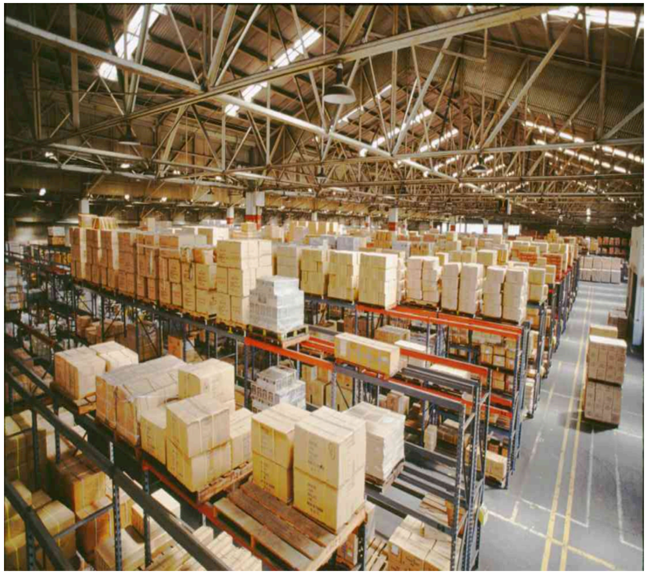


AFRL, www.aeryon.com

Planning in a Dynamic Environment

- Dynamic obstacles + complex tasks in a warehouse





- Q: How to compute an optimal control policy that guarantees a complex, logical task is completed?

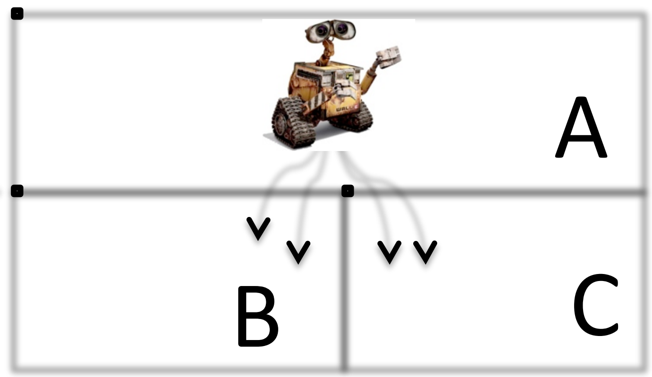
Our Contributions

- **Optimal** control for **non-deterministic** systems with **temporal logic** specifications
- **Polynomial** time controller synthesis
- **Anytime** optimization

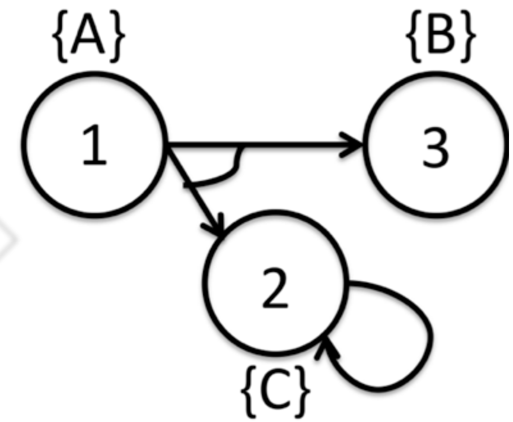
Cost function:	Average	Minimax	Task Cycle
Complexity:	POLY	POLY in task graph	EXP in task graph

Hierarchical Control Architecture

Dynamical system



Discrete abstraction[1]



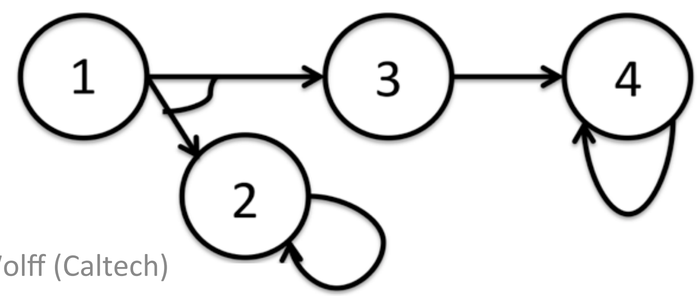
Non-deterministic transition system

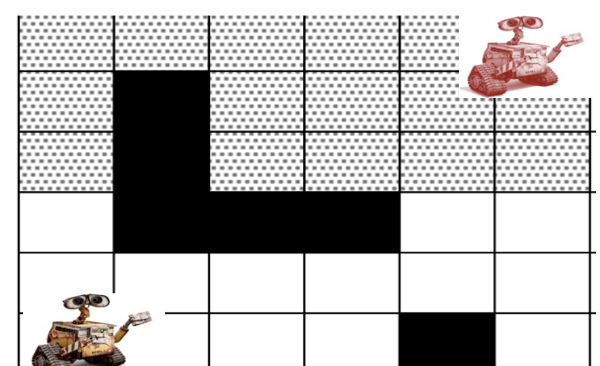
- We focus on the discrete planning layer
- Discrete plan is executed at continuous level

1. AlurHLP00, BeltaH06, HabetsCS06, KaramanF09, KloetzerB08, WongpiromsarnTM12, and more

Non-deterministic Transition Systems

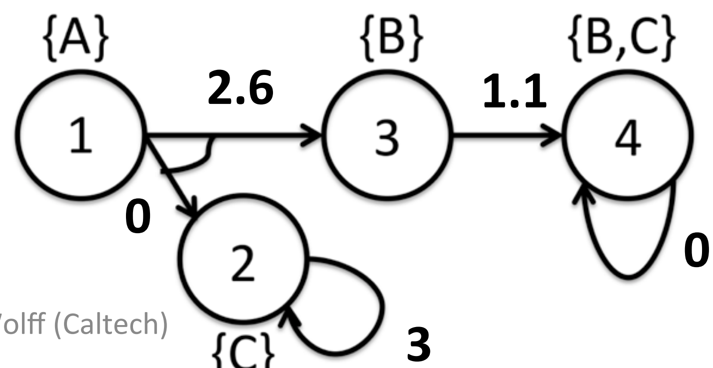
- A **non-deterministic transition system (NTS)** is a tuple T = (S, A, R, s_0, AP, L) with
  - **states** S,
  - **actions** A,
  - **transition** function R: S x A $\rightarrow$ 2^S,
  - initial state s_0,

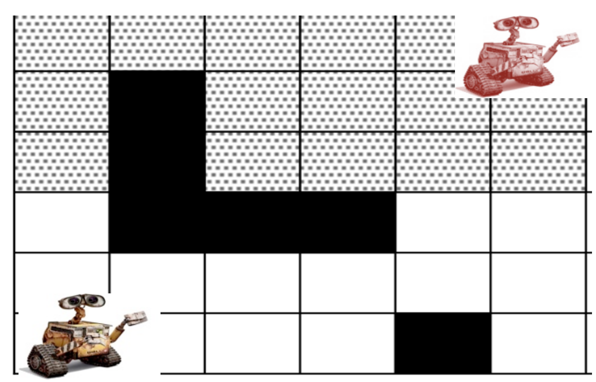




Non-deterministic Transition Systems

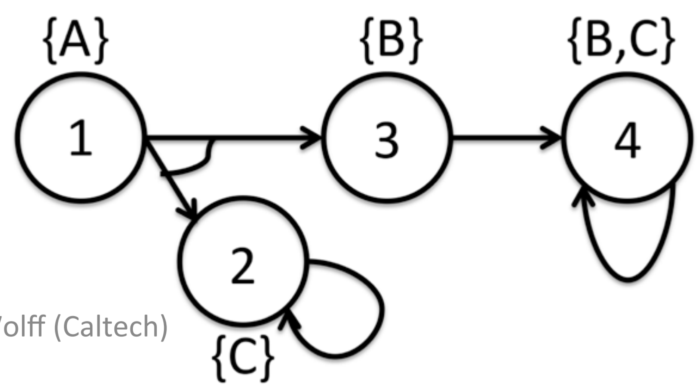
- A **non-deterministic transition system (NTS)** is a tuple $T = (S, A, R, s_0, AP, L)$ with
  - **states** S,
  - **actions** A,
  - **transition** function $R: S \times A \rightarrow 2^S$,
  - initial state s_0,
  - atomic propositions AP,
  - labeling function $L : S \rightarrow 2^{AP}$, and
  - non-negative valued cost function $c : S \times A \times S \rightarrow \mathfrak{R}$.

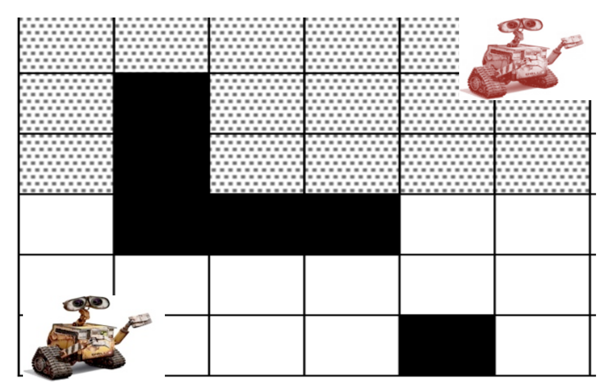




Control Policies

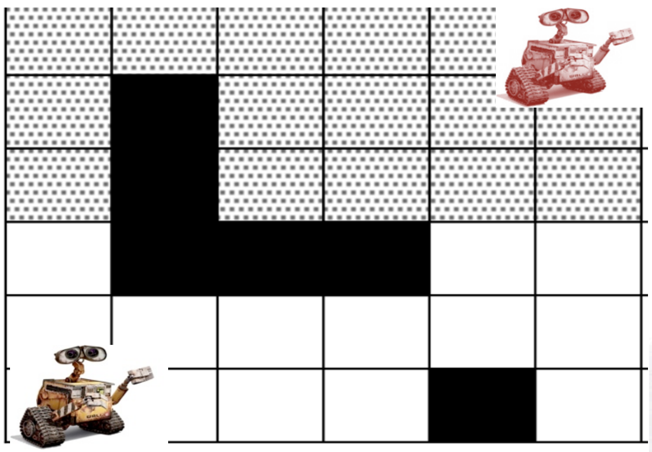
- **Finite-memory control policy:** $\mu: S \times M \rightarrow A \times M$
- **Two-player game**:
  - **System** picks action using control policy
  - **Environment** picks next state
- $T^{\mu}(s)$: set of executions from state s under policy μ





Temporal Logic

- A logic for reasoning about how properties change over time
- Reason about infinite sequences $\sigma = s_0 s_1 s_2 \ldots$ of states
- Propositional logic: $\wedge$ (and), $\vee$ (or), $\implies$ (implies), $\neg$ (not)
- Temporal operators: $\mathcal{U}$ (until), $\bigcirc$ (next), $\Box$ (always), $\Diamond$ (eventually)



Motion Planning

Dangerous liquid handling

Bomb disposal

Complex sequencing of actions

Temporal Logic

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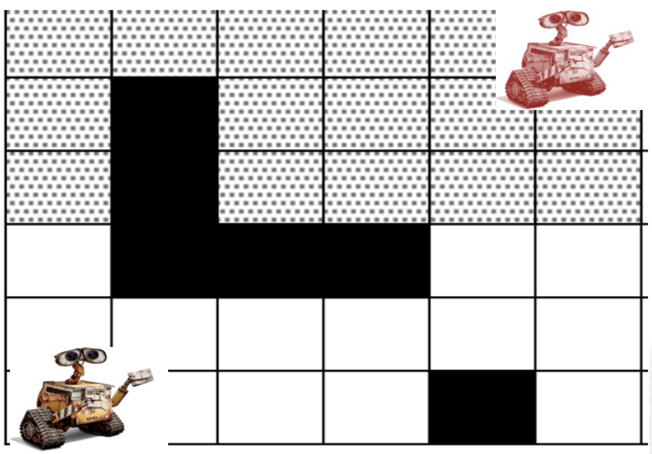
- Temporal operators: $\mathcal{U}$ (until), $\bigcirc$ (next), $\square$ (always), $\diamond$ (eventually)



Motion Planning

Dangerous liquid handling

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INTRACTABLE!

Specification Language

- We consider formulas of the form:

$$\varphi = \varphi_{\text{safe}} \;\wedge\; \varphi_{\text{resp}} \;\wedge\; \varphi_{\text{per}} \;\wedge\; \varphi_{\text{task}} \;\wedge\; \varphi_{\text{resp}}^{\text{ss}},$$

where

$$\varphi_{\text{safe}} := \square\psi_1,$$

Safety

$$\varphi_{\text{resp}} := \bigwedge_{j \in I_2} \square(\psi_{2,j} \implies \bigcirc\phi_{2,j}),$$

Response

$$\varphi_{\text{per}} := \Diamond\,\square\,\psi_3,$$

Stability

$$\varphi_{\text{task}} := \bigwedge_{j \in I_4} \square\,\Diamond\,\psi_{4,j},$$

Repeated tasks

$$\varphi_{\text{resp}}^{\text{ss}} := \bigwedge_{j \in I_5} \Diamond\,\square\,(\psi_{5,j} \implies \bigcirc\phi_{5,j}).$$

Steady-state response

Cost Functions

- Generic cost function **J**

$$J : T^{\mu}(s) \rightarrow \mathbb{R}_{\geq 0}$$

- We consider:
  - Average cost
  - Minimax (bottleneck) cost
  - Average cost-per-task-cycle

Problem Statement

- **Given**:
  - Non-deterministic transition system **T**
  - Temporal logic specification **φ** of the form

$$\varphi = \varphi_{\text{safe}} \wedge \varphi_{\text{resp}} \wedge \varphi_{\text{per}} \wedge \varphi_{\text{task}} \wedge \varphi_{\text{resp}}^{\text{ss}}$$

  - Cost function **J**

- **Problem**: Create control policy **μ** such that that the set of runs $\mathbf{T^{\mu}(s_0)}$ satisfies **φ** and minimizes **J**

$$\min_{\mu} J(T^{\mu}(s_0))$$
$$s.t. \quad T^{\mu}(s_0) \vDash \varphi$$

Related Work

- Automata-based approach [Vardi & Wolper]
  - Construct automaton from specification
  - EXP or 2-EXP in the specification

- Our approach
  - No automaton construction
  - Compute directly on the state space

Related Work

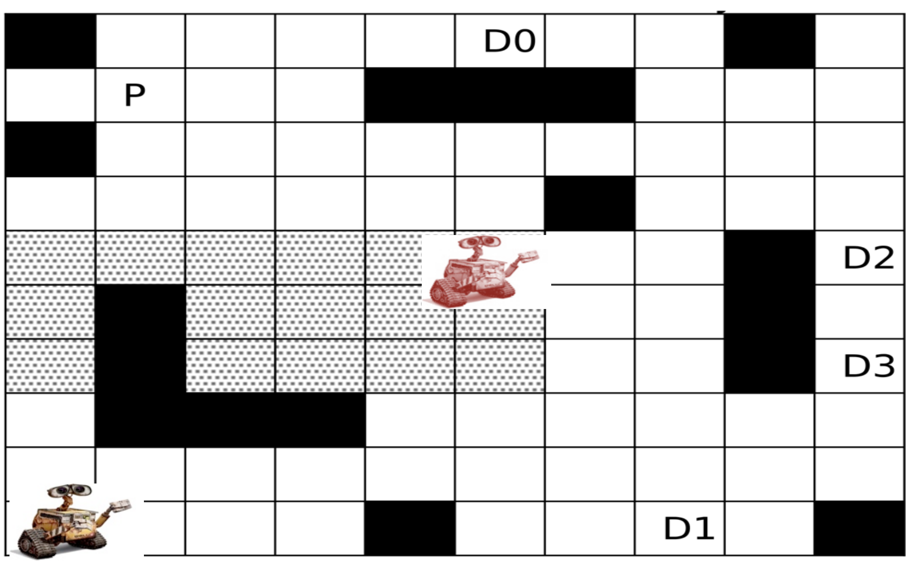
- Related logics:
  - GR(1): PitermanPS06, BloemJPPS12
  - GRabin(1): Ehlers11
  - AlurT04; MalerPS95
- Optimal control: JingEKG13

GR(1) system + stability

- How this work differs:
  - More system properties/tasks than GR(1)
  - Only bounded liveness assumptions on environment

Main Idea

- Optimization boils down to reasoning about worst-case costs between tasks
- Use **value function** and **task graph** for this



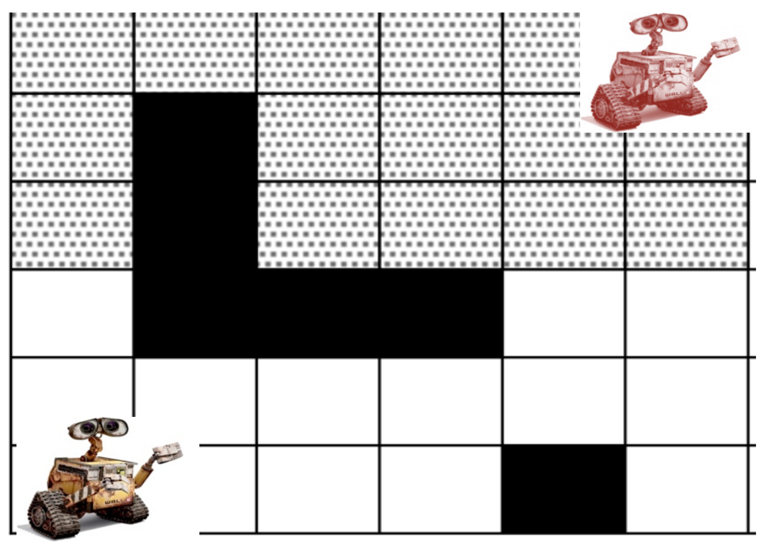
$$\varphi_{\text{task}} := \bigwedge_{j \in I_4} \square \diamond \psi_{4,j}$$

Tasks: P, D0, D1, D2, D3

Value Function and Reachability

- $\mathbf{V^c_B(s)}$: minimum cost to reach set B from state s under all resolutions of the non-determinism

$$V^c_{B,\mathcal{T}}(s) = \min_{a \in A(s)} \boxed{\max_{t \in R(s,a)}} V^c_{B,\mathcal{T}}(t) + c(s,a,t)$$



Value Function and Reachability

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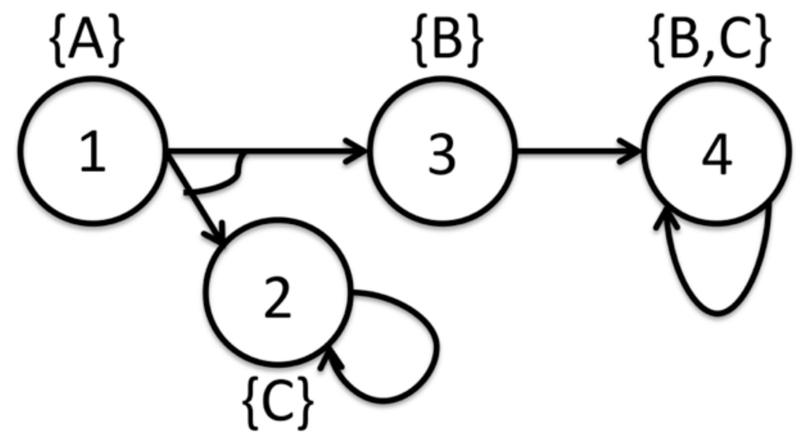
- Example
  - $V^c_4(1) = \infty$
  - $V^c_4(2) = \infty$
  - $V^c_4(3) = 1$
  - $V^c_4(4) = 0$

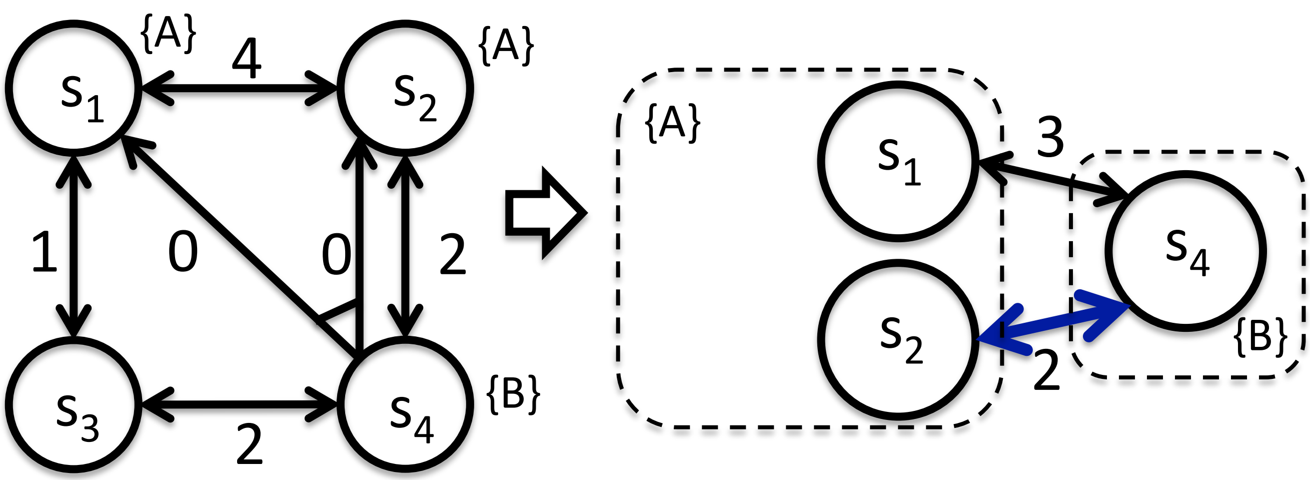


Task Graph

- Create new graph that encodes shortest paths between tasks

Tasks: A,B

Cost = 2

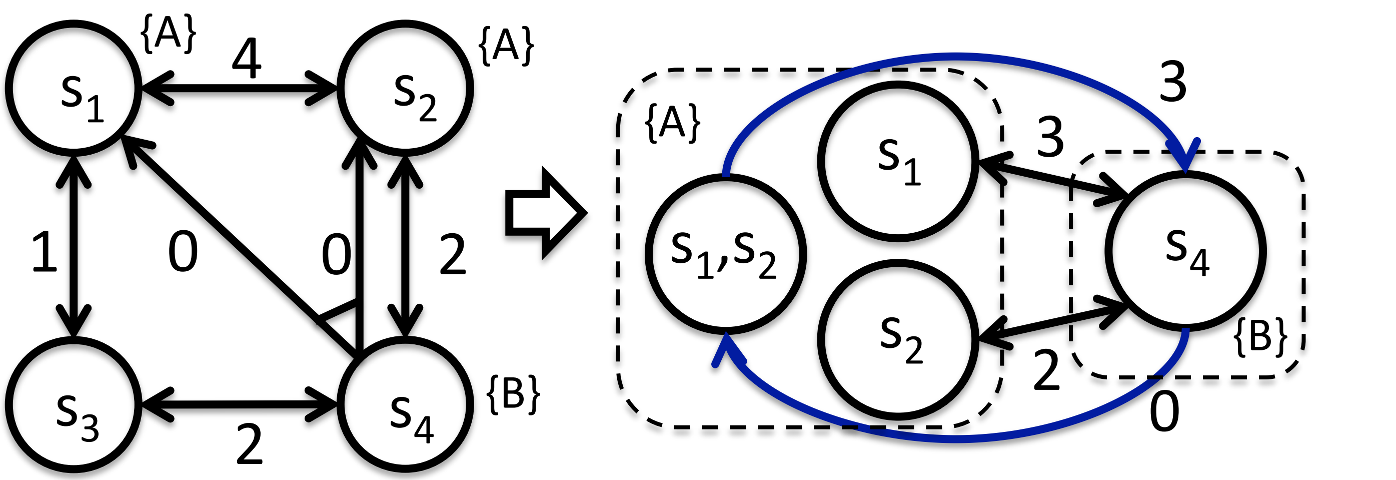


Task Graph

- Create new graph that encodes shortest paths between tasks

Tasks: A,B

Cost = 1.5

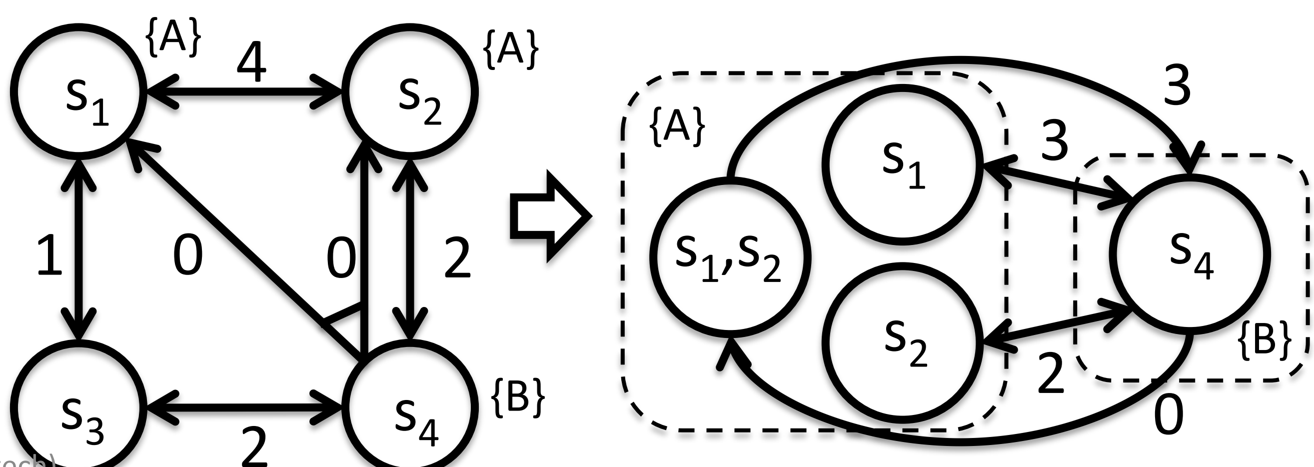


Including subsets in the task graph can reduce cost!

Task Graph

- Create new graph that encodes shortest paths between tasks
- Number of states
  - Deterministic: |F|
  - Non-deterministic: $\sum_{i \in I_4} 2^{|F_i|} - 1$

$$\varphi_{\text{task}} := \bigwedge_{j \in I_4} \square \diamond \psi_{4,j}$$



Average Cost Function

- Average cost of run σ is

$$J'_{\text{avg}}(\sigma, \mu(\sigma)) := \limsup_{n \to \infty} \frac{\sum_{t=0}^{n} c(\sigma_t, \mu(\sigma_t), \sigma_{t+1})}{n}$$

- The average cost function is

$$J_{\text{avg}}(\mathcal{T}^{\mu}(s)) := \sup_{\sigma \in \mathcal{T}^{\mu}(s_0)} J'_{\text{avg}}(\sigma, \mu(\sigma))$$

Average—Solution

- Policy has two parts:
  1) Optimal policy ignoring the tasks
  2) Visit all tasks once
- An optimal policy alternates
  - 12 112 1112….
  - Requires infinite memory
- We adapt an algorithm from Chatterjee, Henzinger, Jurdzinski 2006.
- **Polynomial time**

Minimax (bottleneck) Cost

- Minimax cost of run σ is

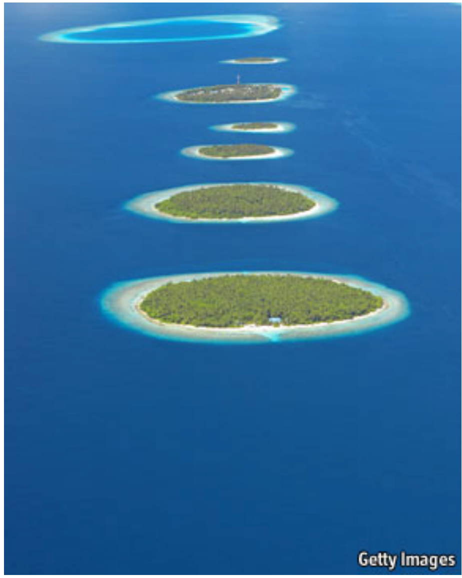
$$J'_{\text{bot}}(\sigma, \mu(\sigma)) := \limsup_{i \to \infty} (\mathbb{T}_{\text{task}}(i+1) - \mathbb{T}_{\text{task}}(i))$$

where $T_{\text{task}}(i)$ is the accumulated cost at the i-th completion of a task

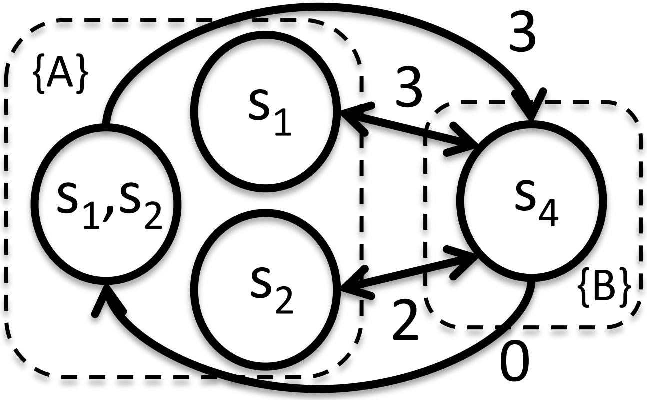
- The minimax cost function is

$$J_{\text{bot}}(\mathcal{T}^{\mu}(s)) := \max_{\sigma \in \mathcal{T}^{\mu}(s)} J'_{\text{bot}}(\sigma, \mu(\sigma))$$

Minimax—Solution



- Approach
  - Fix a cost λ
  - Remove all edges with cost > λ from task graph
  - Is remaining graph have a strongly connected component that includes all tasks?
  - Binary search on λ
- **Polynomial time** in task graph



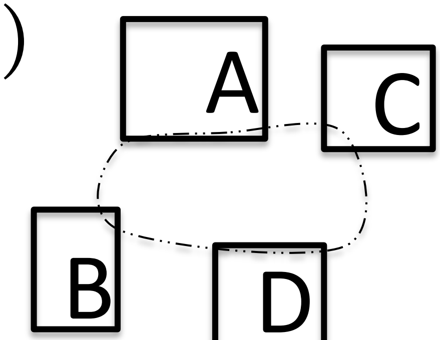
Average Cost-Per-Task-Cycle

- Average cost-per-task-cycle of run σ is

$$J'_{TC}(\sigma, \mu(\sigma)) := \limsup_{n\to\infty} \frac{\sum_{t=0}^{n} c(\sigma_t, \mu(\sigma_t), \sigma_{t+1})}{\sum_{t=0}^{n} I_{TC}(t)}$$

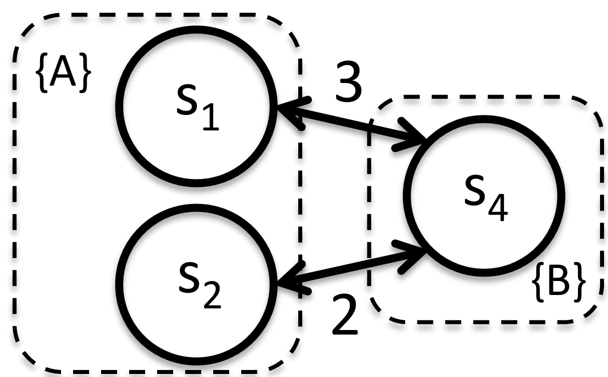
- The average cost-per-task-cycle cost function is

$$J_{TC}(\mathcal{T}^{\mu}(s)) := \max_{\sigma\in\mathcal{T}^{\mu}(s)} J'_{TC}(\sigma, \mu(\sigma))$$



Optimality for Task Cycle is Hard

- **Theorem**: Computing a control policy that is minimizes the average cost-per-task-cycle is NP-hard, even in the deterministic case.
- **Proof**: Construct a generalized traveling salesman problem where tasks are nodes in the TSP graph.

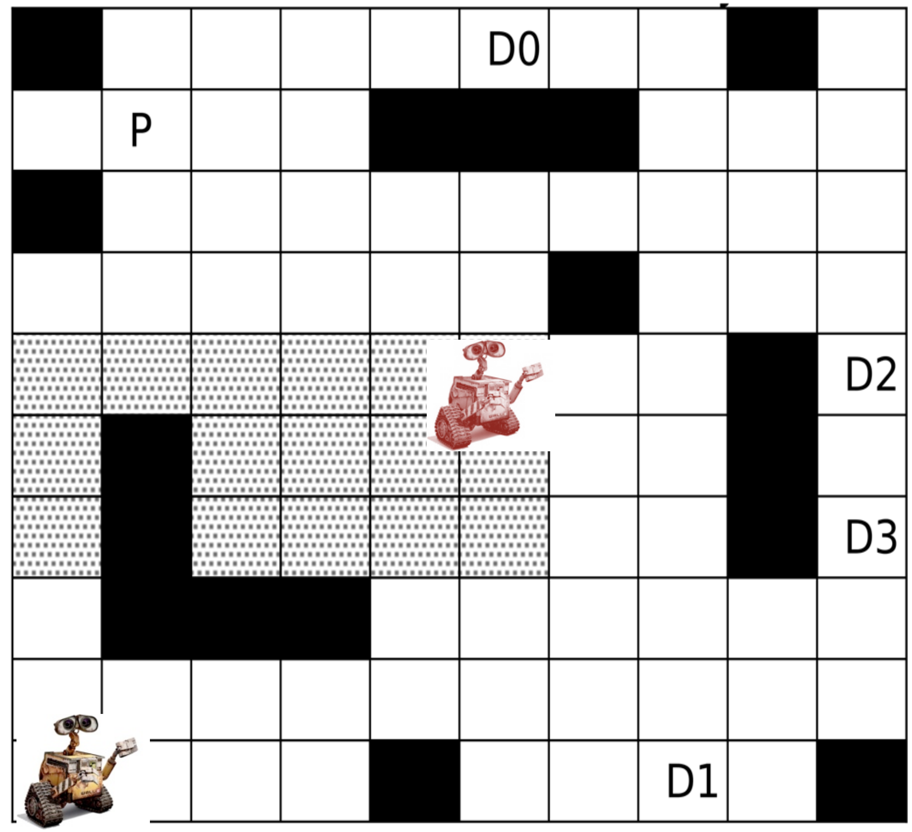


Task Cycle—Solution

- **Assumption**: The task ordering is fixed
- Solve generalized TSP on task graph
  - Use commercial solvers
  - Approximate solutions
- Solution gives optimal task ordering

Example: Pickup and Delivery

- System:
  - Robot and obstacle move to adjacent regions each step
- Specs:
  - Always avoid collisions
  - Repeatedly visit **P**ickup and **D**ropoff locations



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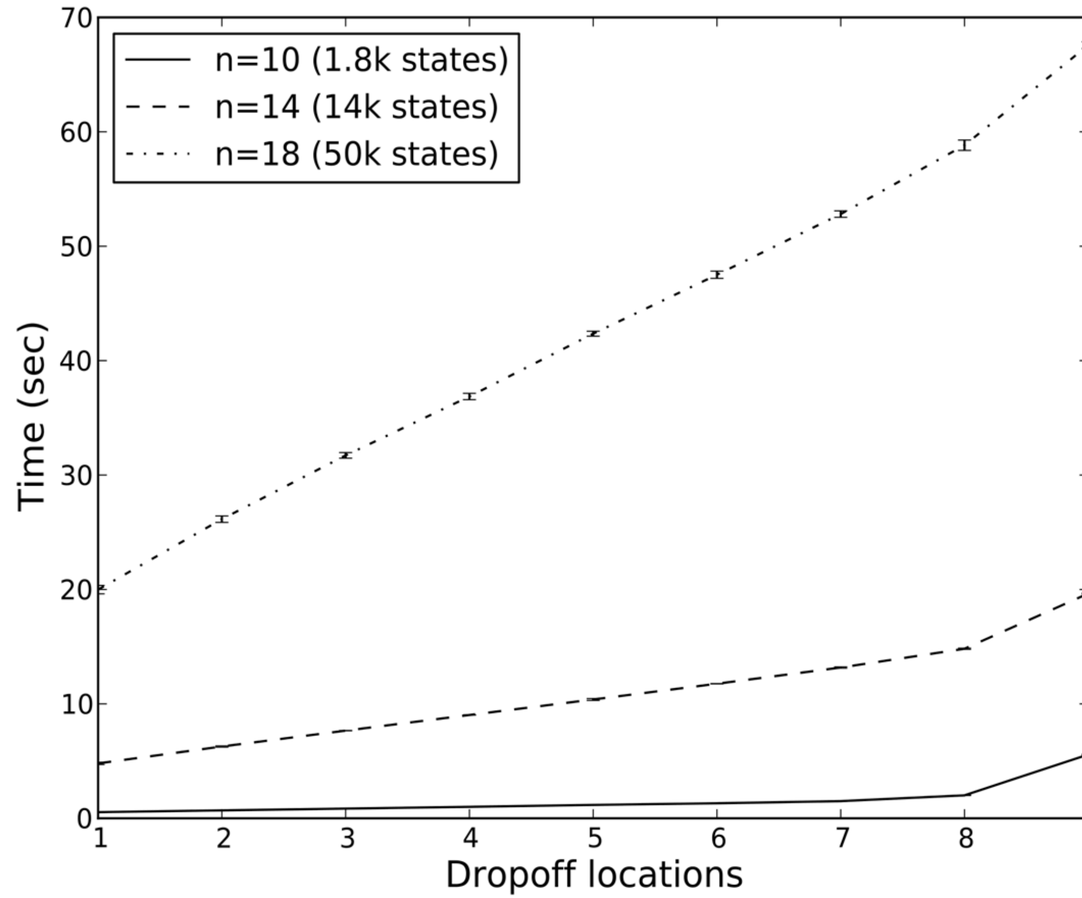
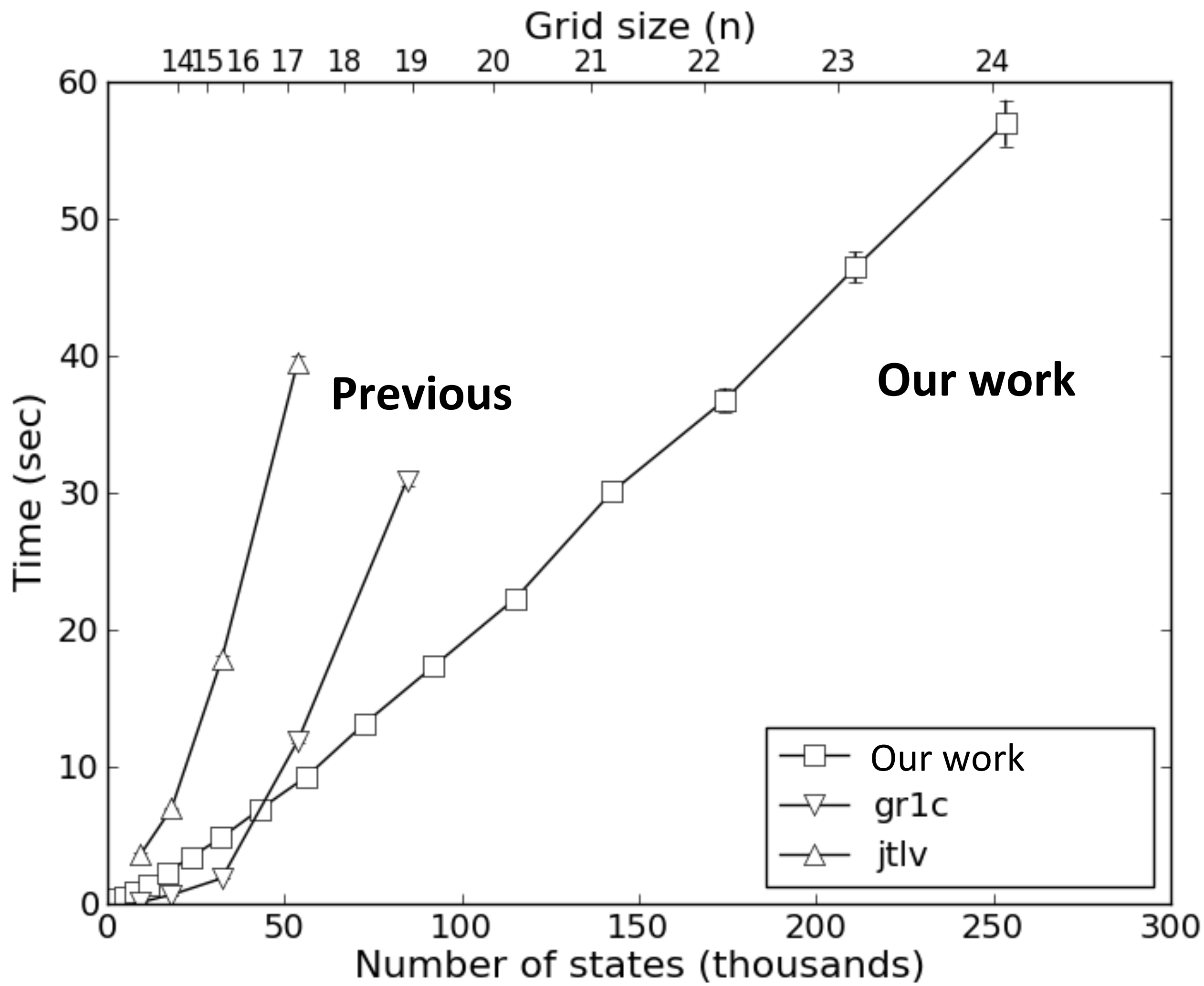
Computation Time—Optimal Controller



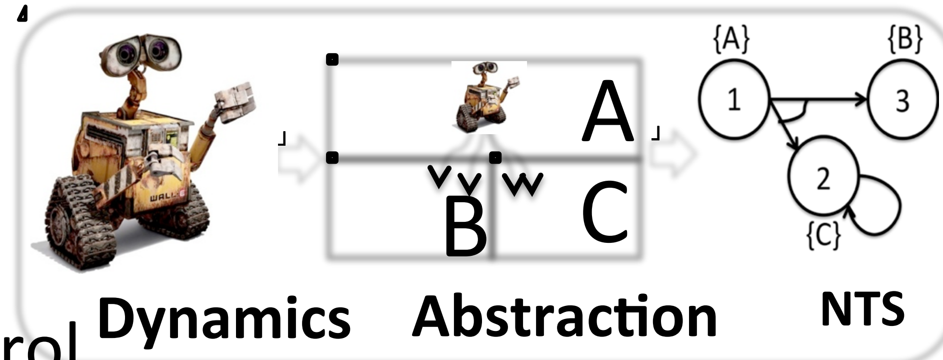
Comparison to GR1 (feasible)



Conclusions

- **Optimal control with temporal logic**

Cost function:	Average	Minimax	Task Cycle
Complexity:	POLY	POLY in task graph	EXP in task graph



- Future work
  - Receding horizon control
  - Removing fixed-ordering assumption

Thank you!

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- Funding: NDSEG fellowship, Boeing, AFOSR